ECE 3317
Fall 2023

Homework #5

Assigned: Thursday, Sep. 28
Due: Thursday, Oct. 5

Please do Probs. 2-4 and 6-10. (You are welcome to do the other problems for practice if you wish.)

Note: In this homework set we are in the sinusoidal steady-state (phasor domain).

1) Use the formula for the load reflection coefficient to show that

$$\left|\Gamma_L\right| \le 1 .$$

The load reflection coefficient formula is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} .$$

Assume that the real part of the load impedance is a positive number, and the characteristic impedance of the line is also a positive real number. What does your conclusion mean physically, in terms of power flowing in the incident and reflected waves? (see Prob. 3 below).

2) A 75 [Ω] coaxial line used for TV has an outer radius of b = 0.25 [cm] and an inner radius of a = 0.039 [cm]. The coax is filled with (nonmagnetic) Teflon ($\varepsilon_r = 2.2$) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic ($\mu = \mu_0$). Assume that the conductivity of copper is 3.0×10^7 [S/m] (this value is for "practical copper" and accounts for surface roughness). Make a table that shows the attenuation in [dB/m] for various frequencies, including 1 KHz, 10 KHz, 100 KHz, 1 MHz, 10 MHz, 100 MHz, 1 GHz, and 10 GHz.

3) Consider a lossless coaxial cable with an inner radius a and an outer radius b, filled with a nonmagnetic ($\mu = \mu_0$) dielectric material having a relative permittivity ε_r. The phasor electric field and the phasor magnetic field inside the coax for a transmission line wave traveling in the positive z direction are given by

$$\underline{E} = \underline{\hat{\rho}}\left(\frac{A}{\rho}\right)e^{-j\beta z}$$

$$\underline{H} = \underline{\hat{\phi}}\left(\frac{B}{\rho}\right)e^{-j\beta z},$$

where $\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}$,

and A and B are (complex) constants. (These constants may be related to the charge density on the inner conductor and the surface current flowing on the inner conductor by using Gauss's law and Ampere's law, respectively, but it is not necessary to do this.) Calculate the complex power P_z flowing down the coax in the positive z direction by using the complex Poynting vector and integrating over the cross section of the coax (as you did in Prob. 10 of HW 2). Then show that this complex power is equal to the complex power predicted by using the formula

$$P_z = \frac{1}{2} V I^* ,$$

where the phasor voltage V between the inner and outer conductors and the phasor current I flowing in the z direction on the inner conductor are given by

$$V = \int_a^b E_\rho \, d\rho$$

$$I = (2\pi a) J_{sz} = (2\pi a)\left(\underline{\hat{\rho}} \times \underline{H}\right) \cdot \underline{\hat{z}}\Big|_{\rho=a} = (2\pi a) H_\phi\Big|_{\rho=a} .$$

4) As a continuation of the previous problem, show that for the lossless coax the time-average power for a wave traveling in the positive z direction is given by

$$\left\langle \mathscr{P}_z(t) \right\rangle = \frac{1}{2} \frac{|V|^2}{Z_0} .$$

Note: Although the previous two problems assumed a lossless coaxial cable, the results are valid for any lossless transmission line. (Hint: Recall that the time-average power comes from the real part of the complex power. Also note that the characteristic impedance Z_0 is the ratio of the voltage phasor to the current phasor, for a wave traveling in the positive z direction. The characteristic impedance Z_0 is real for a lossless transmission line.)

5) Assume that we have a lossless transmission line, with voltage and current given by

$$V(z) = A e^{-jk_z z} + B e^{+jk_z z}$$

$$I(z) = \frac{1}{Z_0}\left(A e^{-jk_z z} - B e^{+jk_z z}\right),$$

where $k_z = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon}$.

Because the line is lossless, the wavenumber k_z and the characteristic impedance Z_0 are both real.

Starting with the expression for complex power flow, $P_z = \frac{1}{2} V I^*$, show that the time-average power flowing on the line in the z direction is

$$\langle \mathscr{P}_z(t) \rangle = \frac{|A|^2}{2Z_0} - \frac{|B|^2}{2Z_0}.$$

Give a physical interpretation of this result. (Hint: Recall that the time-average power comes from the real part of the complex power.)

6) Calculate the input impedance for a lossless transmission line that has a characteristic impedance of 50 [Ω] that is connected to a load impedance of 100 [Ω]. Assume that the length of the transmission line is 10 [m], and that the transmission line is filled with Teflon (nonmagnetic), having a relative permittivity of ε_r = 2.2. Make a table showing the input impedance for the following frequencies: 0 Hz, 60 Hz, 1 [kHz], 1 [MHz], 10 [MHz], 100 [MHz]. Do your results approach what you expect as the frequency tends to zero? Explain.

7) Assume we have the same transmission line and load as in the previous problem. At the input (left) end of the line there is a sinusoidal source having a peak amplitude of 120√2 = 169.68 [V] (corresponding to 120 [V] RMS). Calculate the time-average power absorbed by the load at 60 [Hz] and at 10 [MHz]. Also, calculate the value predicted by circuit theory, which ignores the transmission line (and therefore assumes that the input impedance is the same as the load impedance).

(Hint: The power time-average absorbed by the load is the same as the time-average power going into the transmission line at the input end, since the line is lossless. At the input, the model for the system consists of the source connected to a complex impedance Z_{in}.)

8) A transmission line is terminated in a normalized load of $Z_L^N = Z_L / Z_0 = 0.5 + j1.0$.

(a) Calculate the SWR.

(b) Calculate the position $z_{\min}$ of a voltage minimum (the one that is on the line and closest to the load). Your answer will be in terms of wavelengths (i.e., $z_{\min}/\lambda_d$).

(c) Calculate percentage of power that is reflected by the load. (Hint: Consider how power is related to voltage, as you showed in Prob. 4.)

9) For the previous problem, make an accurate plot of the normalized voltage $|V(z)|/|V^+|$ as a function of z/λ_d on the line (where z is negative). Plot over the range $-1.0 \le z/\lambda_d \le 0$. You may use Matlab or any software that you wish to make the plot.

10) For a normalized load impedance of

$$Z_L^N = Z_L / Z_0 = 0.6 + j0.4,$$

find the location of the first voltage maximum and the first voltage minimum from the load end. (The word "first" means the ones that are the closest to the load.) Repeat for the current. Your answers will be in terms of wavelengths (i.e., z/λ_d).

Note: $\lambda_d = \lambda_0 / \sqrt{\varepsilon_r \mu_r}$ for a lossless line, but you cannot determine the wavelength since you are not given the frequency and the properties of the filling material.

11) A 50 [Ω] transmission line (i.e., one with a characteristic impedance of 50 [Ω]) is terminated in a load impedance of 30 - j20 [Ω]. Carefully draw the crank diagram for this system, showing values at intervals of $\lambda_d/16$. Using the crank diagram (and a ruler), draw the voltage standing wave pattern on the line.

Note: $\lambda_d = \lambda_0 / \sqrt{\varepsilon_r \mu_r}$ for a lossless line, but you cannot determine the wavelength since you are not given the frequency and the properties of the filling material.