ECE 3317
Fall 2022

Homework #9

Assigned: Thursday, Nov. 9
Due: Thursday, Nov. 16

Do Probs. 3–9. You may do Probs. 1 and 2 on your own if you wish, but please turn in only Probs. 3–9.

1) A plane wave in free space has an electric field given by

$$\underline{E} = \left(4\underline{\hat{x}} - 4\underline{\hat{y}} - \sqrt{0.8}\,\underline{\hat{z}}\right) e^{-j\left(k_x x + k_y y + k_z z\right)},$$

where $k_x = 0.4k_0$ and $k_y = 0.2k_0$.

Determine k_z and the $\underline{k}$ vector, and verify that the electric field is perpendicular to the $\underline{k}$ vector. Then determine the magnetic field vector in the phasor domain from Maxwell's equations, and verify that it is also perpendicular to the $\underline{k}$ vector. Assume that the wave is either propagating or decaying in the positive *z* direction (you should determine which is the case). This means that k_z is either a positive real number or an imaginary number that has a negative imaginary part.

2) A plane wave is in a nonmagnetic lossless region with $\varepsilon = \varepsilon_0 \varepsilon_r$. The field components are

$$\underline{E} = \underline{E}_0\, e^{-j\left(k_x x + k_y y + k_z z\right)}$$

and

$$\underline{H} = \underline{H}_0\, e^{-j\left(k_x x + k_y y + k_z z\right)} \;,$$

where

$$\underline{E}_0 = 2\underline{\hat{x}} + 3\underline{\hat{y}} + 4\underline{\hat{z}}$$

and

$$\underline{H}_0 = \frac{1}{\eta_0}\left(7\underline{\hat{x}} - 6\underline{\hat{y}} + \underline{\hat{z}}\right).$$

The wavenumbers k_x, k_y, k_z are all real numbers.

(a) Determine ε_r. Hint: Remember that the magnitude of the magnetic field is related to the magnitude of the electric field for a lossless plane wave by η.

(b) Determine k_x, k_y, k_z, and the $\underline{k}$ vector in terms of k_0. Calculate a real unit vector $\underline{\hat{p}}$ that specifies the direction the wave is traveling (the direction of power flow). Hint: For a lossless plane wave, the power flow (Poynting vector) is in the direction of the $\underline{k}$ vector, and the magnitude of the $\underline{k}$ vector is k.

(c) Calculate the angles (θ, ϕ) in spherical coordinates that describe the direction of propagation (the direction of power flow).

3) Calculate the critical angle for a plane wave going from:

(a) Water $(\varepsilon_r = 81, \mu_r = 1)$ to air.

(b) Silicon $(\varepsilon_r = 11.68,\ \mu_r = 1)$ to air.

4) White light is composed of the entire visible spectrum. The index of refraction n for most materials is a weak function of wavelength λ, often described over a limited frequency region by Cauchy's equation,

$$n = A + B/\lambda^2 .$$

A beam of white light is incident at 35° (with respect to the normal) on a piece of glass with $A = 1.4$ and $B = 6\times10^{-15}\ \text{m}^2$.

What are the transmitted angles for colors violet (400 [nm]), blue (450 [nm]), green (550 [nm]), yellow (600 [nm]), orange (650 [nm]), and red (700 [nm])? This separation of colors is the dispersive effect of a prism.

5) A plane wave travels from medium 1 $(\mu_1 = \mu_0, \varepsilon_1 = 4\varepsilon_0)$ to medium 2, which is air $(\mu_2 = \mu_0, \varepsilon_2 = \varepsilon_0)$.

(a) Find the critical angle.

(b) If the angle of incidence is 45º, find k_{xi} and k_{zi} in terms of k_0. Assume the same geometry that was used in the class notes.

(c) Find k_{zt} in terms of k_0.

(d) Find the normalized distance z_0 / λ_0 in the second medium at which the field is down by 10 dB from what it is just below the interface. What would the distance z_0 (in meters) be if the wavelength was 600 [nm]?

6) A microwave signal at 2.0 GHz from a cell-phone base station bounces off a building, which is modeled as a lossless nonmagnetic material having a relative permittivity of $\varepsilon_r = 4$. Assume that the angle of incidence is 45°.

a) What is the percent power reflected if the wave is a TM_z polarized wave?

b) What is the percent power reflected if it is a TE_z polarized wave?

c) What is the percent power reflected if it has equal powers in both polarizations?

(Hint: For part (c) you may assume that the incident power density is 1 [W/m^2] if you wish, but the final answer will not depend on the power density of the incident wave. If you make this assumption, then each type of incident wave, TM_z and TE_z, will have a power density of 0.5 [W/m^2]. The total power in the reflected wave will be the sum of the power in the reflected TM_z wave and the power in the reflected TE_z wave.)

7) A 1.575 GHz GPS signal from a satellite is a RHCP polarized wave. It thus has equal power densities in the TM_z and TE_z polarizations (and the two corresponding electric field components are also 90° out of phase from each other, though this is not important for the present problem). The signal is incident at an angle of 45° on ocean water, which is nonmagnetic with a relative permittivity of $\varepsilon_r = 81$ and a conductivity of $\sigma = 4$ [S/m]. What percentage of the power is reflected from the surface of the ocean? Do you think the reflected wave will be circularly polarized? You do not have to do any calculations, but please justify your answer.

(Hint: Use the same hint given in Prob. 6.)

8) Sunlight contains equal power densities in both the parallel and perpendicular polarizations (i.e., the TM_z and TE_z polarizations). Assume that sunlight is incident at a 45° angle on a puddle of water. Because the water is relatively pure fresh water, it is nonconductive and has a dielectric constant (relative permittivity) of $\varepsilon_r = 1.78$ at optical frequencies. The water is nonmagnetic.

(a) What percentage of the reflected wave power is in the parallel polarization?

(b) What percentage of the reflected wave power is in the perpendicular polarization?

(c) If a person is wearing polarized sunglasses that screen out any light that is polarized horizontally (TE_z), what percentage of the incident sunlight power density gets through the glasses when the person looks at the reflected light coming from the puddle?

(Hint: Use the same hint given in Prob. 6.)

9) Calculate the Brewster angle for the problem above (the sunlight going from air to water) and then answer the same three questions in the previous problem if the sunlight is now incident at the Brewster angle instead of at 45^o.