ECE 3318
Applied Electricity and Magnetism

Spring 2023

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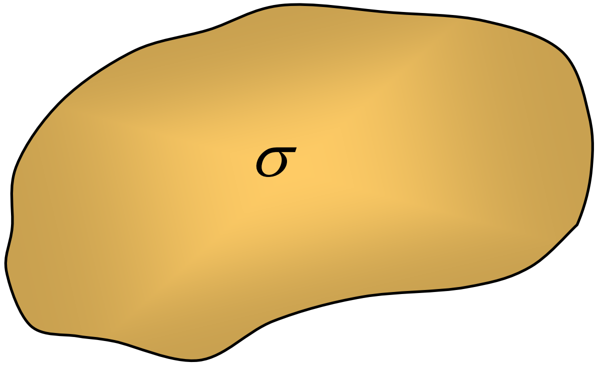
Dept. of ECE

Notes 12
Conductors

Conductors

Ohm's law

$$\underline{J} = \sigma \underline{E}$$



Good electric conductors: $\sigma >> 1$

Perfect electric conductor (PEC): $\sigma \rightarrow \infty$

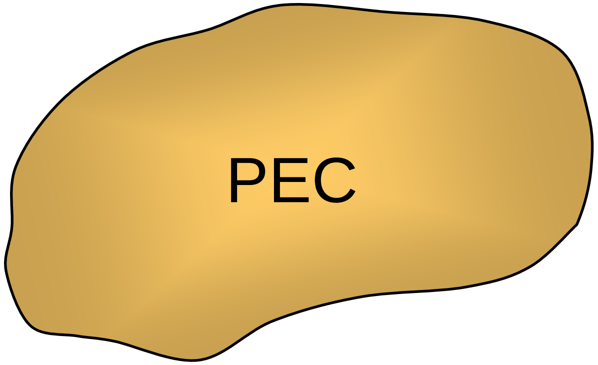
Material	σ [S/m]
Silver	6.3×10^7
Copper	6.0×10^7
Copper (annealed)	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Zinc	1.7×10^7
Brass	1.6×10^7
Nickel	1.4×10^7
Iron	1.0×10^7
Tin	9.2×10^6
Steel (carbon)	7.0×10^6
Steel (stainless)	1.5×10^6

Note:
Many of the properties derived for PECs hold very accurately for good conductors.

http://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

Perfect Electric Conductors

Inside a perfect conductor, the electric field must be zero.



Ohm's law:

$$\underline{J} = \sigma \underline{E}$$

or

$$\underline{E} = \underline{J} / \sigma$$

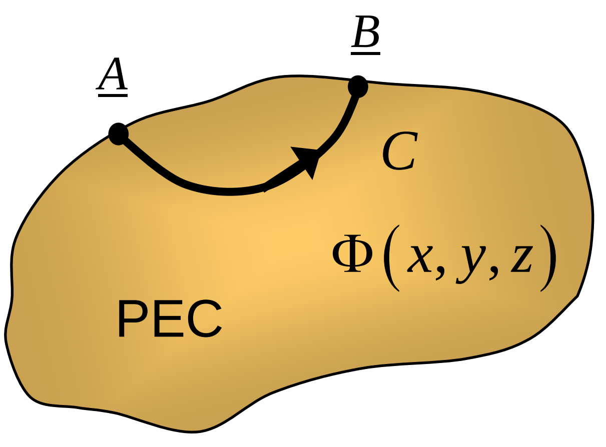
PEC: $\sigma \to \infty$

$\Rightarrow$ $\underline{E} \to \underline{0}$

Note:
We assume that the current density cannot go to infinity.

Perfect Electric Conductors (cont.)

Inside and on a perfect conductor, the potential must be constant.



$$\underline{E} = \underline{0}$$

Hence

$$V_{AB} = \int_{A}^{B} \underline{E} \cdot \underline{dr} = 0$$

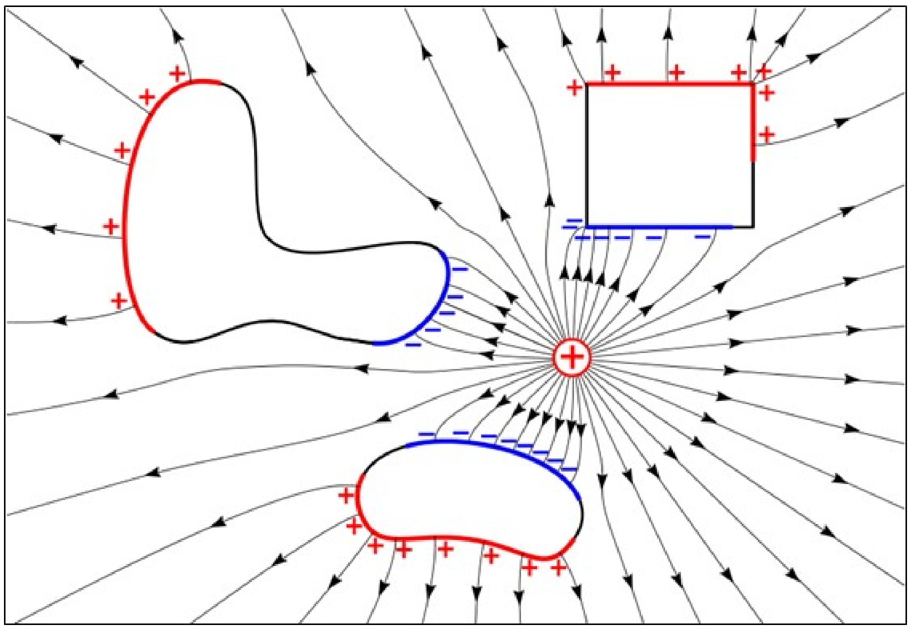
$\Rightarrow$ $\Phi = \text{constant}$

Note:
The voltage drop integral is independent of the path, so we can choose a path that stays inside the conductor (where the electric field is zero).

Perfect Electric Conductors (cont.)

Electric lines of flux must enter or leave a conductor <u>perpendicular</u> to it.

In other words: $\underline{E}_t$ = tangential electric field = $\underline{0}$

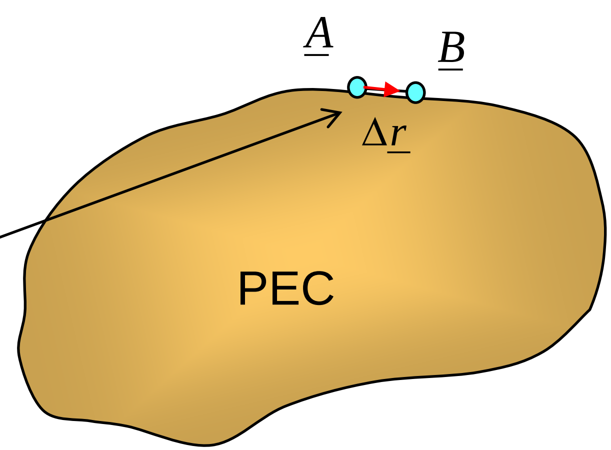


On surface of PEC:

$$V_{AB} = \int_{A}^{B} \underline{E} \cdot d\underline{r} \approx \underline{E} \cdot \Delta \underline{r} = 0$$

Hence, $\underline{E}$ is perpendicular to $\Delta \underline{r}$ for any small $\Delta \underline{r}$ along the surface.

We choose a small path $\Delta \underline{r}$ in a tangential direction along the surface.



Perfect Electric Conductors (cont.)

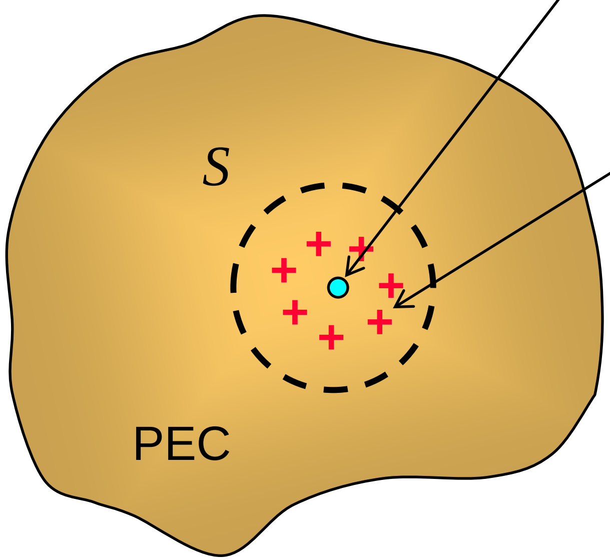
Inside a PEC $\rho_v = 0$

Proof: <u>Assume</u> a point inside where $\rho_v > 0$

$\rho_v > 0$ inside small volume

(since ρ_v is a continuous function).

$$\Rightarrow \; Q_{encl} > 0$$

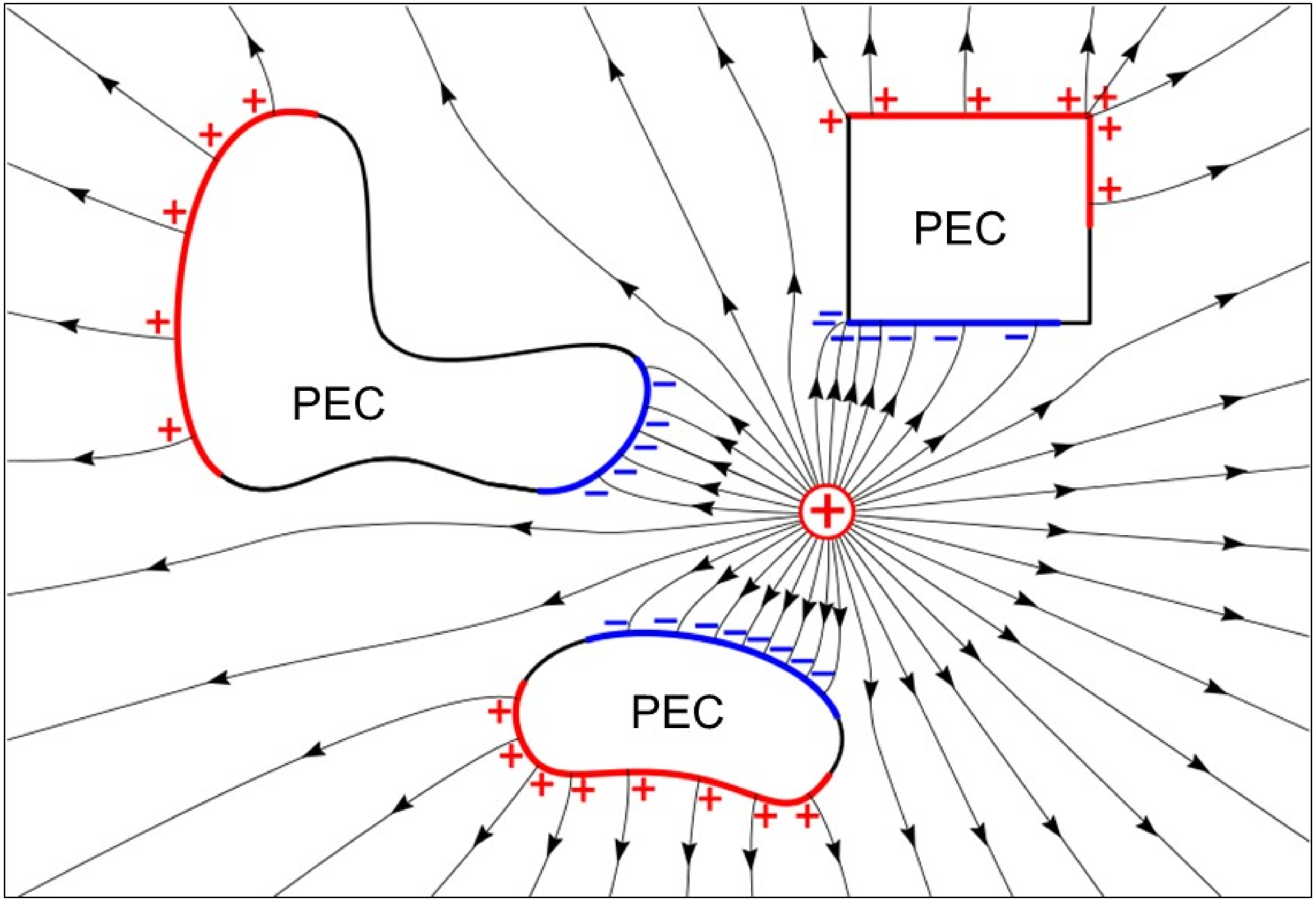


$$\oint_S \underline{D} \cdot \underline{\hat{n}} \; dS = Q_{encl}$$

Hence $\quad 0 = Q_{encl} > 0 \quad$ Contradiction !

Perfect Electric Conductors (cont.)

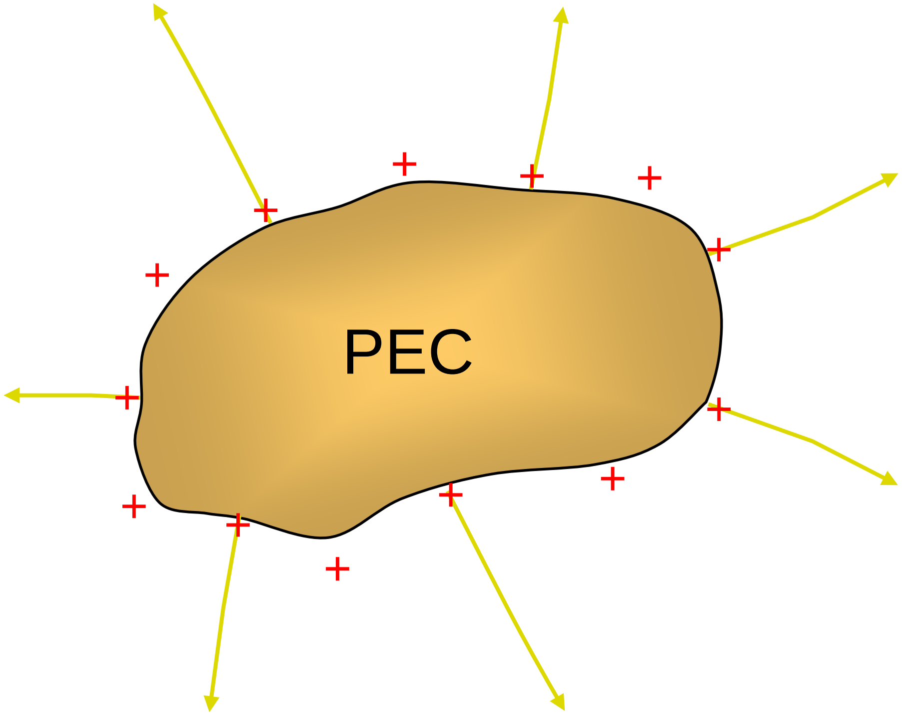
Only ρ_s on the <u>surface</u> is allowed for a PEC.



http://en.wikipedia.org/wiki/Electrostatics

Properties of Conductors

Summary of Properties for PEC



$\underline{E} = \underline{0}$ (inside)

$\Phi = \text{constant}$ (inside and on surface)

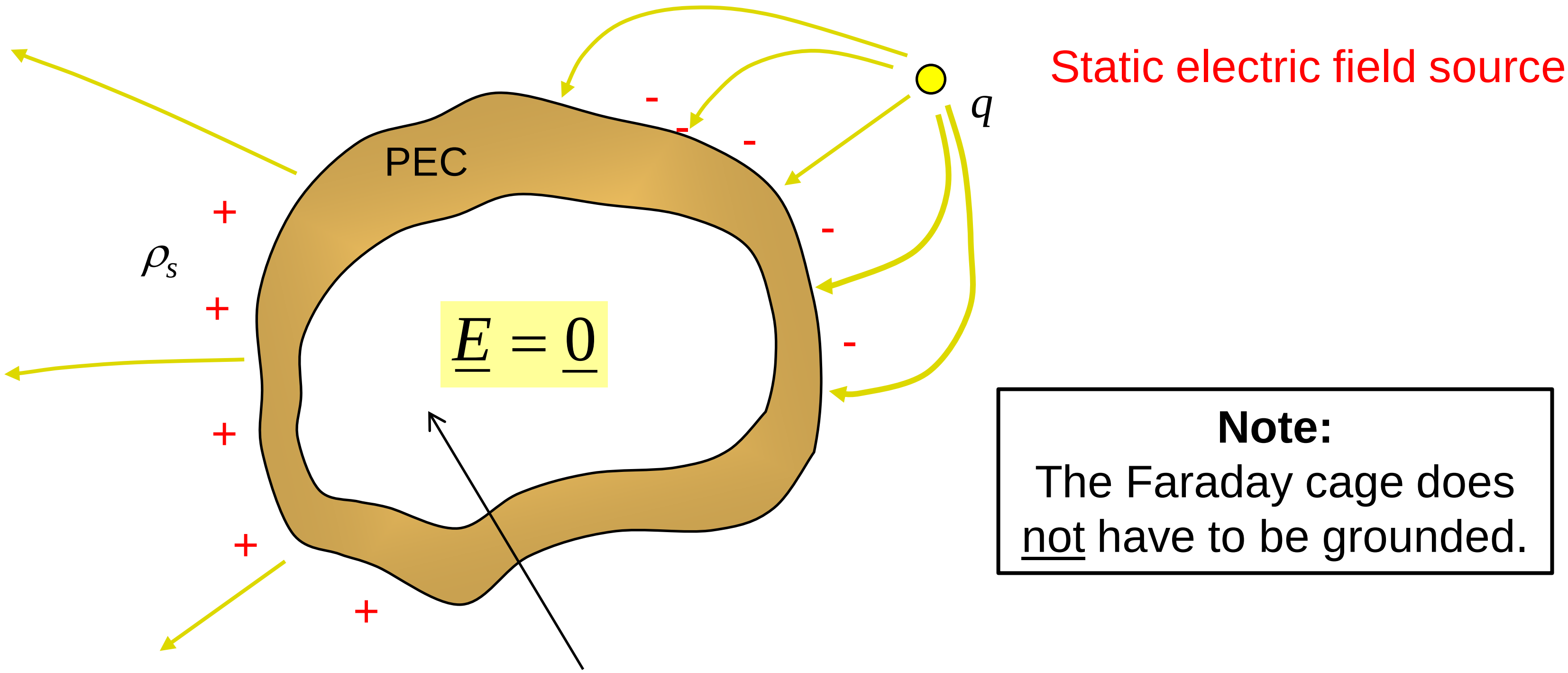
$\rho_v = 0$ (inside)

$\underline{E}$ is $\perp$ to surface (on surface)

Faraday Cage Effect

Inside of a hollow PEC shell in statics:

- There is no electric field.
- There is no ρ_s on the inner surface.

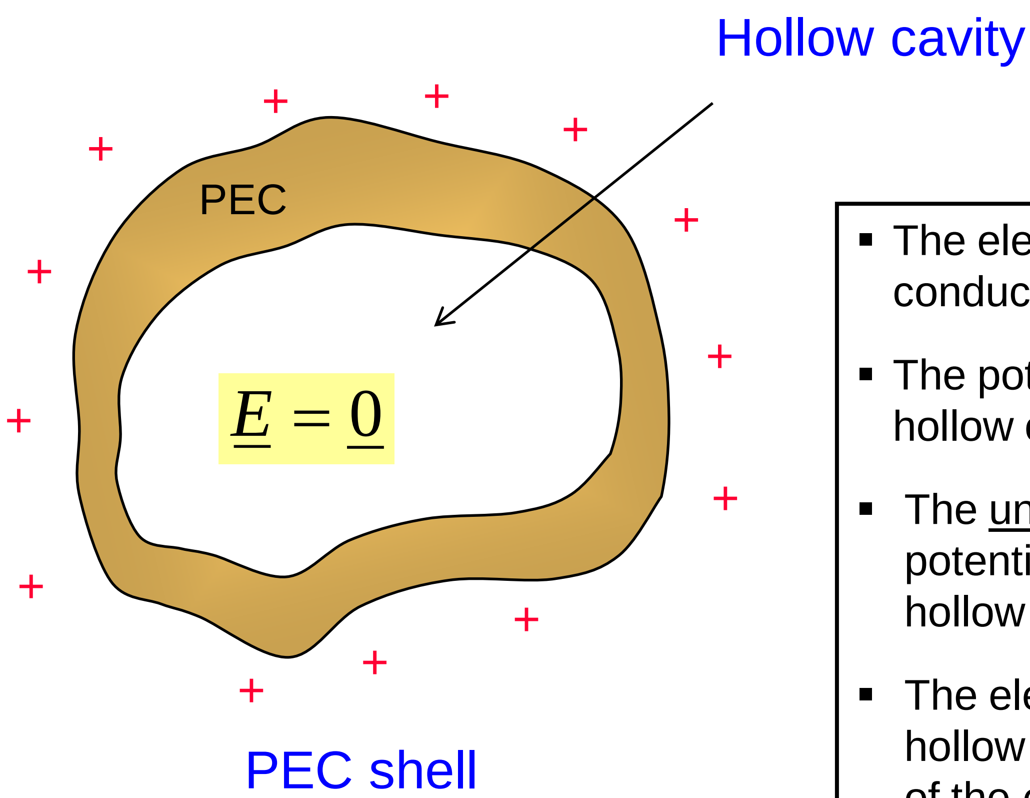


Note:
The Faraday cage does
not have to be grounded.

This is a shielded region (no electric field).

Faraday Cage Effect (cont.)

Proof of Faraday cage effect



Outline of proof:

- The electric field is zero inside the conductor.
- The potential on the boundary of the inner hollow cavity is constant.
- The uniqueness theorem then says that the potential must be a constant throughout the hollow cavity region (see the next slide).
- The electric field is then zero inside the hollow cavity as it comes from the gradient of the electric field.

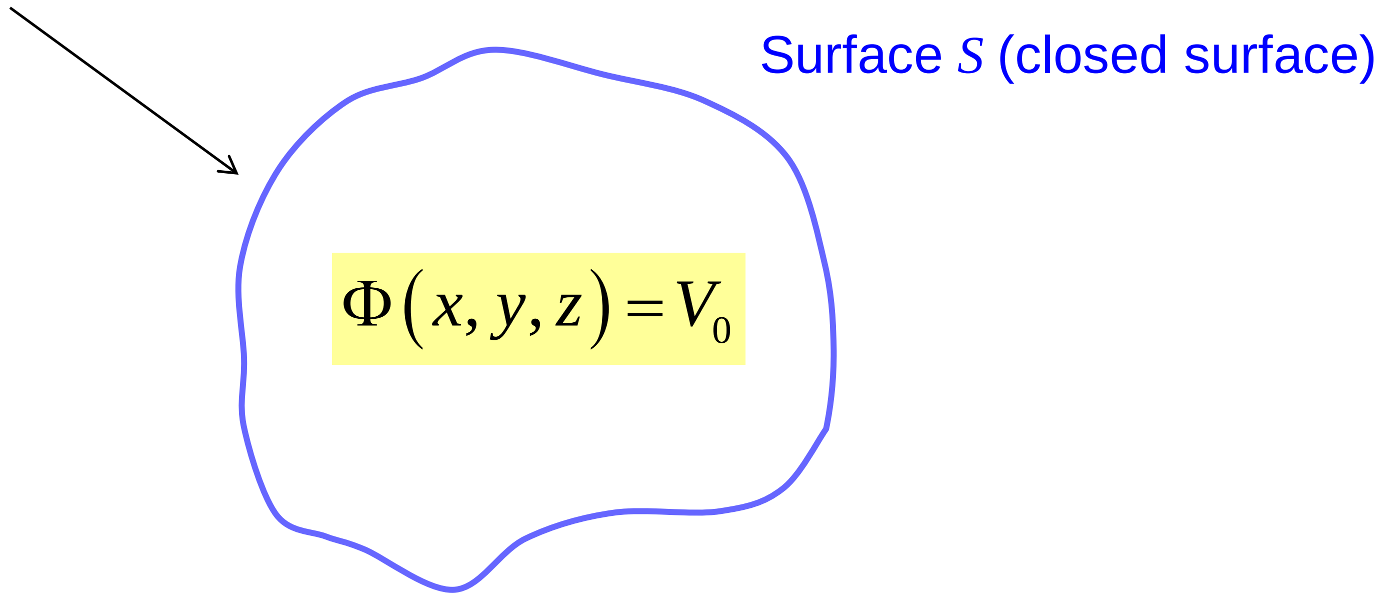
Note: The uniqueness theorem is discussed on the next slide.

Faraday Cage Effect (cont.)

Uniqueness Theorem

If $\Phi = V_0$ (a constant) everywhere on the boundary S, then $\Phi(x,y,z) = V_0$ everywhere inside the region.

Given: $\Phi = V_0$ (a constant) on the boundary



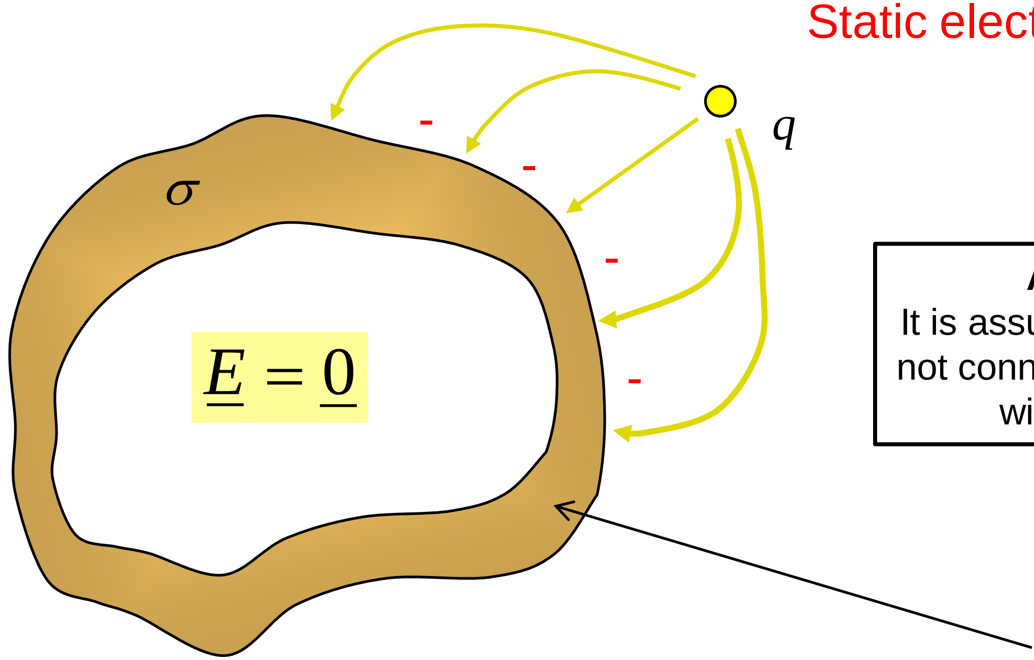
$$\Phi(x, y, z) = V_0$$

Faraday Cage: Practical Conductor

In <u>static equilibrium</u>, the Faraday cage still works, even with a practical <u>good</u> conductor (finite conductivity):

$$\text{From Ohm's law: } \underline{J} = \underline{0} \text{ inside metal} \Rightarrow \underline{E} = \underline{0}$$

Static electric field source



Assumption:
It is assumed that the shell is not connected to any external wires or sources.

$\underline{E} = \underline{0}$ inside metal

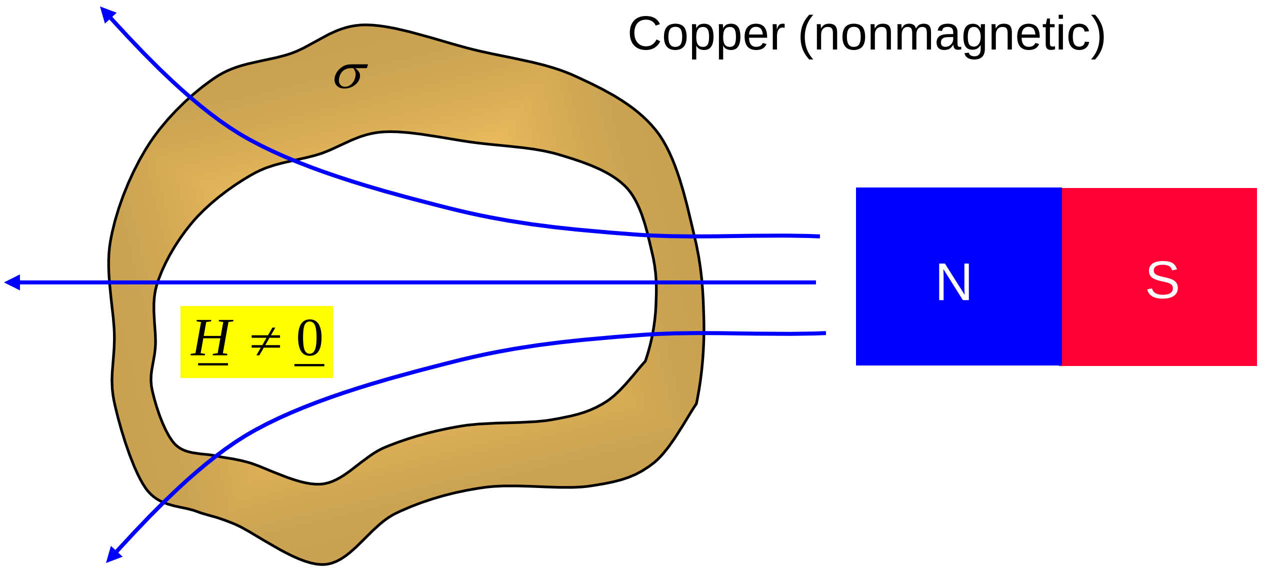
The rest of the proof works the same way.

Conducting metal shell (finite conductivity)

At equilibrium: $\underline{J} = \underline{0}$

Faraday Cage: Note on Magnetic Field

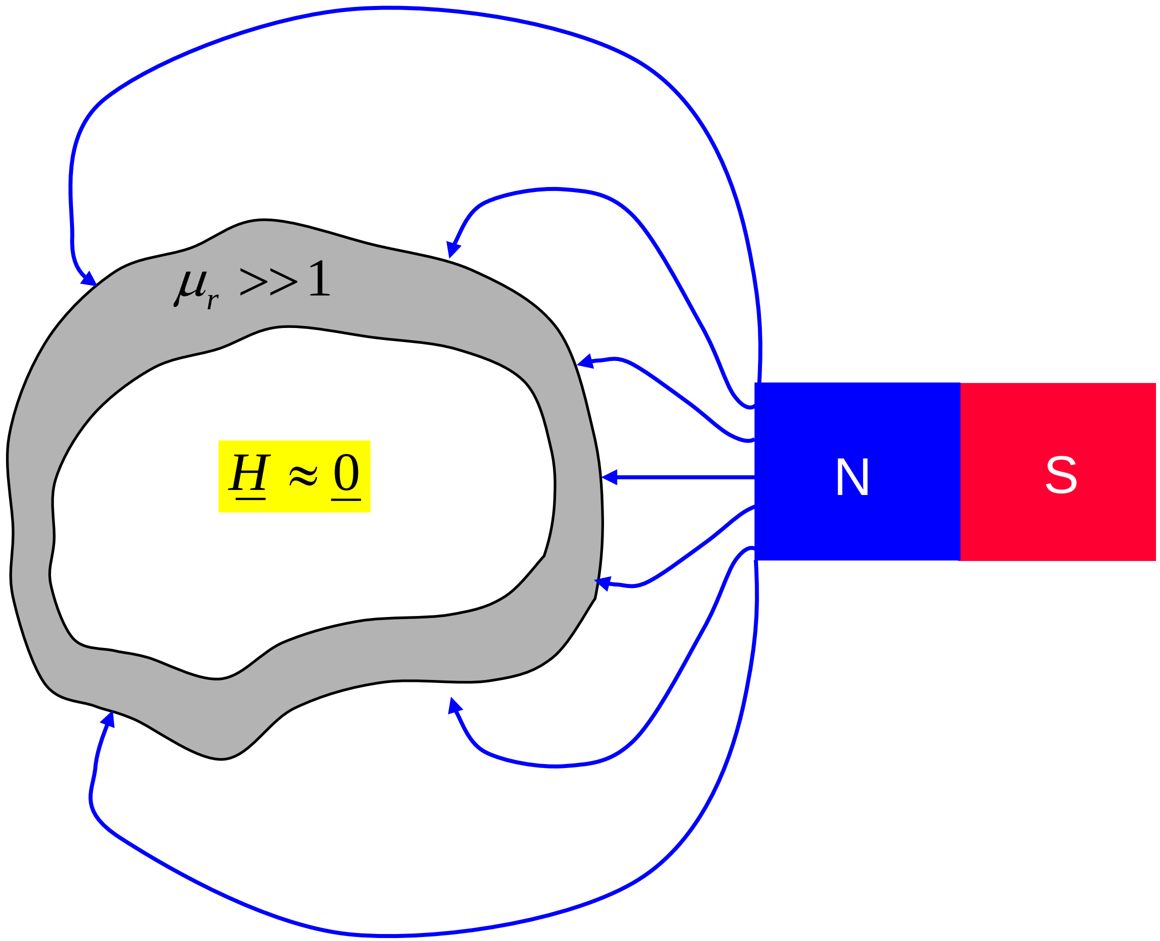
A Faraday cage made of a good conductor does not block a static magnetic field!



Note:
A high permeability material will act as a good shield for a static magnetic field (e.g., Mu-metal, $\mu_r = 100{,}000$).

Faraday Cage: Note on Magnetic Field

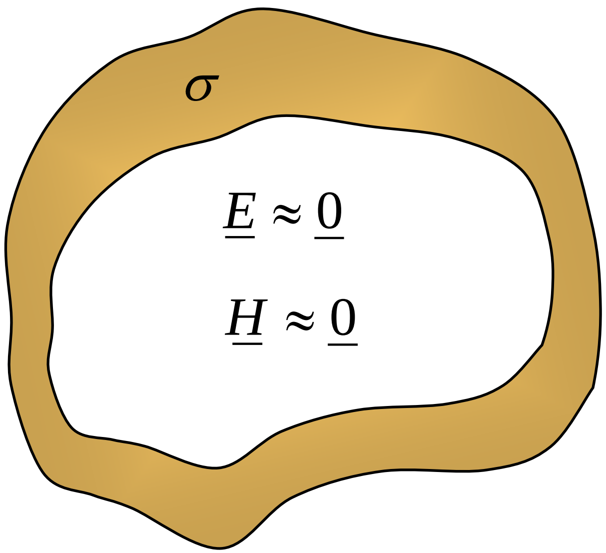
High permeability shield $(\mu_r >> 1)$

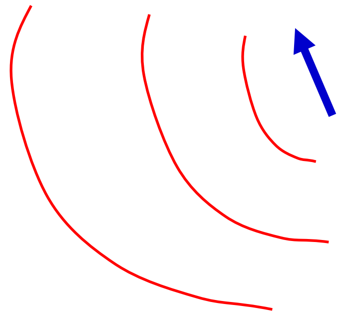


A Faraday cage made of a <u>high permeability</u> material will act as a good shield for a static magnetic field.

High-Frequency Shielding

At <u>high frequencies</u>, both electric and magnetic fields are blocked by the <u>skin effect</u>.





Radiating antenna source

Thickness >> δ

Skin depth:

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

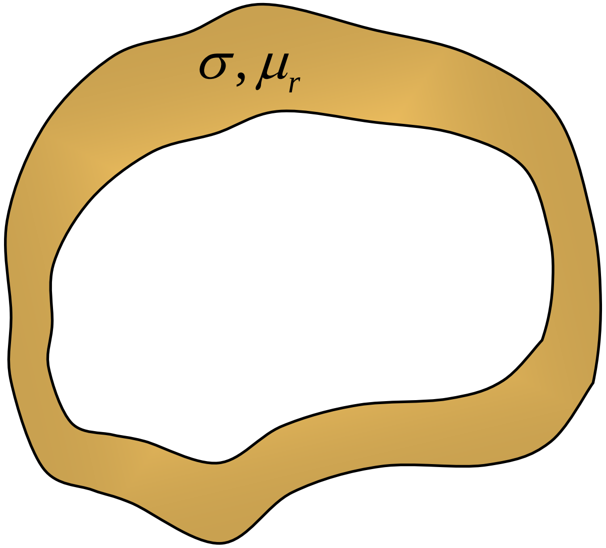
To be a good shield, the thickness of the shield should be large compared to a skin depth.

This situation is discussed in ECE 3317 (skin-depth effect).

Summary on Shielding

When will a closed metal shield be an effective shield?

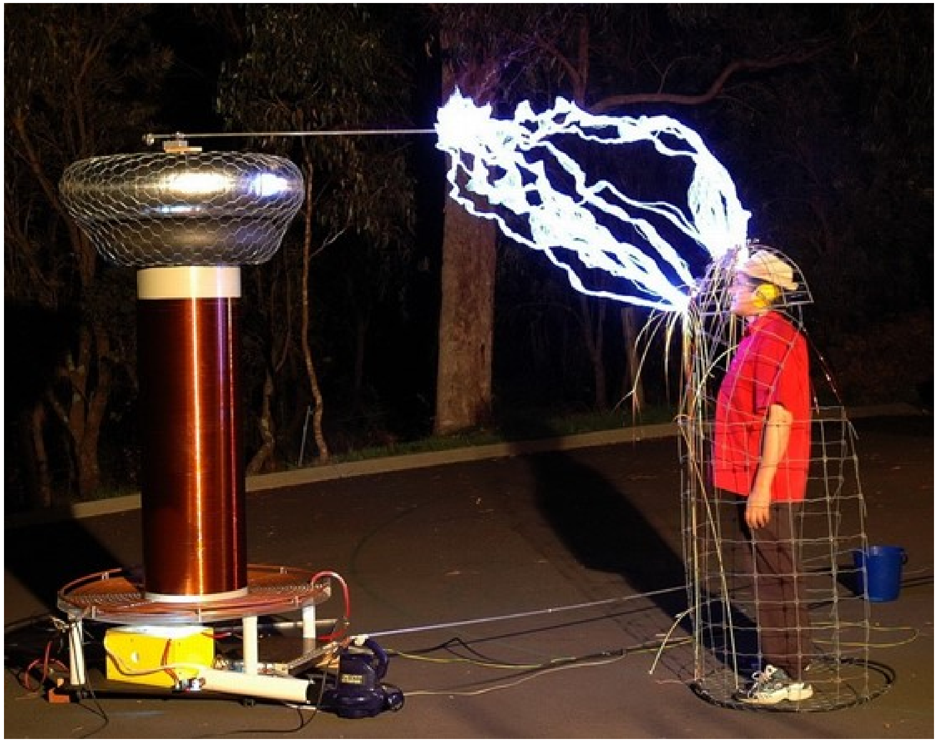
- ❖ A good conductor ($\sigma >> 1$) will block all high-frequency signals (skin effect).
- ❖ In statics, a good conductor will block the electric field (Faraday Cage effect).
- ❖ In statics, a good conductor will not necessarily block the magnetic field.
- ❖ In statics, a high-permeability material ($\mu_r >> 1$) will block the magnetic field.



Conductive shield

Faraday Cage Effect (cont.)

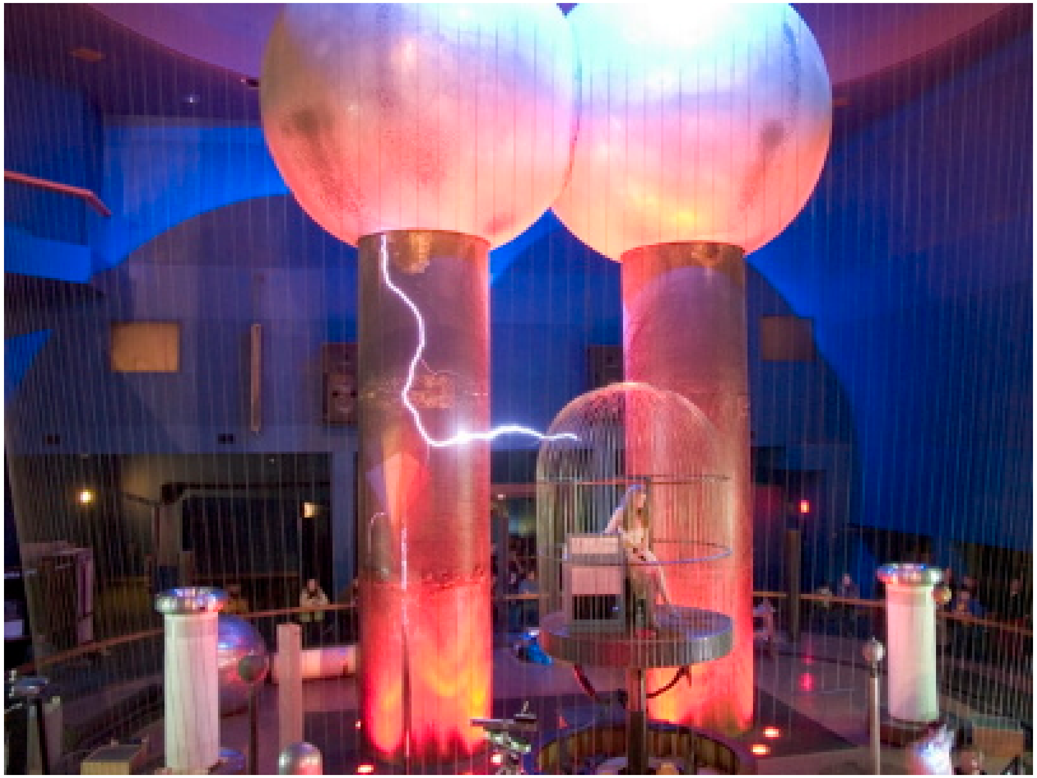
Faraday-cage effect

They are safe from the Tesla coil!

Faraday Cage Effect (cont.)

Faraday-cage effect



She is safe from the Van de Graaff generator!

(Boston Science Museum)

Faraday Cage Effect (cont.)

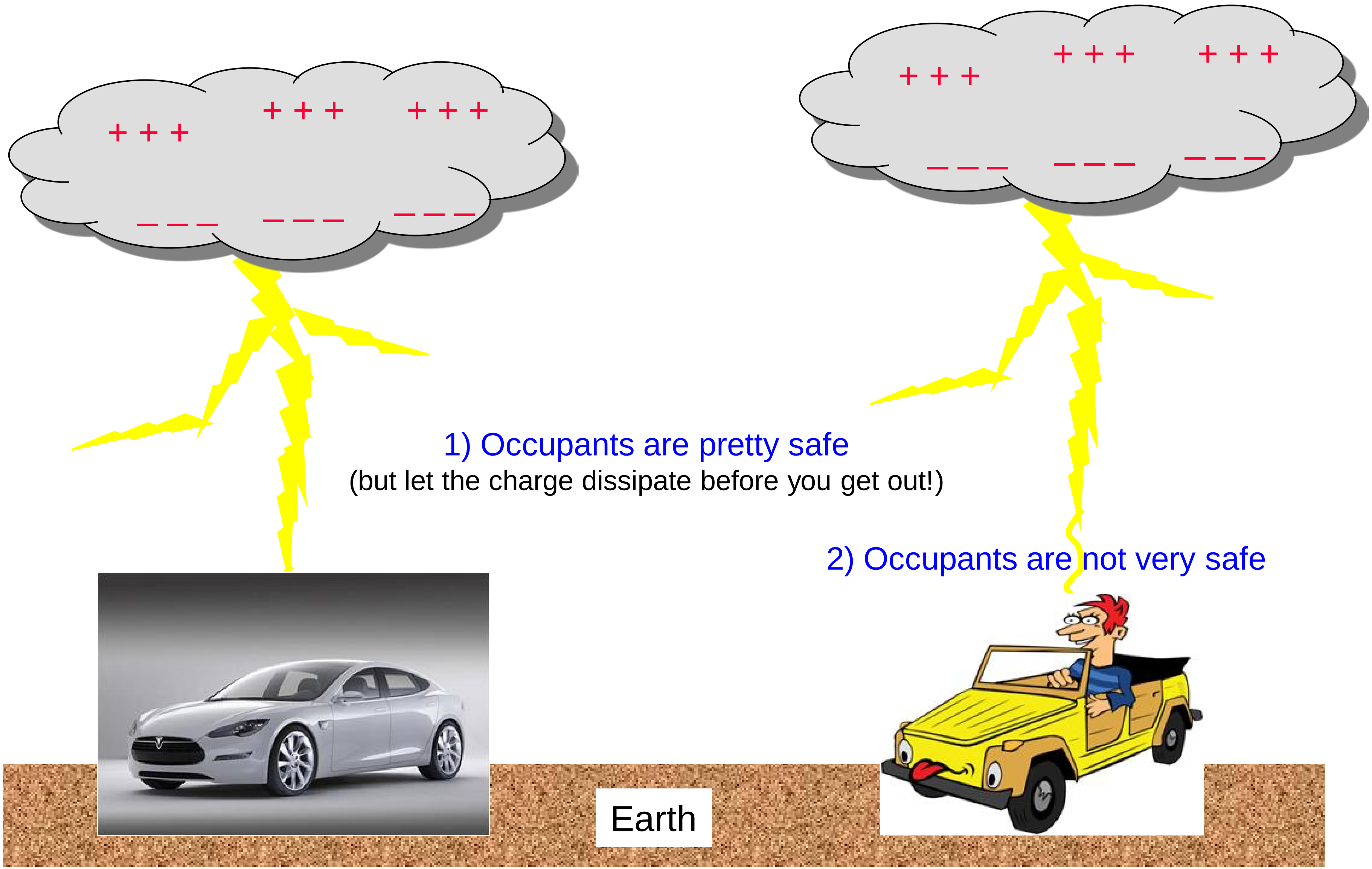
The importance of being insulated:



She is insulated from ground!

(Boston Science Museum)

Faraday Cage Effect (cont.)



Faraday Cage Effect (cont.)

Faraday Cage Shielded Room

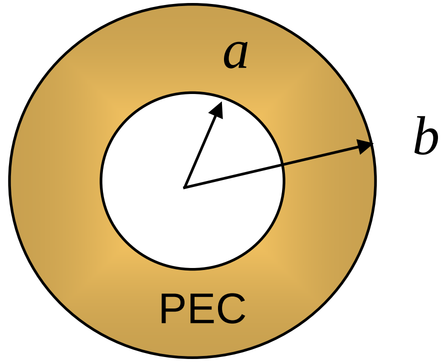
Self-standing modular Faraday cage
Superior screening of RF/LF/HF signals, e.g. for R&D, TEMPEST and Testing purposes. The modular Faraday cage is designed to meet or even exceed the vast majority of shielding requirements. The system is constructed of shielded modular panels, available in either standard-sized or custom-designed panels to meet exact specifications in government, industry, research and development, university or hospital use. The system is completely self-standing (independent of the host building). The modular panels can be shipped and assembled by the customer or under supervision of our engineers, anywhere in the world.

Applications
RF/LF/HF test and measurements
EMC test labs
Wireless product testing
EMI/RFI shielded server rooms
TEMPEST / Sensitive information protection
HEMP & EMP protection
Neuroscience laboratories
Cellular communication devices
Immunity & emission test chambers
Anechoic chambers
MRI rooms
Neurology lab
etc...

http://www.directindustry.com/prod/holland-shielding-systems-bv/faraday-cages-modular-35048-1031291.html

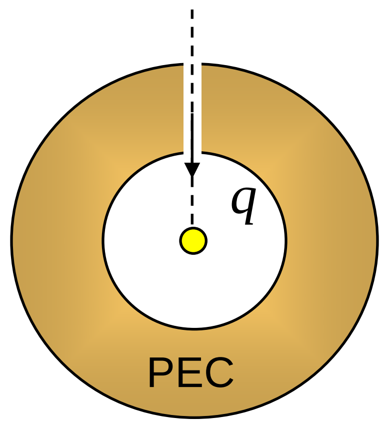
Shielding and Grounding

Start with Spherical
PEC shell



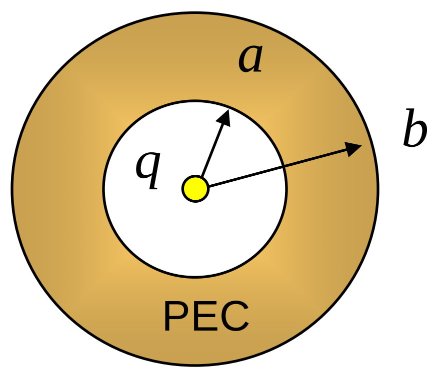
Neutral (no charge)

Drill hole and
insert point charge,
then solder hole.



Note:
The principles illustrated here apply to any shape cavity, but for modelling purposes it is simplest to use a spherical cavity.

Find $\underline{E}$:



Neutral shell (no charge)

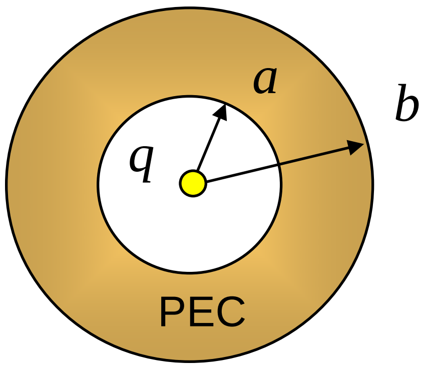
Shielding and Grounding (cont.)

(a) $r < a$

$$\oint_S \underline{D} \cdot \underline{\hat{n}}\, dS = Q_{encl}$$

$$D_r \left(4\pi r^2\right) = q$$

$$\underline{E} = \underline{\hat{r}} \left(\frac{q}{4\pi\varepsilon_0 r^2} \right)$$



Neutral shell

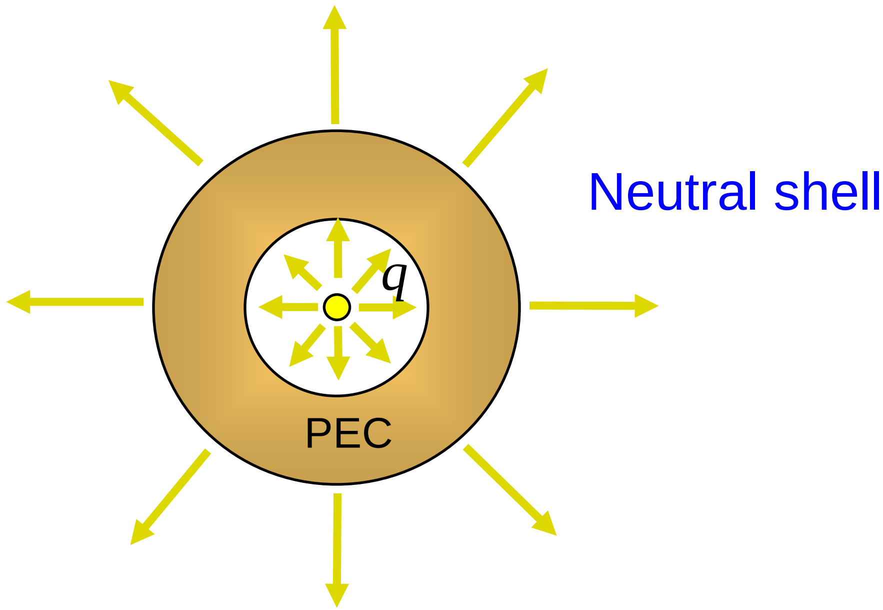
(b) $a < r < b$ $\underline{E} = \underline{0}$ (PEC)

(c) $r > b$

$$D_r \left(4\pi r^2\right) = q$$

$$\underline{E} = \underline{\hat{r}} \left(\frac{q}{4\pi\varepsilon_0 r^2} \right)$$

Shielding and Grounding (cont.)



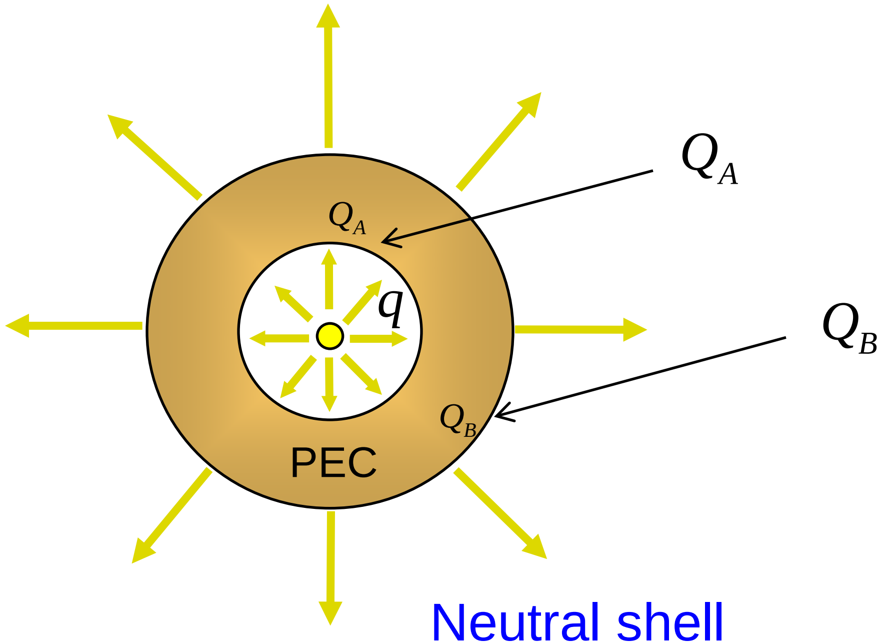
$$\underline{E} = \underline{\hat{r}} \left(\frac{q}{4\pi\varepsilon_0 r^2} \right) \quad \text{(outside metal)}$$

The neutral metal shell does not block the static electric field <u>coming from the inside</u> !

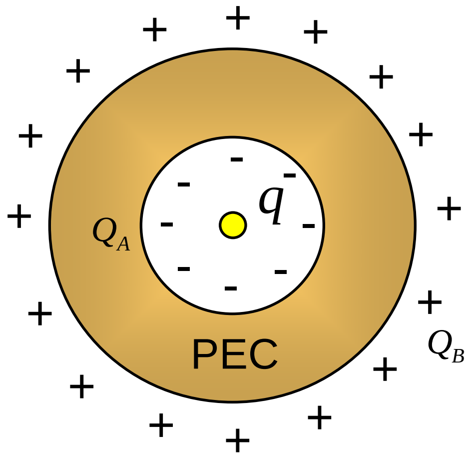
(The shell would be a good shield for <u>high-frequency</u> fields coming from the inside, however, due to the skin-depth effect.)

Shielding and Grounding (cont.)

Find Q_A, Q_B:

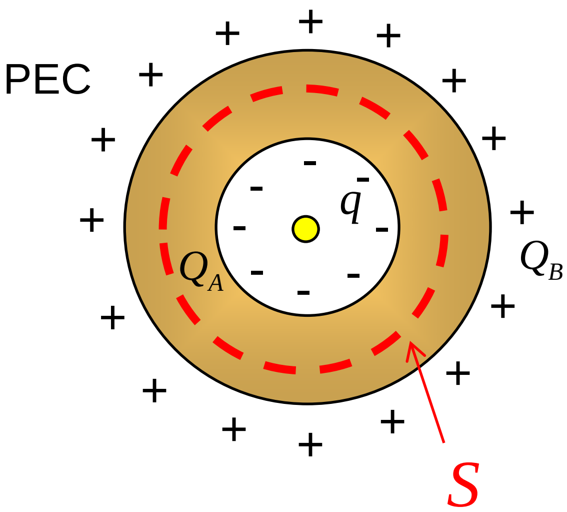


Neutral shell



Neutral shell: $Q_A = -Q_B$

Shielding and Grounding (cont.)



A Gaussian surface is chosen <u>inside</u> the metal shell.

$$\oint_S \underline{D} \cdot \underline{\hat{n}} \, dS = Q_{encl}$$

$$\Rightarrow \quad D_r \left(4\pi r^2\right) = Q_{encl}$$

The electric field is zero inside of a PEC.

so $\quad Q_{encl} = 0$

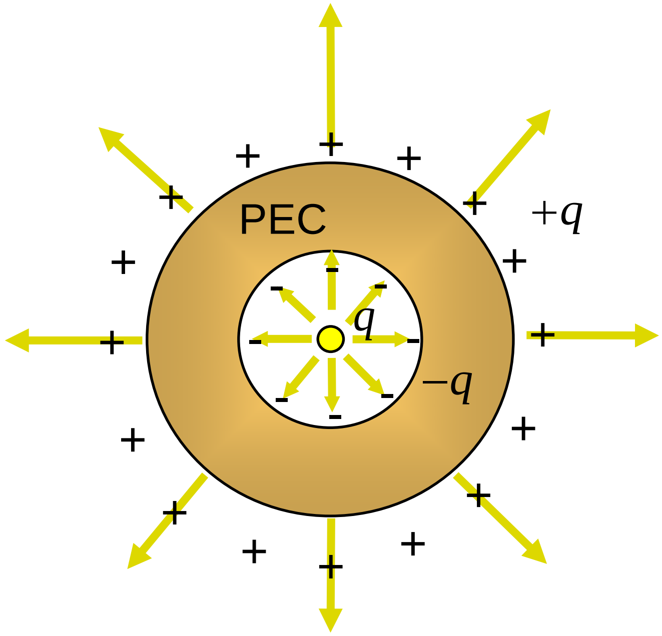
or $\quad Q_A + q = 0$

Hence

$$Q_A = -q$$

We then also have $\quad Q_B = +q \quad$ (neutral shell)

Shielding and Grounding (cont.)



Flux picture showing the charge on the surfaces

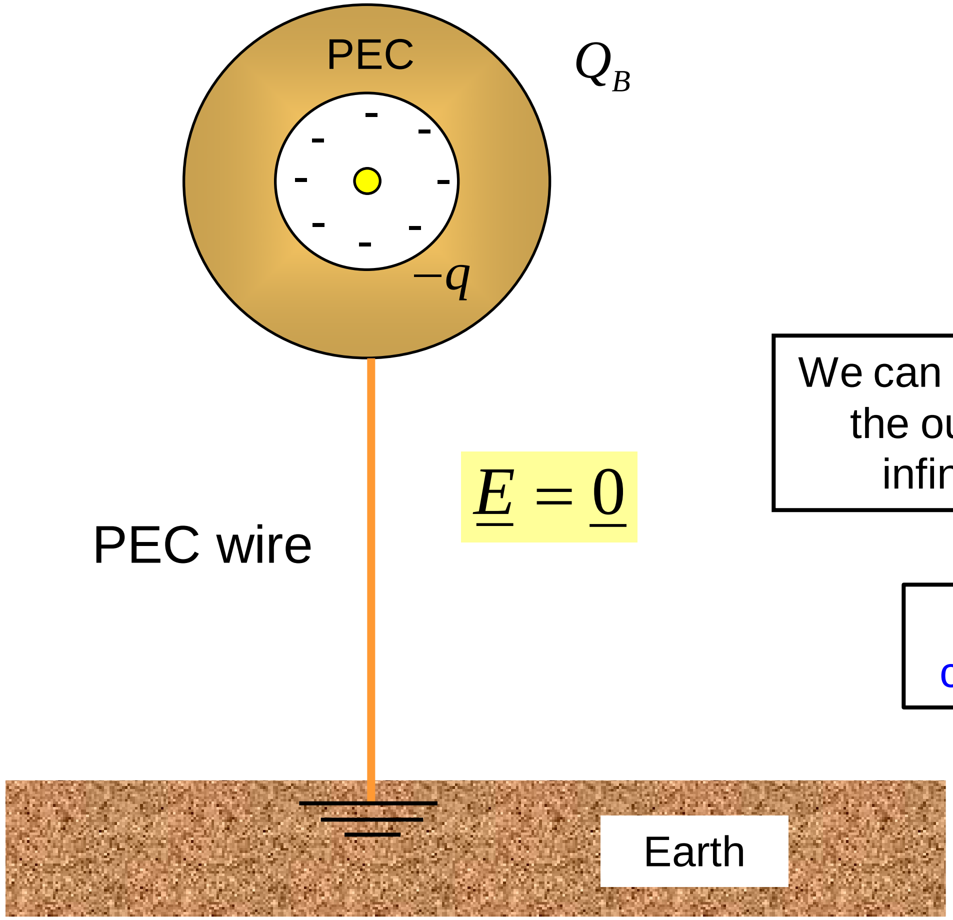
Note: Flux lines go from positive charges to negative charges.

(Also, they go from higher potential to lower potential.)

Shielding and Grounding (cont.)

Next, "ground" the shell:

$$r > b: \quad \underline{E} = \underline{0}$$



Proof:

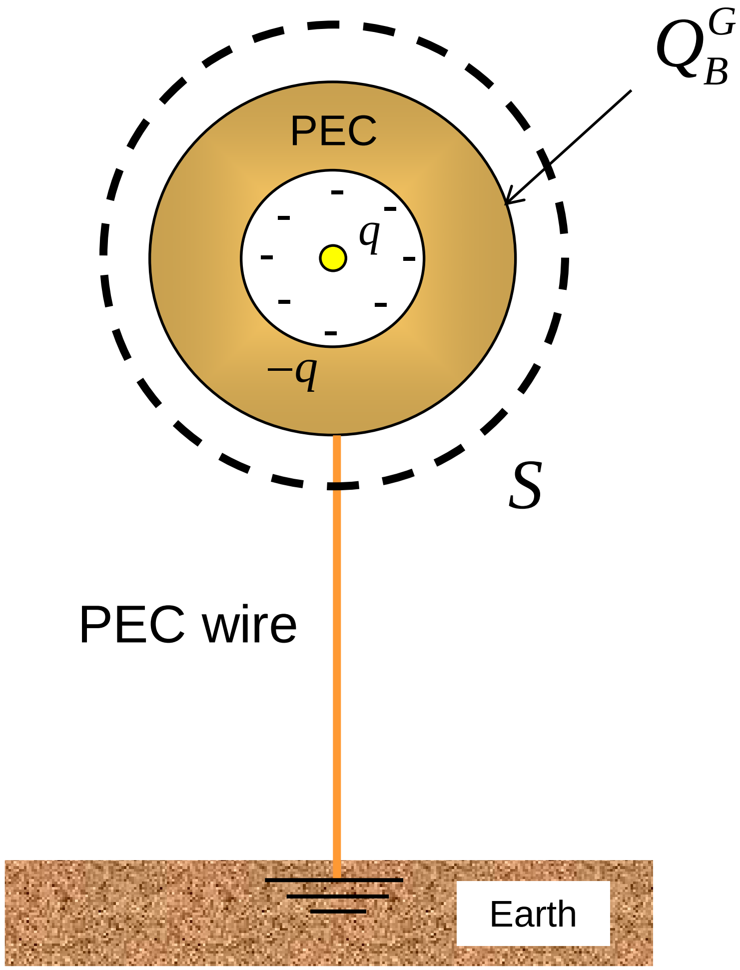
We can invoke the uniqueness theorem, considering the outside of the shell, the wire, the earth, and infinity to form the boundary S of our region.

Note: Grounding has not affected the charge on the inner surface of the shell.

The earth is modeled as a big conductor.

Shielding and Grounding (cont.)

Charge on outer surface:



$$r > b: \quad \underline{E} = \underline{0}$$

$$\Rightarrow \oint_S \underline{D} \cdot \underline{\hat{n}} \, dS = Q_{encl}$$

$$\Rightarrow Q_{encl} = 0$$

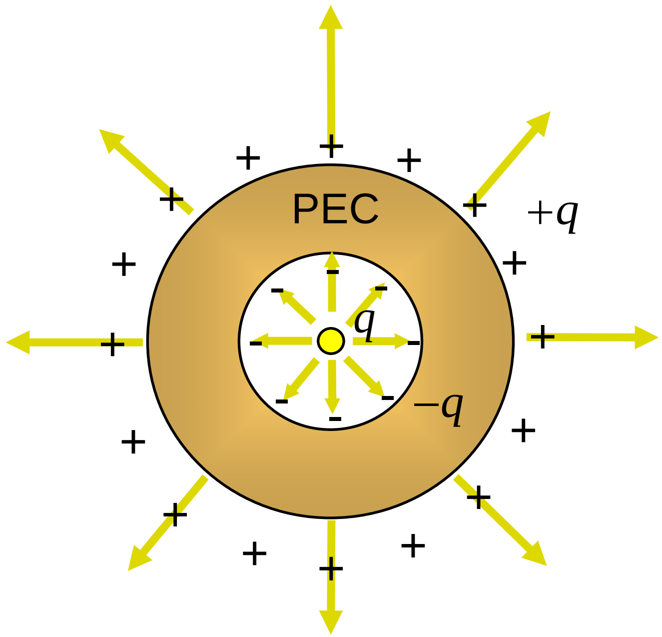
$$\Rightarrow q + (-q) + Q_B^G = 0$$

Hence

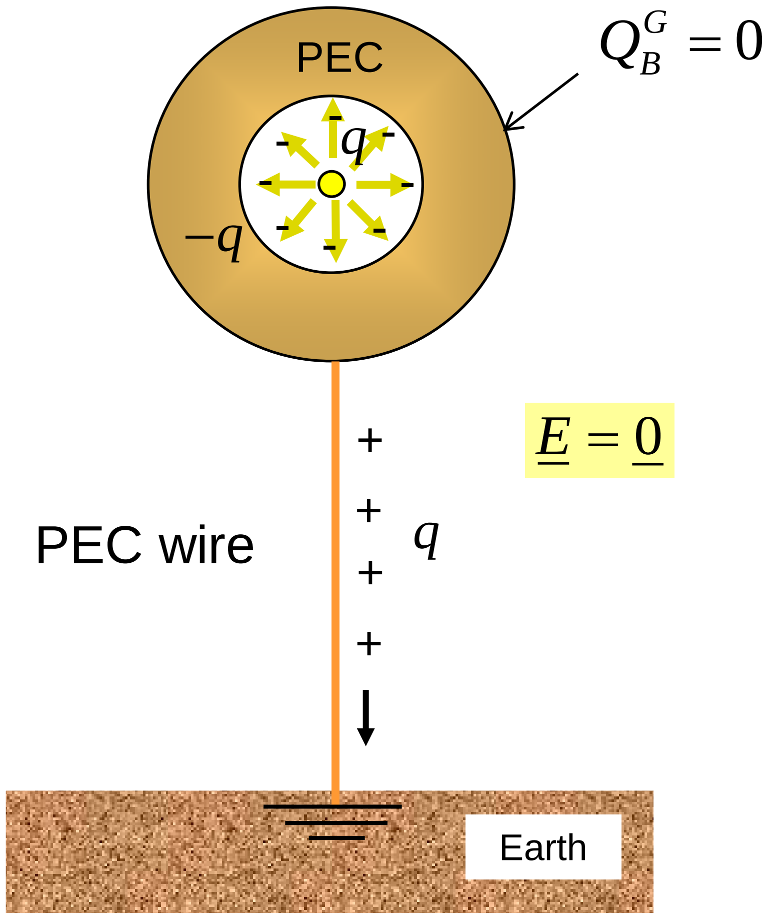
$$Q_B^G = 0$$

Shielding and Grounding (cont.)

Charge on outer surface (cont.):



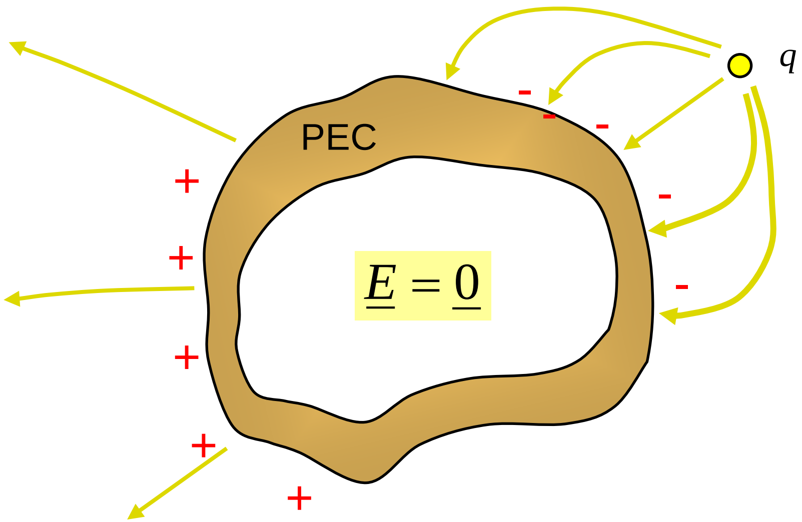
Neutral shell before grounding



After grounding

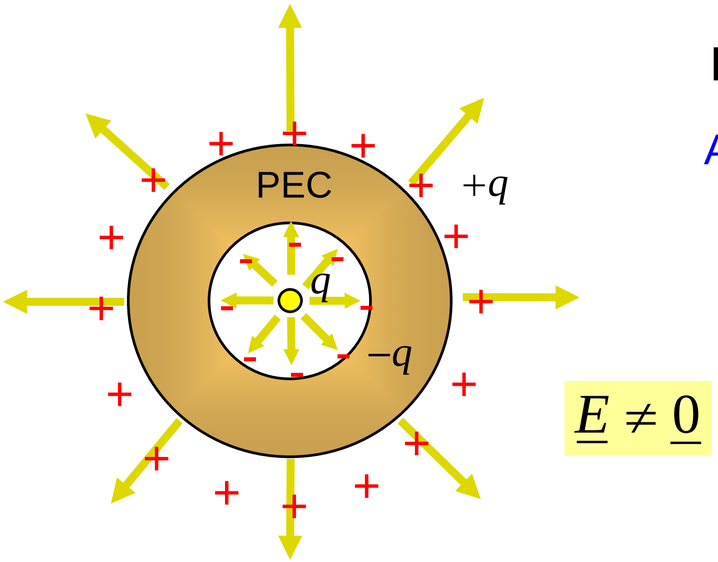
The charge q on the outer surface has flowed down to ground.

Summary of Shielding & Grounding in Statics



Effect # 1: Faraday cage effect

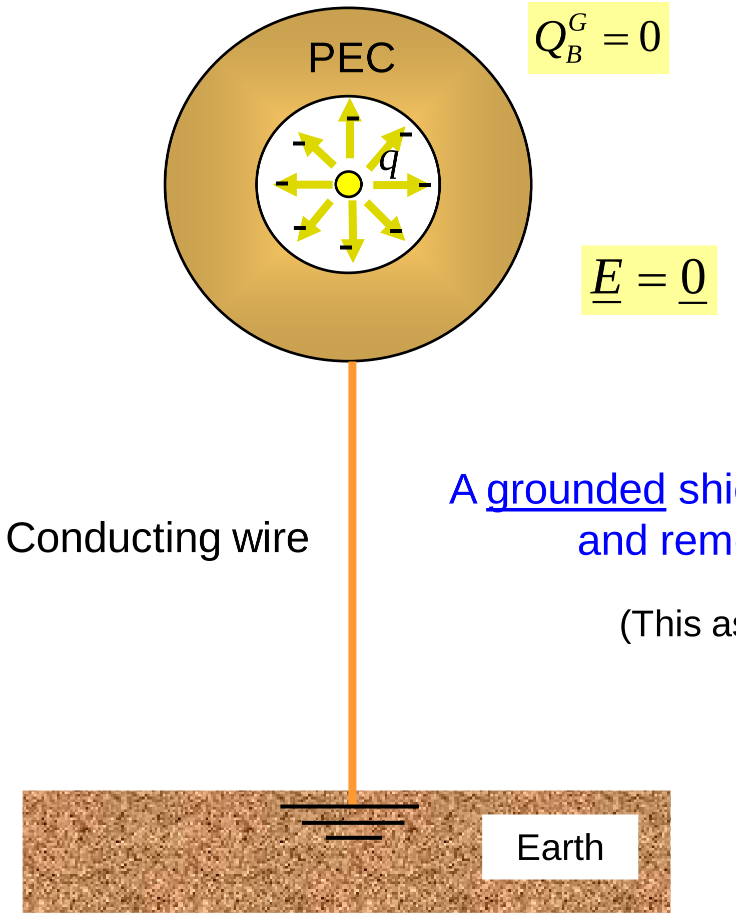
There is no static field on the inside (does not require grounding).



Effect # 2: Static field penetration

An ungrounded shield does not block the static electric field of an inside source.

Summary of Shielding & Grounding in Statics (cont.)

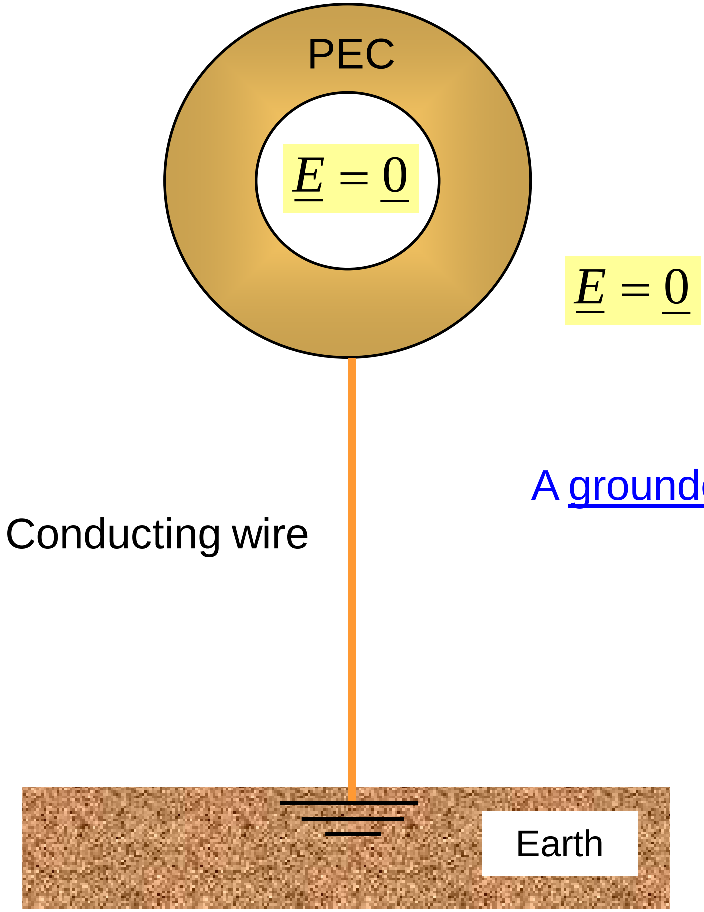


Effect # 3: Grounding

A <u>grounded</u> shield removes the static electric field in the exterior region and removes static charge buildup from the outer surface.

(This assumes that there are no charges in the outside region.)

Summary of Shielding & Grounding in Statics (cont.)



Grounded Faraday cage

(effect #1 + effect #3)

A <u>grounded</u> hollow Faraday cage has no static field anywhere.

Comments on Grounding*

Static Fields

- Grounding removes static charge build up on objects.
  - Grounding removes electric fields produced from any static charge build up.
  - This is good for safety (e.g., when operating in explosive environments.)
  - This is good for reducing noise in sensitive measurements.

Power line frequency

- Grounding is good for the safety of AC equipment (preventing shocks from the AC line voltage).

*This topic was also discussed in Notes 5.