ECE 3318
Applied Electricity and Magnetism

Spring 2023

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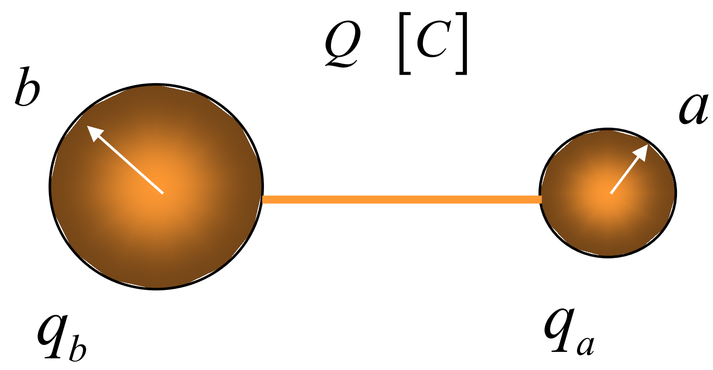
Dept. of ECE

Notes 16
Sharp Point Property

Sharp Point Property

A sharp point produces a strong electric field.

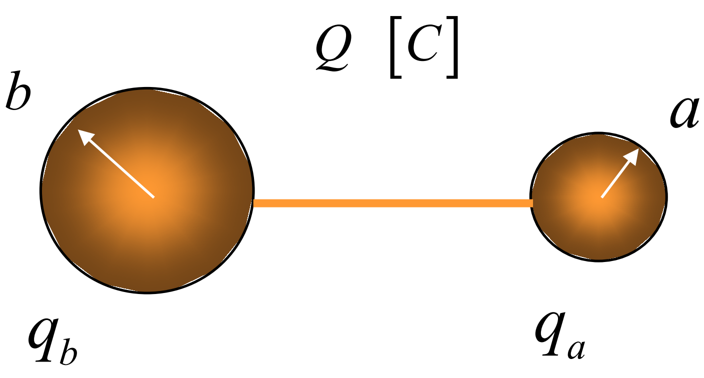


$$\Phi_a = \Phi_b$$

The system is charged with Q [C].

Determine how much charge goes to each sphere, and the electric field at the surface of each sphere.

Sharp Point Property (cont.)



$$\Phi_a = \Phi_b$$

$$\Rightarrow \quad \frac{q_a}{4\pi\varepsilon_0 a} = \frac{q_b}{4\pi\varepsilon_0 b}$$

We ignore the effects of the wire on the charge distribution, and we ignore the interaction between the two spheres.

$$\Rightarrow \quad \frac{q_a}{q_b} = \frac{a}{b}$$

Also, $q_a + q_b = Q$

Solution:

$$q_a = Q\left(\frac{a}{a+b}\right) \qquad q_b = Q\left(\frac{b}{a+b}\right)$$

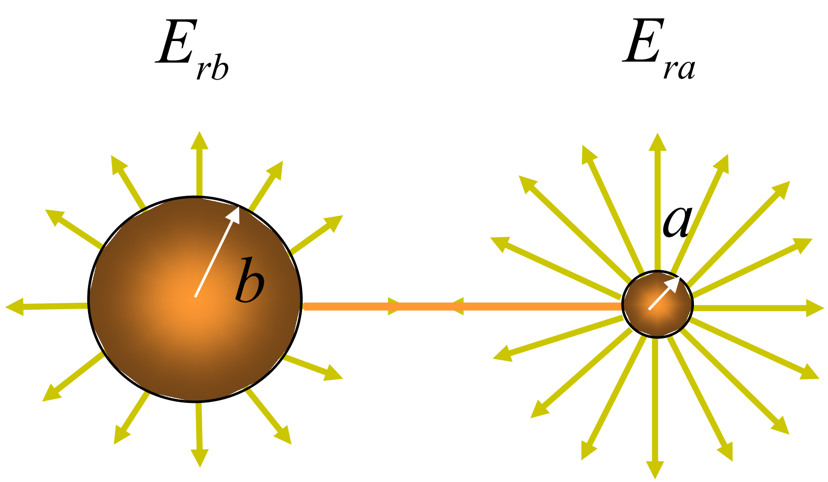
Sharp Point Property (cont.)

$$E_{ra} = \frac{q_a}{4\pi\varepsilon_0 a^2}$$

$$E_{rb} = \frac{q_b}{4\pi\varepsilon_0 b^2}$$

so

$$\frac{E_{ra}}{E_{rb}} = \left(\frac{b}{a}\right)^2 \left(\frac{q_a}{q_b}\right)$$



Also, from the last slide, $\frac{q_a}{q_b} = \frac{a}{b}$

Hence,

$$\frac{E_{ra}}{E_{rb}} = \left(\frac{b}{a}\right)$$

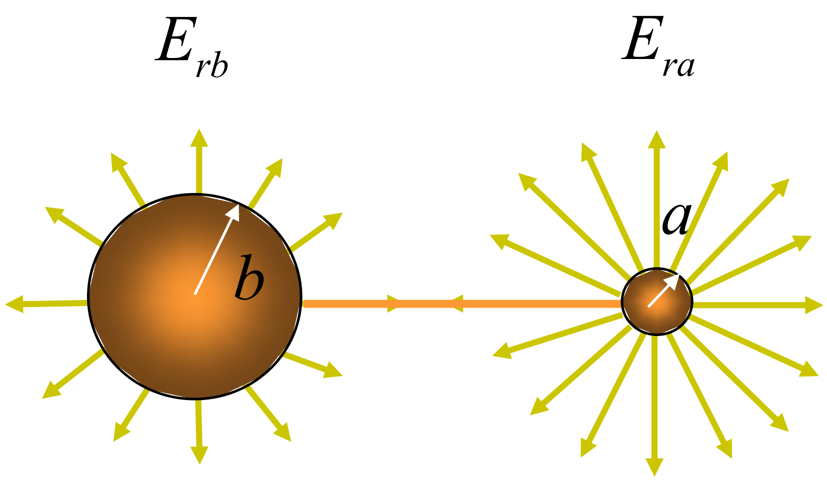
A stronger electric field exists on the smaller sphere.

Note: $|E_{ra}| \to \infty$ as $a \to 0$

Sharp Point Property (cont.)

Conclusion:

If there is any charge on a metallic system, the electric field well be very high at sharp points or corners.

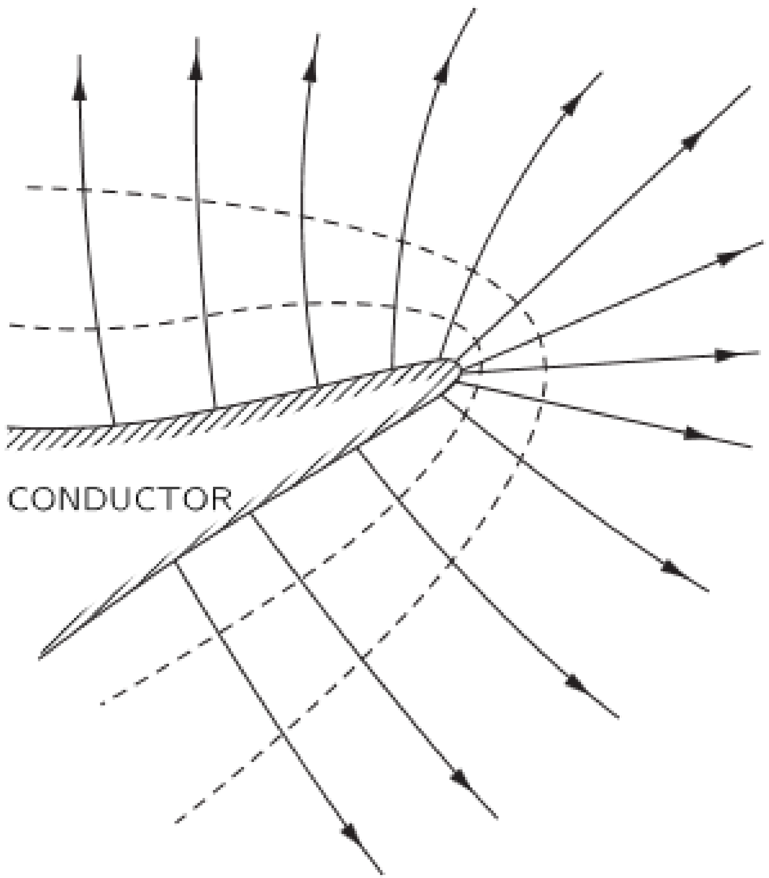


Note:

Grounding the metallic system will remove the charge that is built up on the system ($Q = 0$).

Sharp Point Property (cont.)

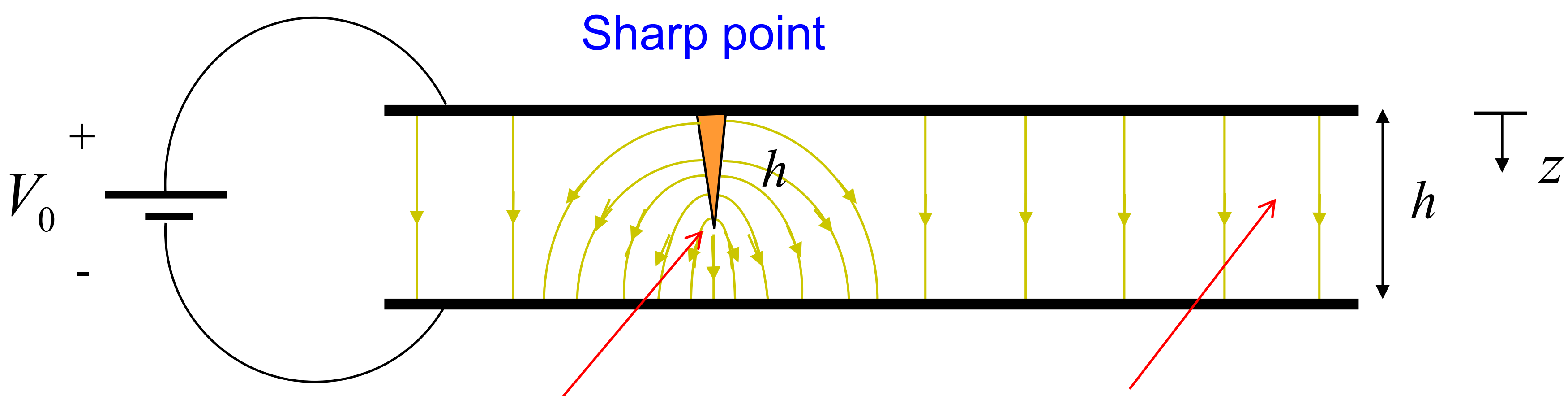
The field is strong as we approach a sharp bend or point on a practical conductor that has a charge on it.



http://www.feynmanlectures.info/docroot/II_06.html

Sharp Point Property (cont.)

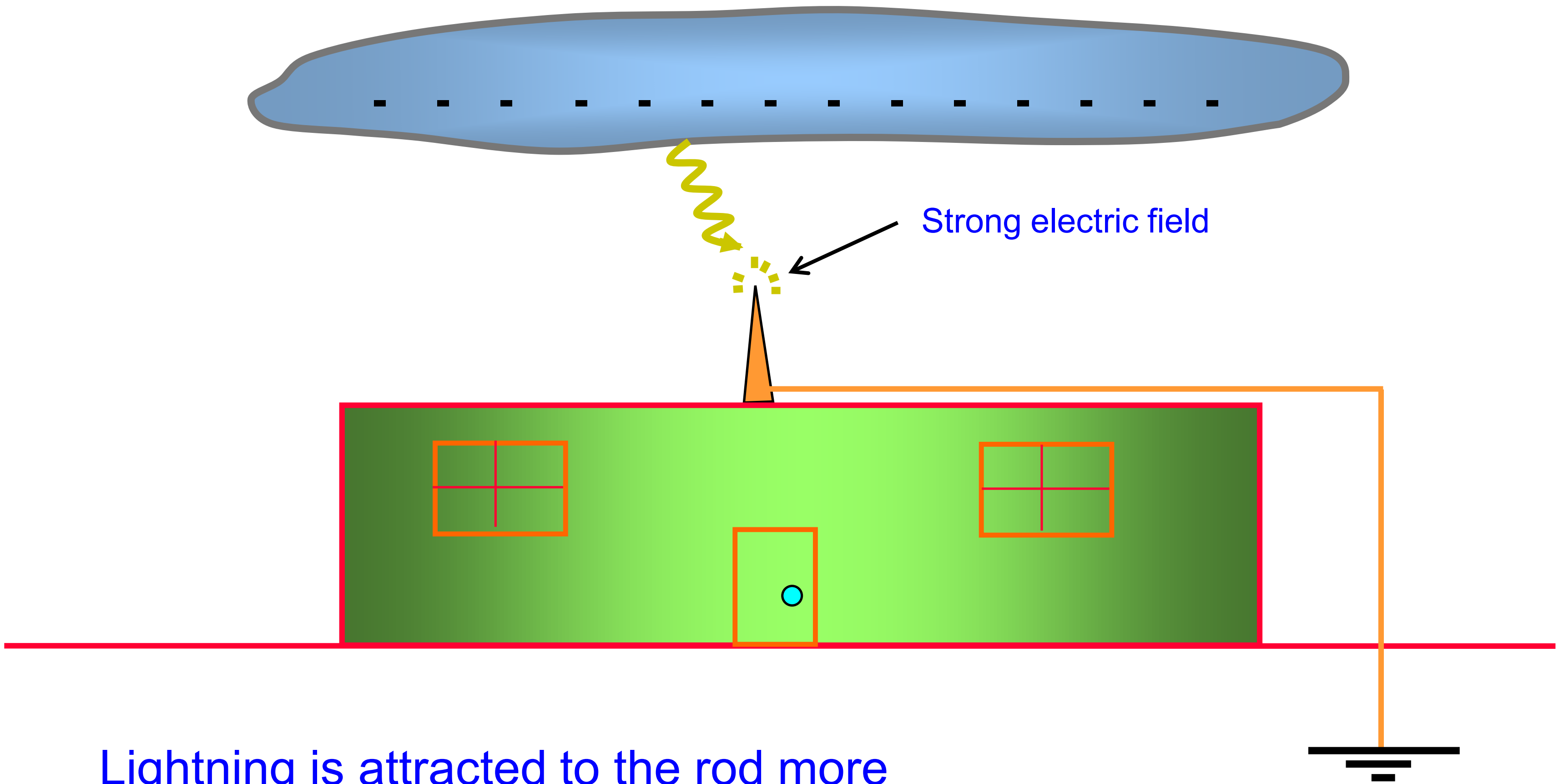
The electric field near a sharp metal point may be much higher than the surrounding field.



$$E_z >> \frac{V_0}{h}$$

$$E_z \approx \frac{V_0}{h}$$

Lightning Rod



Lightning is attracted to the rod more than the rest of the building:

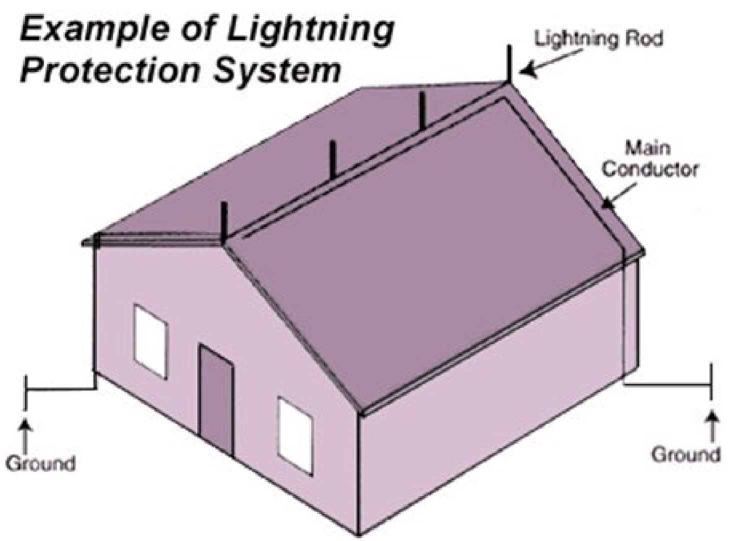
a) Because it is taller
b) Because of the sharp point

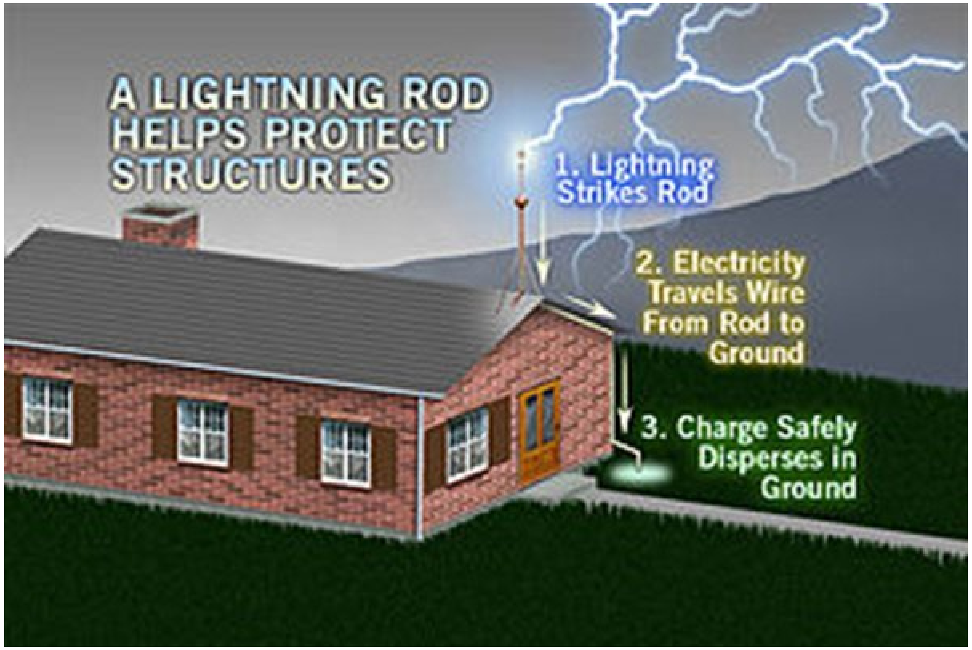
Important to have it well grounded !

Lightning Rod (cont.)

A lighting rod in action

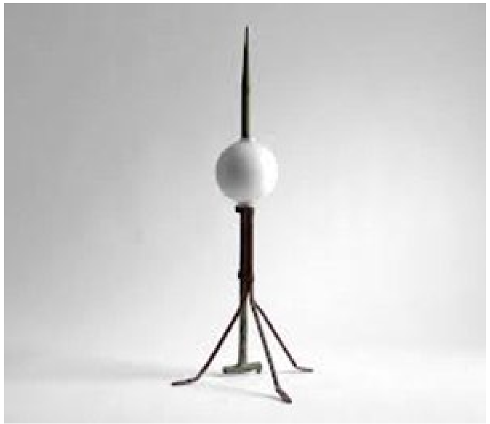
Lightning Rod (cont.)





For larger structures, it is good to have more than one rod, and/or make them as high as possible.

Lightning Rod (cont.)



Examples of lightning rods on homes and barns.

Lightning Rod (cont.)

A lightning rod on top of the “Purple Parking Garage” at IAH airport.

Lightning Rod (cont.)

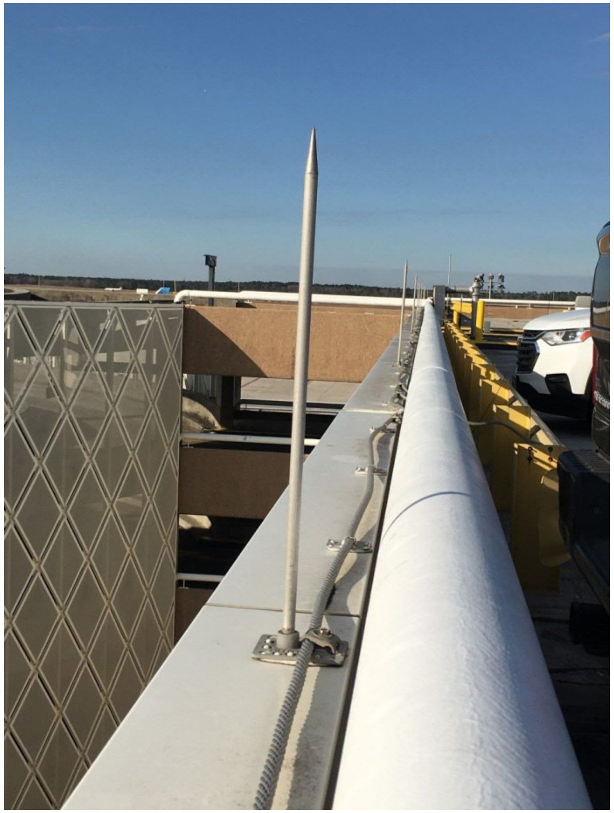
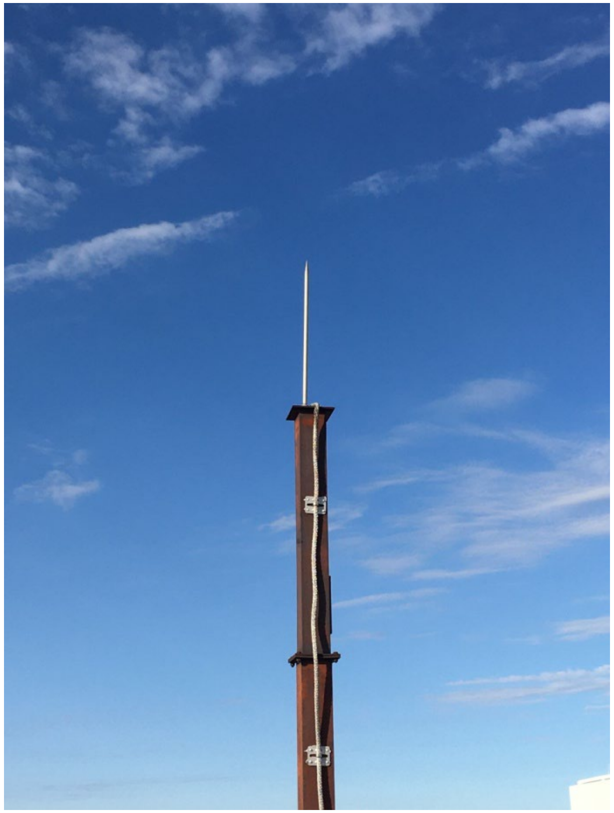
A close-up of the grounding system at the “Purple Parking Garage” at IAH airport.

Lightning Rod (cont.)

George Bush
Intercontinental Airport
(IAH)
Purple Parking Garage

Lighting rods are also on top of the light posts.

Lightning Rod (cont.)

George Bush Intercontinental Airport (IAH)
Brown Parking Garage

Static Discharge Wicks

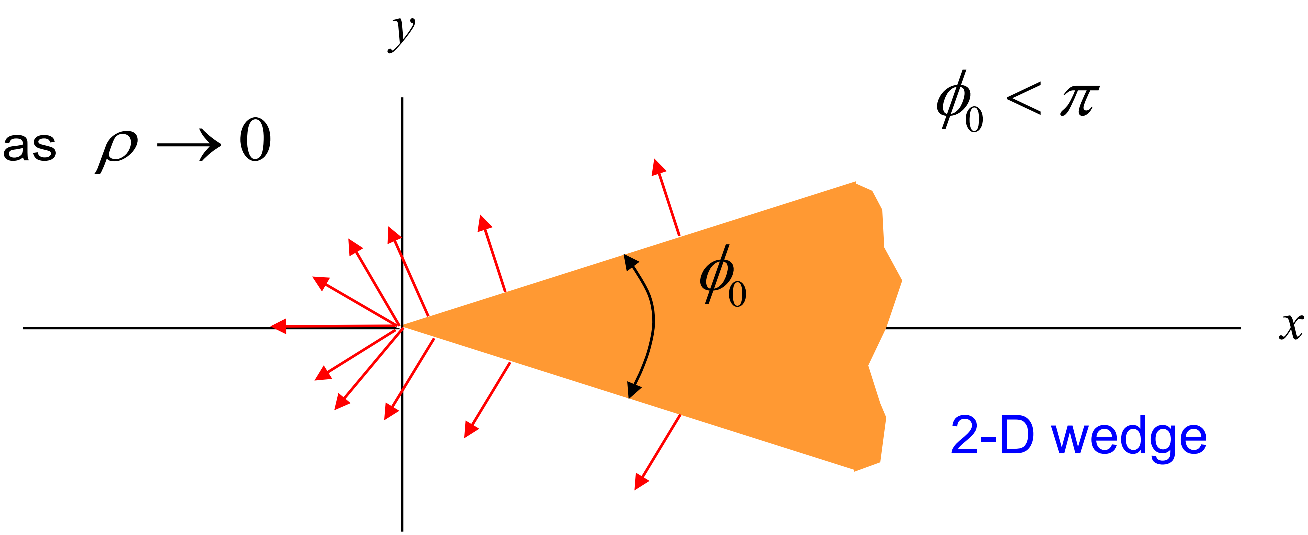
"Static discharge wicks" are used on aircraft to help bleed off any charge buildup due to friction with particles (ice, dust, etc.) the atmosphere. Static charge buildup on antennas (and subsequent corona) can cause interference with radio communications.

http://en.wikipedia.org/wiki/Static_discharger

Edge on a Wedge

The field approaches infinity as we approach the corner of a conducting wedge.

$\left|\underline{E}\right| \to \infty$ as $\rho \to 0$



On wedge surface:

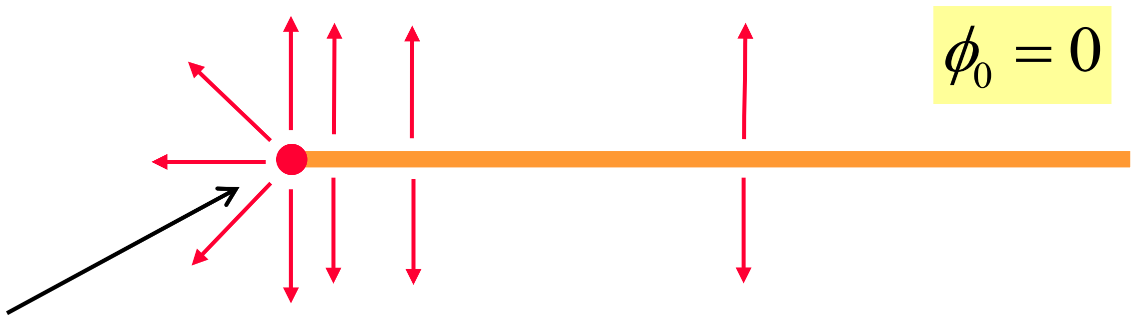
$$\left|E_\phi\right| \propto 1/\rho^{\alpha}$$

$$\alpha = \frac{1/2 - \phi_0/(2\pi)}{1 - \phi_0/(2\pi)}$$

ϕ_0	α
0^{o}	0.5
30^{o}	0.455
60^{o}	0.4
90^{o}	0.333

Knife Edge

The field approaches infinity as we approach the tip of a flat metal plate.



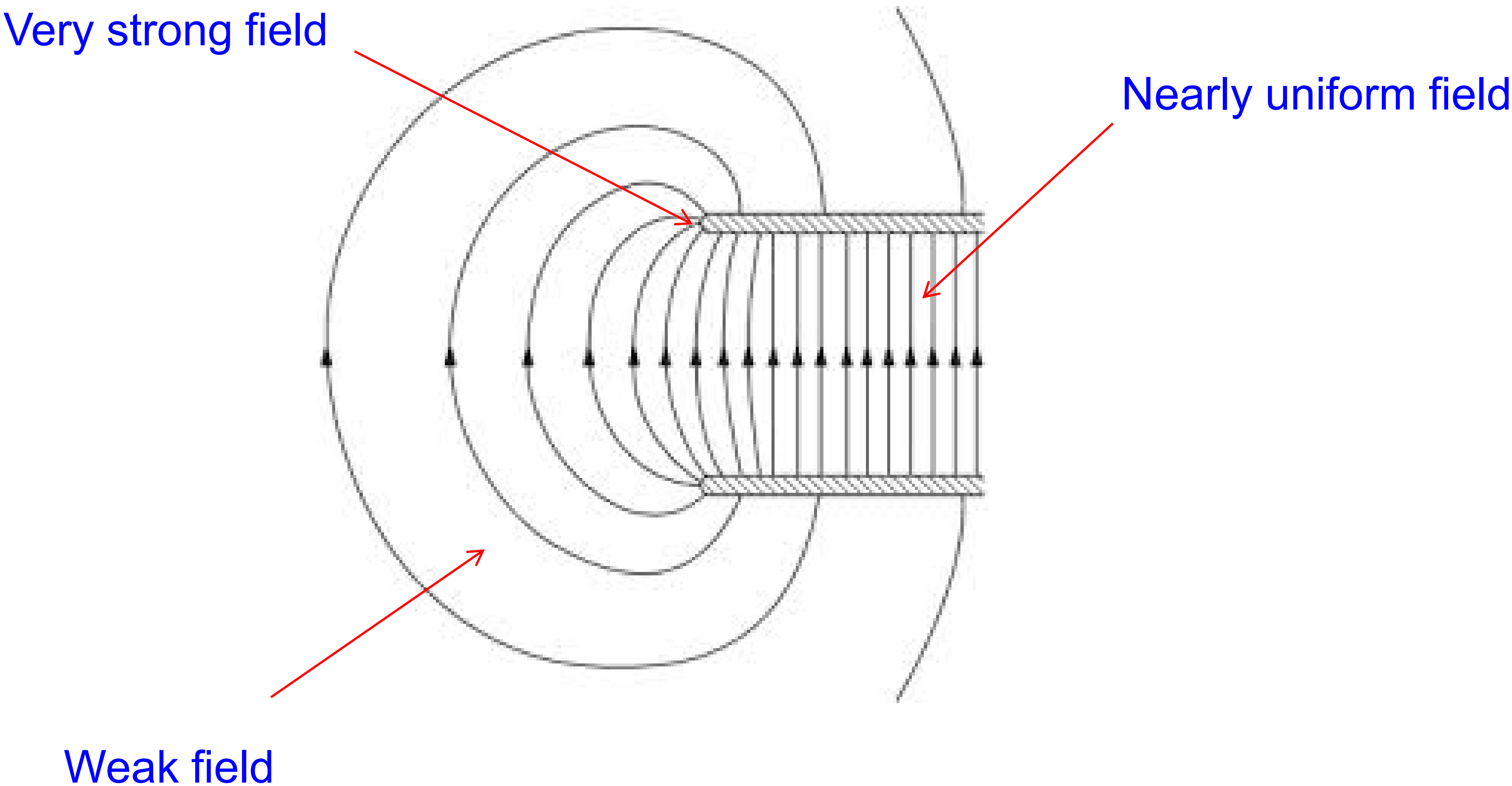
"Knife edge"

$$\alpha = \frac{1/2 - \phi_0/(2\pi)}{1 - \phi_0/(2\pi)} \quad \Rightarrow \quad \alpha = 1/2$$

$$\left|\underline{E}\right| \propto \frac{1}{\sqrt{\rho}}$$

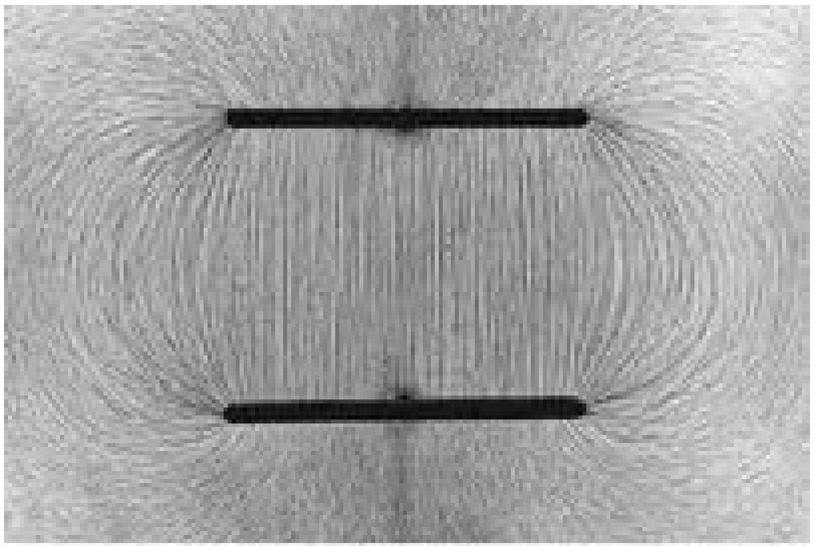
Knife Edge (cont.)

Field near the edge of a parallel-plate capacitor

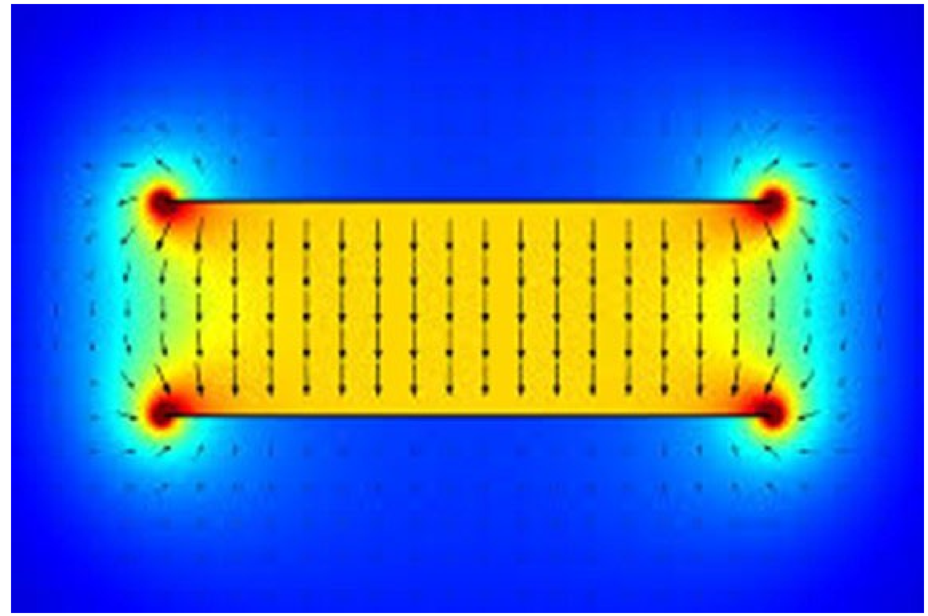


Credit: Feynman Lectures on Physics

Knife Edge (cont.)



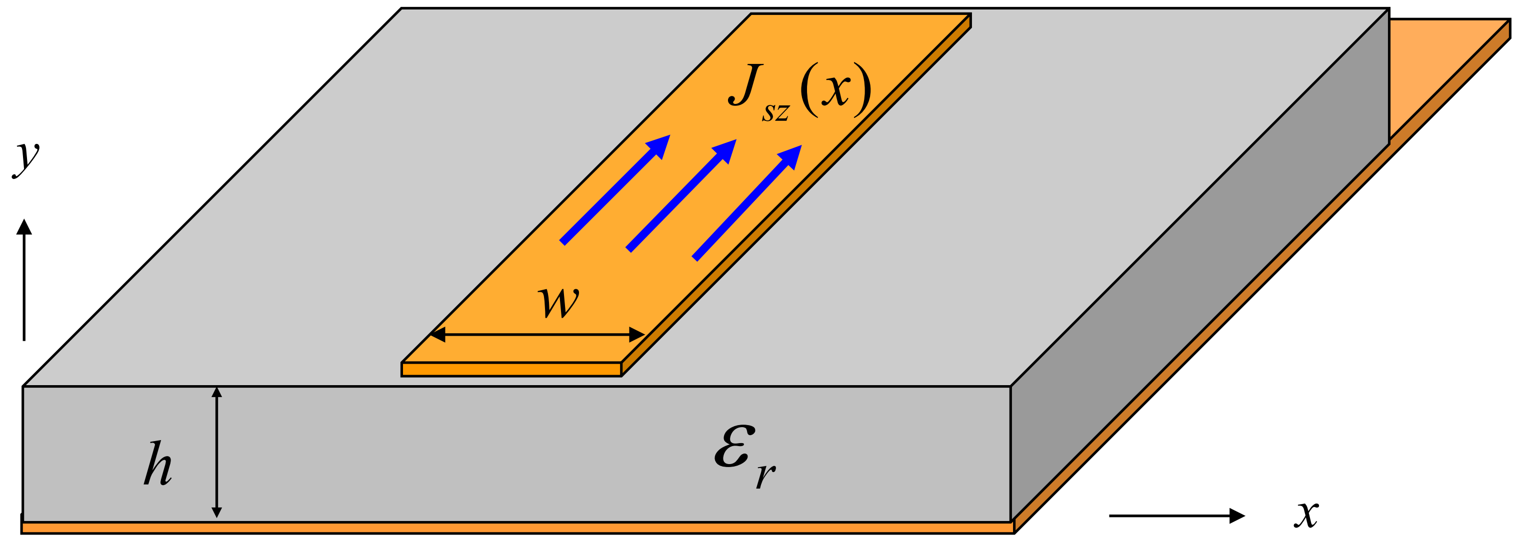
Actual physical flux plot

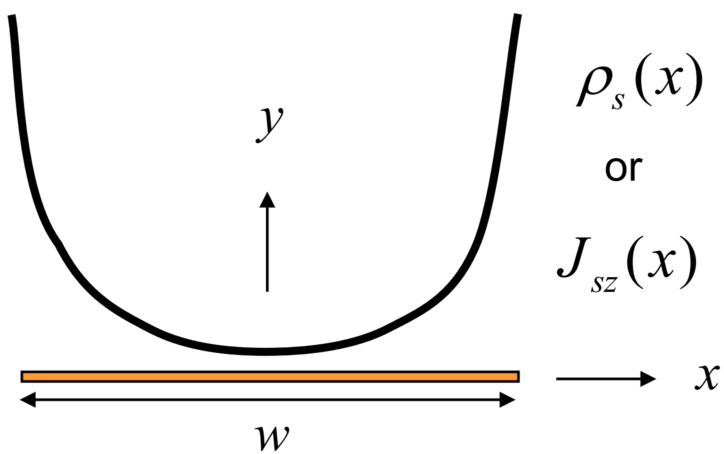


Simulation:
color scale shows field strength

Microstrip Line

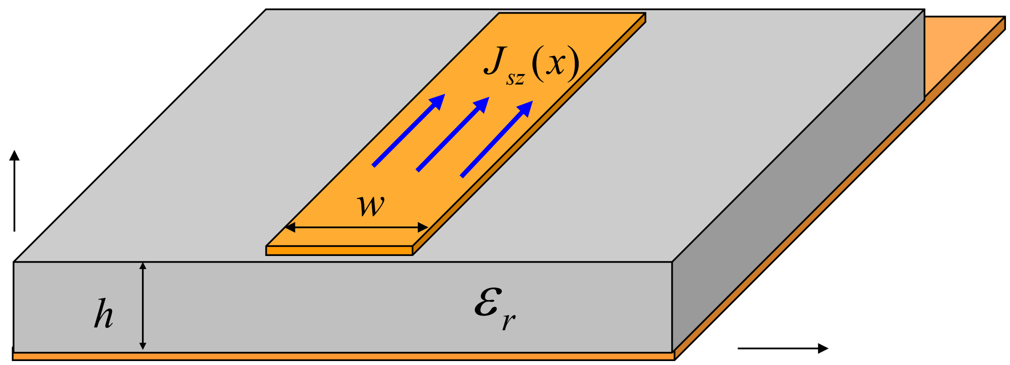
The field and current approach infinity as we approach the <u>edges</u> of the line.





The edge singularity causes increased conductor loss, compared with transmissions lines such as coaxial cables. It also lowers the power handling capability due to dielectric breakdown.

Microstrip Line



Near the edges we have (proof omitted):

$$\left|J_z\right| \propto \frac{1}{\sqrt{x^2 - \left(\frac{w}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(x - w/2\right)\left(x + w/2\right)}}$$

Distance to right/left edge