ECE 3317
Fall 2023

Homework #8

Assigned: Thursday, Nov. 2
Due: Thursday, Nov. 9

1) Calculate the skin depth and the surface impedance for aluminum at a frequency of 2.45 [GHz] (this is the frequency of a microwave oven). The conductivity of aluminum is taken as 2.0×10^7 [S/m]. Aluminum is nonmagnetic ($\mu = \mu_0$).

2) A sheet of aluminum foil has a thickness of about 1 mil (a mil is 0.001 inches). The conductivity of aluminum is taken as 2.0×10^7 [S/m]. Calculate the dB of attenuation of a plane wave that goes through the sheet of aluminum foil, at a frequency of 2.45 [GHz]. Is the sheet of aluminum foil a good shield for the microwaves that are inside a microwave oven operating at this frequency?

3) A "twin-lead"" transmission line consists of two round wires of radius a that are in free space ($\varepsilon_r = 1$), separated by distance h between the centers. Assume that the metal wires have a surface resistance R_s. Derive an approximate expression for resistance per unit length at high frequencies, assuming that the wires are far enough apart that the surface current on each wire is constant.

4) Consider a wire that is being used as a vertical "via" interconnect on a printed circuit board (PCB). The via is made of "practical" copper (with a conductivity of 3.0×10^7 [S/m]). The via has a diameter of 0.25 [mm] and a length of 1.5 mm. What is the impedance (in Ohms) of the via at a frequency of 10 GHz?

5) Find the polarization (linear, circular, or elliptical) and handedness (left-handed or right-handed) for the following fields (using the graphical rotation rule, based on plotting the phasors in the complex plane):

   (a) $\underline{E} = \left(\underline{\hat{x}} - j\underline{\hat{y}}\right)e^{-jkz}$

   (b) $\underline{E} = \left[\left(3 - j2\right)\underline{\hat{x}} + \left(1 + j\right)\underline{\hat{z}}\right]e^{-jky}$

   (c) $\underline{E} = \left[\left(j3\underline{\hat{y}} - j2\underline{\hat{z}}\right)\right]e^{jkx}$

   (d) $\underline{E} = \left[\left(1 + j\right)\underline{\hat{x}} + \left(1 - j\right)\underline{\hat{y}}\right]e^{jkz}$

6) A plane wave that is propagating in the positive z direction has the following electric field at $z = 0$:

$$\underline{\mathscr{E}} = \left[\underline{\hat{x}}(2.5)\cos(\omega t) + \underline{\hat{y}}(1.5)\sin\left(\omega t - 10^{\circ}\right) \right].$$

a) Determine the constants a, b, and β.

b) Determine what type of wave this is (RHEP, RHCP, LHEP, LHCP, linear), using the graphical rotation rule.

c) Determine the tilt angle τ and the axial ratio AR of this wave. Use the formulas given in Notes 17.

d) Select various values of ωt and make a plot of the electric field vector in the xy plane, for $z = 0$. Measure the tilt angle and the axial ratio from the ellipse on your plot, and verify that you have (approximately) the same results as what you calculated in the previous step. (Recall that when you use the formula in the class notes for the tilt angle τ, you may have to add or subtract 90° to the result.)

7) A wire antenna that is oriented along the x-axis is used to receive a signal from an incoming wave, which may be modeled as a plane wave traveling in the z direction in free space. The open-circuit (phasor) voltage V_T at the terminals of the antenna (which is the Thévenin voltage for the receiving antenna) can be expressed as

$$V_{\mathrm{T}} = h_{\mathrm{eff}} E_x^{\mathrm{inc}} \;[\mathrm{V}],$$

where h_{eff} is the "effective height" of the antenna and E_x^{inc} is the x component of the electric field of the incident plane wave (in the phasor domain).

The <u>normalized</u> Thévenin voltage of the receive antenna in the phasor domain is defined from the Thévenin voltage V_T as

$$\overline{V}_{\mathrm{T}} \equiv V_{\mathrm{T}} / h_{\mathrm{eff}} = E_x^{\mathrm{inc}} \;[\mathrm{V/m}].$$

Calculate the magnitude of the normalized Thévenin voltage in the phasor domain for the following incident plane waves, assuming that each plane wave has an incident time-average power density of 1 [W/m^2].

a) A linearly polarized plane wave, polarized in the x direction and traveling in the z direction.

b) A RHCP wave, traveling in the z direction.

c) A LHCP wave, traveling in the z direction.

Hint: For each of the cases, first calculate what the expression for the electric field vector of the incident plane wave should be in the phasor domain, using the fact that the power density in the plane wave is known (1 [W/m^2]). The time-average power density in the plane wave is

$$\underline{S} = \frac{1}{2\eta_0}\hat{\underline{z}}\left(\left|E_x^{\text{inc}}\right|^2 + \left|E_y^{\text{inc}}\right|^2\right)\ \left[\text{W/m}^2\right].$$

8) A circularly-polarized antenna is designed to receive a RHCP wave traveling in the z direction. It consists of two wire antennas, one oriented in the x direction and one oriented in the y direction. The signals are combined from each antenna with a 90º phase shift between the two antennas, so that the total open-circuit voltage that is received is

$$V_{\text{T}} = h_{\text{eff}}\left(E_x^{\text{inc}} + jE_y^{\text{inc}}\right)\ \left[\text{V}\right].$$

The normalized Thévenin voltage is defined as

$$\overline{V}_{\text{T}} \equiv V_{\text{T}} / h_{\text{eff}} = E_x^{\text{inc}} + jE_y^{\text{inc}}.$$

Calculate the magnitude of the normalized Thévenin voltage for the following incident plane waves listed below, assuming that each plane wave has an incident time-average power density of 1 [W/m^2].

a) A linearly polarized plane wave, polarized in the x direction and traveling in the z direction.

b) A RHCP wave, traveling in the z direction.

c) A LHCP wave, traveling in the z direction.

Hint: For each of the cases, first calculate what the expression for the electric field vector of the incident plane wave should be in the phasor domain, using the fact that the power density in the plane wave is known (1 [W/m^2]). The time-average power density in the plane wave is

$$\underline{S} = \frac{1}{2\eta_0}\hat{\underline{z}}\left(\left|E_x^{\text{inc}}\right|^2 + \left|E_y^{\text{inc}}\right|^2\right)\ \left[\text{W/m}^2\right].$$