ECE 3318
Applied Electricity and Magnetism

Spring 2023

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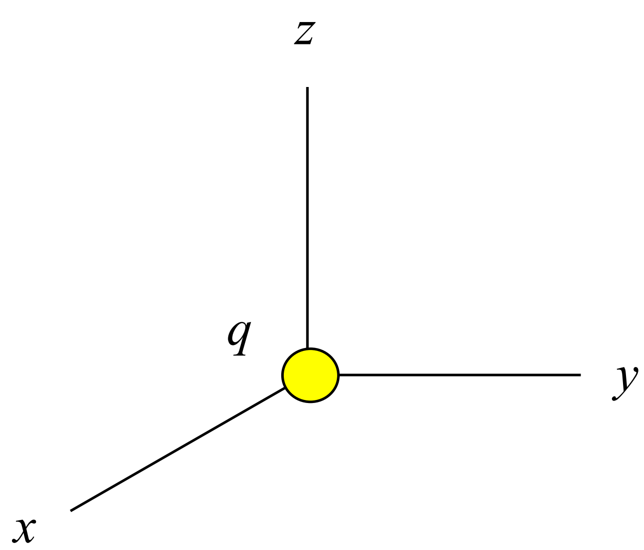
Dept. of ECE

Notes 18
Faraday's Law

Example (cont.)

Find curl of $\underline{E}$ from a static point charge



$$\underline{E} = \underline{\hat{r}}\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)$$

$$\nabla \times \underline{E} = \underline{\hat{r}}\frac{1}{r\sin\theta}\left[\frac{\partial\left(E_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial E_\theta}{\partial\phi}\right] + \underline{\hat{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial\left(rE_\phi\right)}{\partial r}\right] + \underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(rE_\theta\right)}{\partial r} - \frac{\partial E_r}{\partial\theta}\right]$$

$$= \underline{0}$$

Example (cont.)

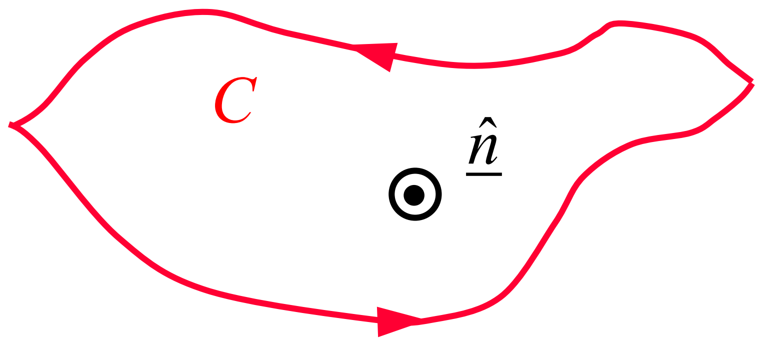
Note:
If the curl of the electric field is zero for the field from a static point charge, then by superposition it must be zero for the field from any static charge density.

This gives us **Faraday's law**:

$$\nabla \times \underline{E} = \underline{0}$$

(in statics)

Faraday's Law in Statics (Integral Form)



Stokes's theorem:

$$\oint_C \underline{E} \cdot d\underline{r} = \int_S \left(\nabla \times \underline{E}\right) \cdot \hat{\underline{n}}\; dS = 0$$

Here S is any "bowl" surface that is attached to C.

Hence

$$\oint_C \underline{E} \cdot \underline{dr} = 0$$

Faraday's Law in Statics (Differential Form)

We show here how the integral form also implies the differential form.

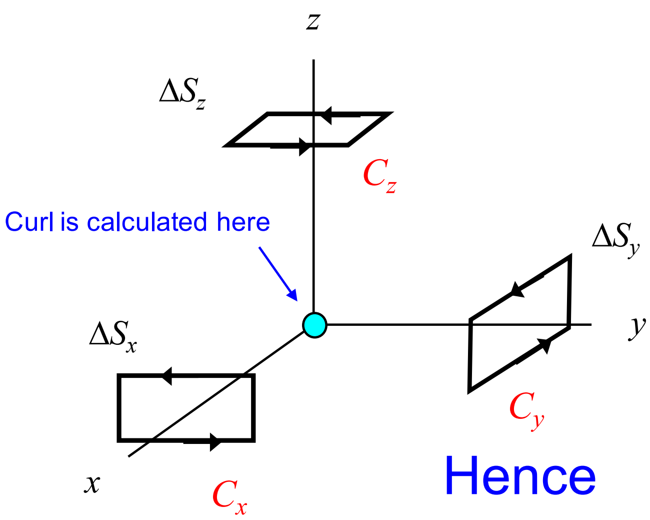
Assume $\oint_C \underline{E} \cdot \underline{dr} = 0$ (for any path C)

We then have (definition of curl):

$$\underline{\hat{x}} \cdot curl\ \underline{E} \equiv \lim_{\Delta s \to 0} \frac{1}{\Delta S_x} \oint_{C_x} \underline{E} \cdot \underline{dr} = 0$$

$$\underline{\hat{y}} \cdot curl\ \underline{E} \equiv \lim_{\Delta s \to 0} \frac{1}{\Delta S_y} \oint_{C_y} \underline{E} \cdot \underline{dr} = 0$$

$$\underline{\hat{z}} \cdot curl\ \underline{E} \equiv \lim_{\Delta s \to 0} \frac{1}{\Delta S_z} \oint_{C_z} \underline{E} \cdot \underline{dr} = 0$$



Hence

$$\nabla \times \underline{E} = \underline{0}$$

Faraday's Law in Statics (Summary)

$$\nabla \times \underline{E} = \underline{0}$$

Differential (point) form of Faraday's law

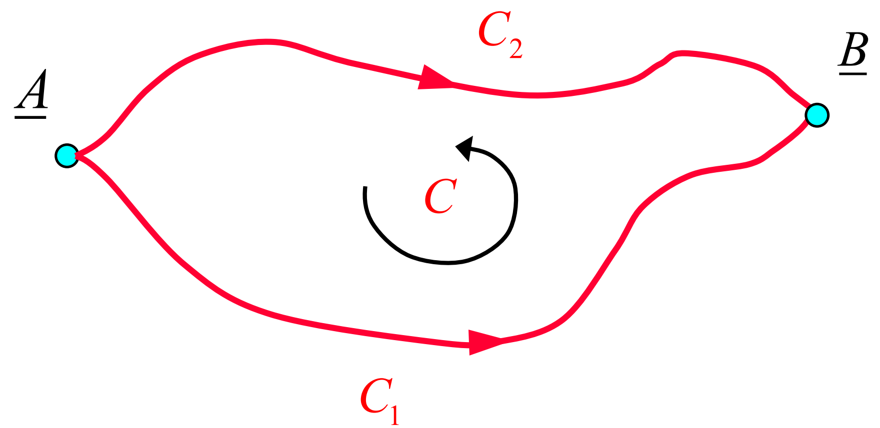
Stokes's theorem ↓ ↑ Definition of curl

$$\oint_C \underline{E} \cdot d\underline{r} = 0$$

Integral form of Faraday's law

Path Independence and Faraday's Law

The integral form of Faraday's law is equivalent to
path independence of the voltage drop calculation in statics.



Proof:

$$\oint_C \underline{E} \cdot d\underline{r} = 0 \text{ (in statics)}$$

Also,

$$\oint_C \underline{E} \cdot d\underline{r} = \int_{C_1} \underline{E} \cdot d\underline{r} - \int_{C_2} \underline{E} \cdot d\underline{r}$$

Hence,

$$\oint_C \underline{E} \cdot d\underline{r} = 0 \quad \Leftrightarrow \quad \int_{C_1} \underline{E} \cdot d\underline{r} = \int_{C_2} \underline{E} \cdot d\underline{r}$$

Summary of Path Independence

Equivalent properties of an electrostatic field

Path independence for V_{AB}

Equivalent

$$\oint_C \underline{E} \cdot d\underline{r} = 0$$

$$\nabla \times \underline{E} = \underline{0}$$

Summary of Electrostatics

Here is a summary of the important equations related to the electric field in statics.

$$\nabla \cdot \underline{D} = \rho_v$$

Electric Gauss law

$$\nabla \times \underline{E} = \underline{0}$$

Faraday's law

$$\underline{D} = \varepsilon_0 \underline{E}$$

Constitutive equation (free space)

Faraday's Law: Dynamics

Experimental Law (dynamics):

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

This is the general Faraday's law in dynamics.

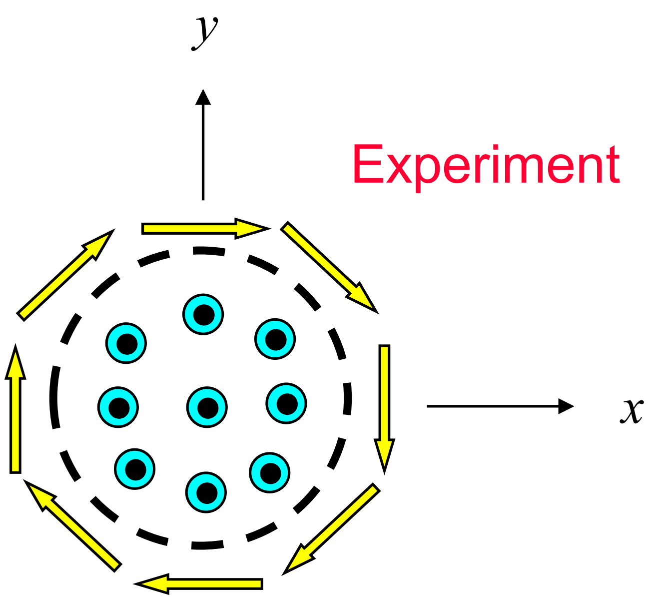
Michael Faraday*

(from Wikipedia)

$\underline{B} =$ magnetic flux density $\left[\mathrm{Webers/m^2}\right]$

*Ernest Rutherford stated: "When we consider the magnitude and extent of his discoveries and their influence on the progress of science and of industry, there is no honour too great to pay to the memory of Faraday, one of the greatest scientific discoverers of all time".

Faraday's Law: Dynamics (cont.)



Magnetic field B_z (increasing with time)

Electric field $\underline{E}$

The changing magnetic field produces an electric field.

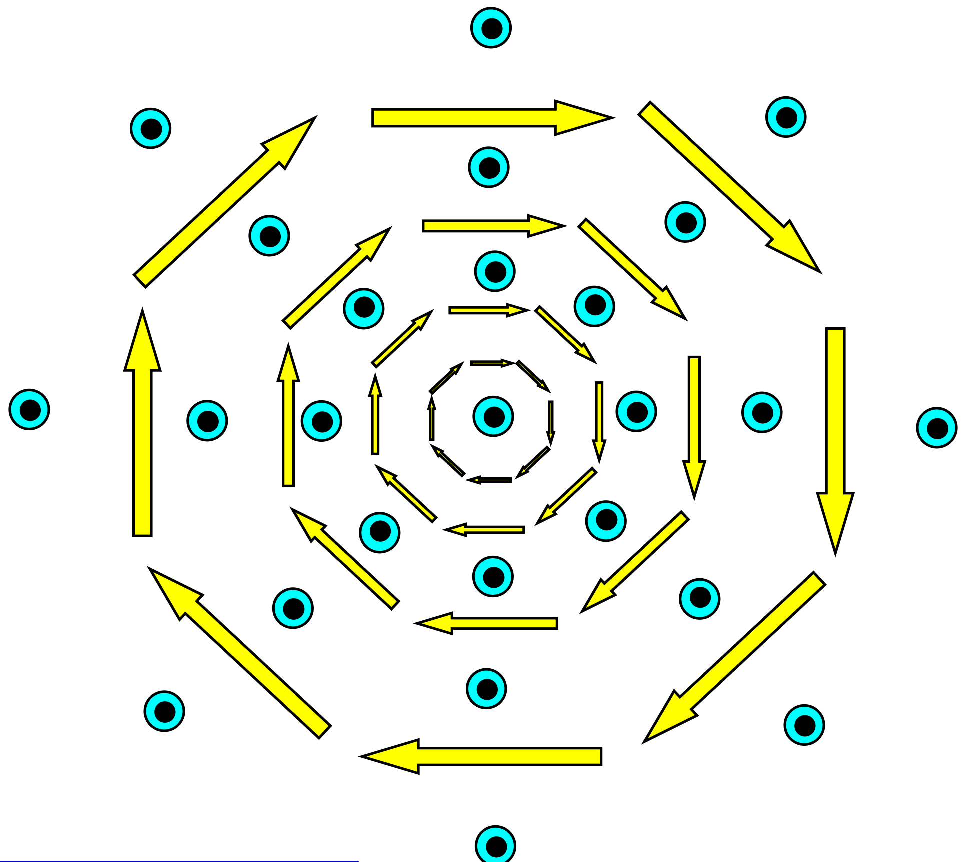
Assume a B_z field that <u>increases</u> with time:

$$\begin{cases} \underline{B} = \underline{\hat{z}} B_z(t) \\ \dfrac{dB_z}{dt} > 0 \end{cases}$$

$$\underline{\hat{z}} \cdot \left(\nabla \times \underline{E} \right) = -\frac{\partial B_z}{\partial t} < 0$$

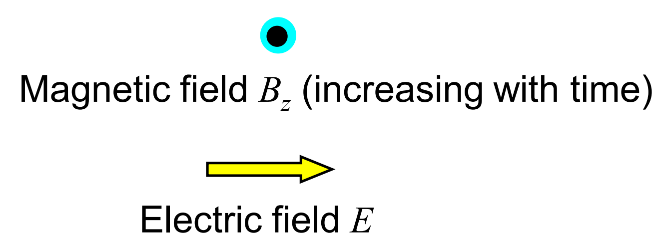
Faraday's Law: Dynamics (cont.)

A changing magnetic field produces a circulating electric field.



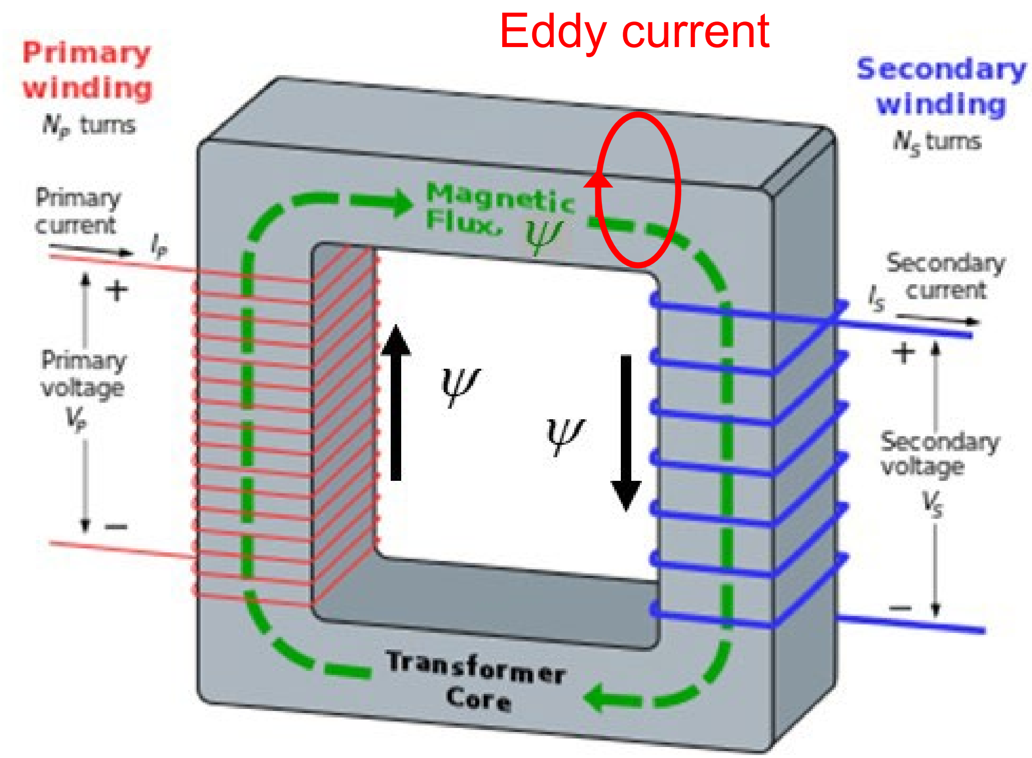
$$\underline{\hat{z}} \cdot \left(\nabla \times \underline{E} \right) < 0$$

Note:
The circulation of the electric field about the z axis is opposite to the right hand, due to the minus sign in Faraday's law.



Eddy Currents

Eddy currents are currents that flow inside a transformer core (or other conducting object) due to a changing magnetic field.



Eddy current cause power loss.

Eddy Currents (cont.)

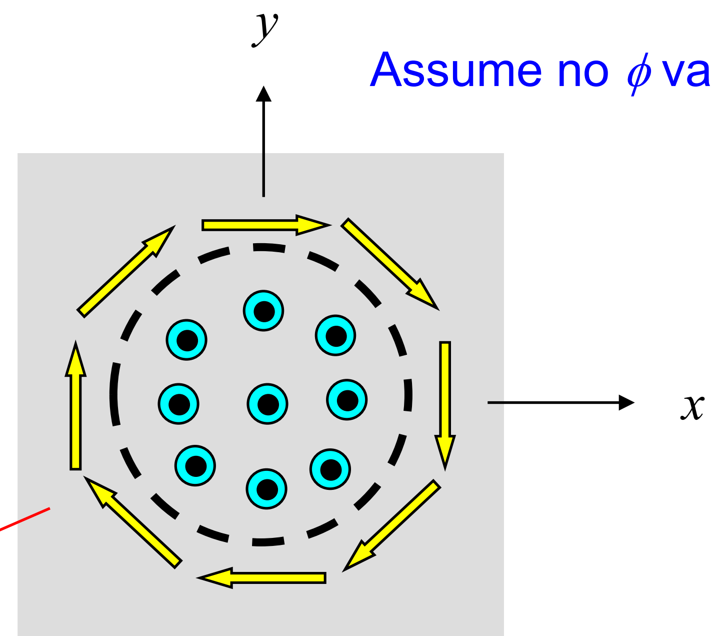
$$\underline{\hat{z}} \cdot \left(\nabla \times \underline{E} \right) = -\frac{\partial B_z}{\partial t} < 0$$

Cylindrical coordinates

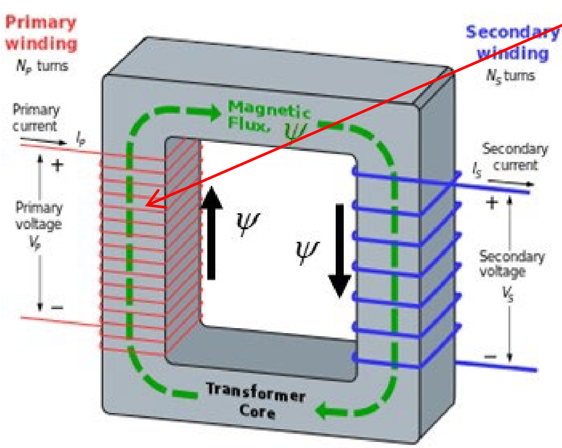
$$\frac{1}{\rho}\left(\frac{\partial\left(\rho E_\phi\right)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi} \right) = -\frac{\partial B_z}{\partial t}$$

Assume no ϕ variation

Assume no ϕ variation.



Transformer core



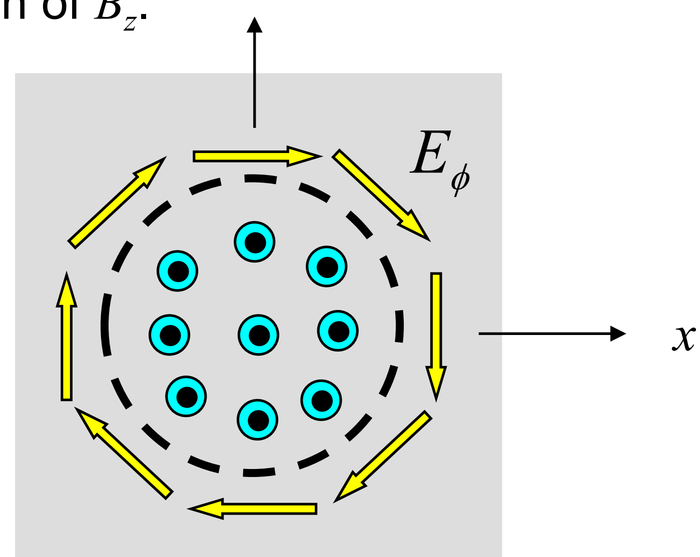
Magnetic field B_z (increasing with time)

Electric field $\underline{E}$

Eddy Currents (cont.)

$$\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial \rho}\right) = -\frac{\partial B_z}{\partial t}$$

Assume no ρ variation of B_z.



Transformer core

$$\frac{\partial E_\phi}{\partial \rho} + \frac{1}{\rho} E_\phi = -\frac{\partial B_z}{\partial t}$$

$$\frac{\partial E_\phi^p}{\partial \rho} + \frac{1}{\rho} E_\phi^p = -j\omega B_z^p$$ (phasor domain)

$B_z(t) = B_{z0}\cos(\omega t + \phi)$ (time domain)

$B_z^p = B_{z0}\, e^{j\phi}$ (phasor domain)

Assume:

$$E_\phi^p(\rho) = A\rho^\alpha$$

$$A\alpha\rho^{\alpha-1} + A\rho^{\alpha-1} = -j\omega B_z^p$$

$$A\rho^{\alpha-1}(\alpha+1) = -j\omega B_z^p$$

Solution:

$$\alpha = 1, \qquad A = -\frac{j\omega}{2} B_z^p$$

(no ρ variation)

Eddy Currents (cont.)

$$E_{\phi}^{p}(\rho) = -\left(\frac{j\omega B_z^p}{2}\right)\rho$$

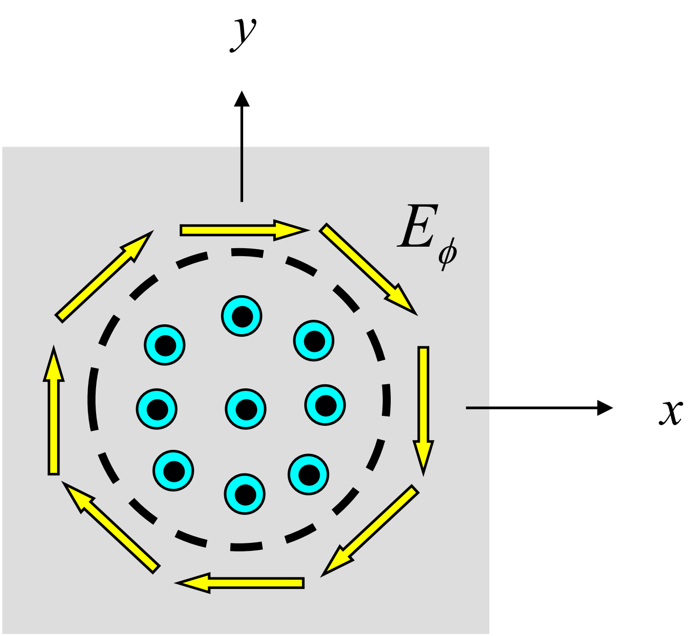
Also, we have Ohm's law:

$$\underline{J} = \sigma \underline{E}$$

Hence we have:

$$J_{\phi}^{p}(\rho) = -\left(\frac{j\omega\sigma B_z^p}{2}\right)\rho$$

Eddy currents*



Transformer core

$$B_z(t) = B_{z0}\cos(\omega t + \phi) \quad \text{(time domain)}$$

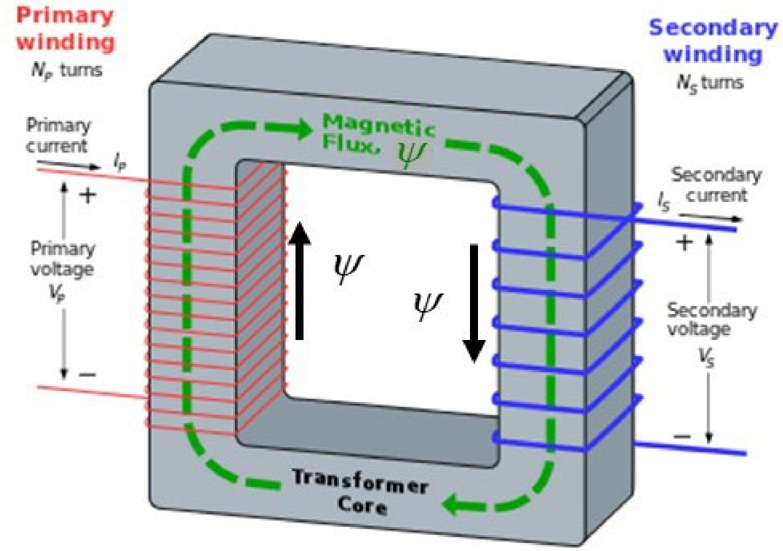
$$B_z^p = B_{z0}\, e^{j\phi} \quad \text{(phasor domain)}$$

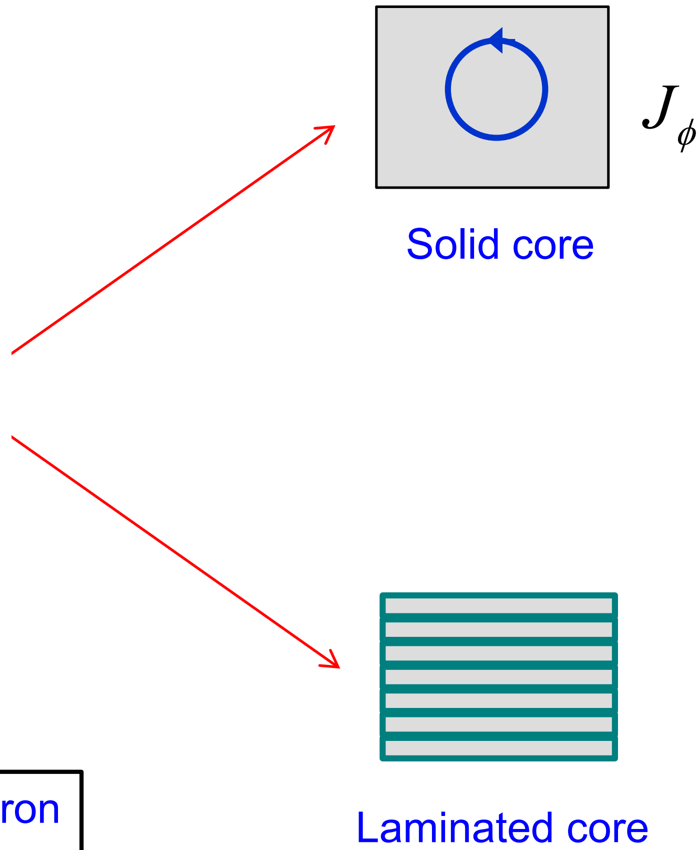
Note that the eddy currents increase with frequency!

*The currents look like eddies or whirlpools in water.

Eddy Currents (cont.)

Laminated cores are used to reduce eddy currents.





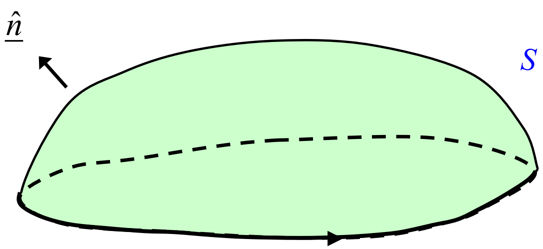
The laminated core consists of layers of iron material coated with electrical insulation.

Faraday's Law: Integral Form

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Integrate both sides over an arbitrary open surface (bowl) S:

$$\int_S \left(\nabla \times \underline{E}\right) \cdot \underline{\hat{n}}\, dS = \int_S \left(-\frac{\partial \underline{B}}{\partial t}\right) \cdot \underline{\hat{n}}\, dS$$



Apply Stokes's theorem for the LHS:

$$\oint_C \underline{E} \cdot d\underline{r} = \int_S \left(-\frac{\partial \underline{B}}{\partial t}\right) \cdot \underline{\hat{n}}\, dS$$

Note:
The right-hand rule determines the direction of the unit normal, from the direction along C.

Faraday's law in integral form

Faraday's Law: Integral Form (cont.)

$$\oint_C \underline{E} \cdot d\underline{r} = \int_S \left(-\frac{\partial \underline{B}}{\partial t} \right) \cdot \underline{\hat{n}}\, dS$$

Assume that the surface and the path are not changing with time:

$$\oint_C \underline{E} \cdot d\underline{r} = -\frac{d}{dt} \int_S \underline{B} \cdot \underline{\hat{n}}\, dS$$

Define magnetic flux through the surface S:

$$\psi \equiv \int_S \underline{B} \cdot \underline{\hat{n}}\, dS$$

We then have

$$\oint_C \underline{E} \cdot d\underline{r} = -\frac{d\psi}{dt}$$

Note:
The right-hand rule determines the direction of the unit normal in the flux calculation, from the direction along C.

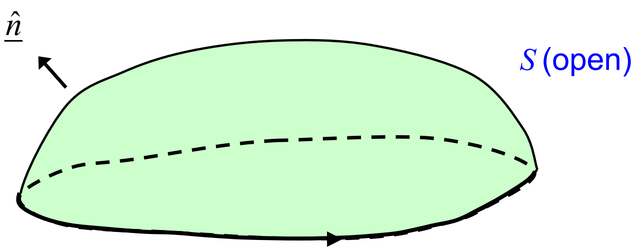
Faraday's Law: Summary

Summary

$$\oint_C \underline{E} \cdot d\underline{r} = -\frac{d\psi}{dt}$$

where

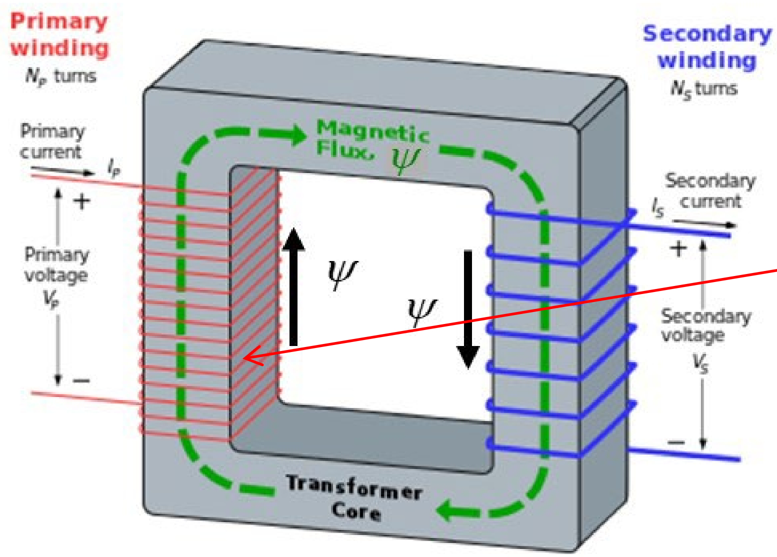
$$\psi \equiv \int_S \underline{B} \cdot \underline{\hat{n}}\, dS$$

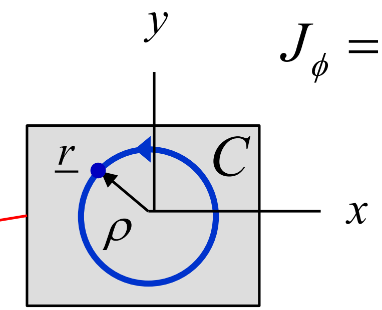


The voltage drop around a closed path is equal to the rate of change of magnetic flux through the path.

Note: The closed path can be anything: in free space, inside of a metal wire, etc.

Eddy Currents (Revisited)





$$J_\phi = \sigma E_\phi$$

Solid core

$B_z(t) = B_{z0}\cos(\omega t + \phi)$ (time domain)

$B_z^p = B_{z0}\, e^{j\phi}$ (phasor domain)

Integral form of Faraday's law:

$$\oint_C \underline{E} \cdot d\underline{r} = -\frac{d\psi}{dt} = -\frac{d}{dt}\left(\pi\rho^2 B_z\right)$$

$$\oint_C \underline{E}^p \cdot d\underline{r} = -j\omega\pi\rho^2 B_z^p$$

$$E_\phi^p\left(2\pi\rho\right) = -j\omega\pi\rho^2 B_z^p$$

$\underline{J} = \sigma \underline{E}$

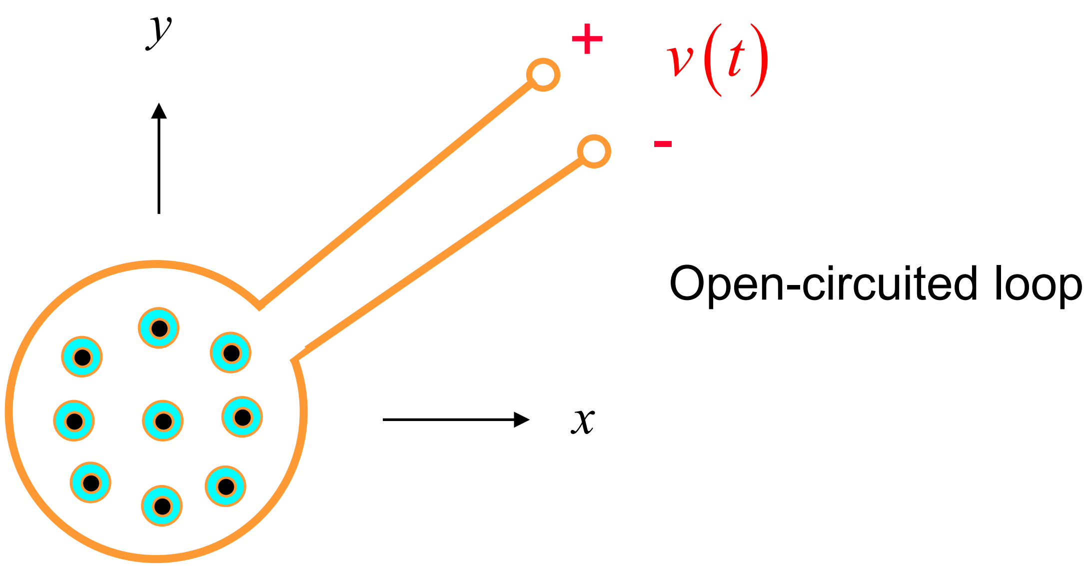
$$J_\phi^p = -j\left(\frac{\sigma\omega\rho}{2}\right)B_z^p$$

This is the same result we obtained before, using the differential form of Faraday's law.

Faraday's Law for a Loop

We measure a voltage across a loop due to a changing magnetic field inside the loop.

(This is the basis for how AC generators and transformers work.)

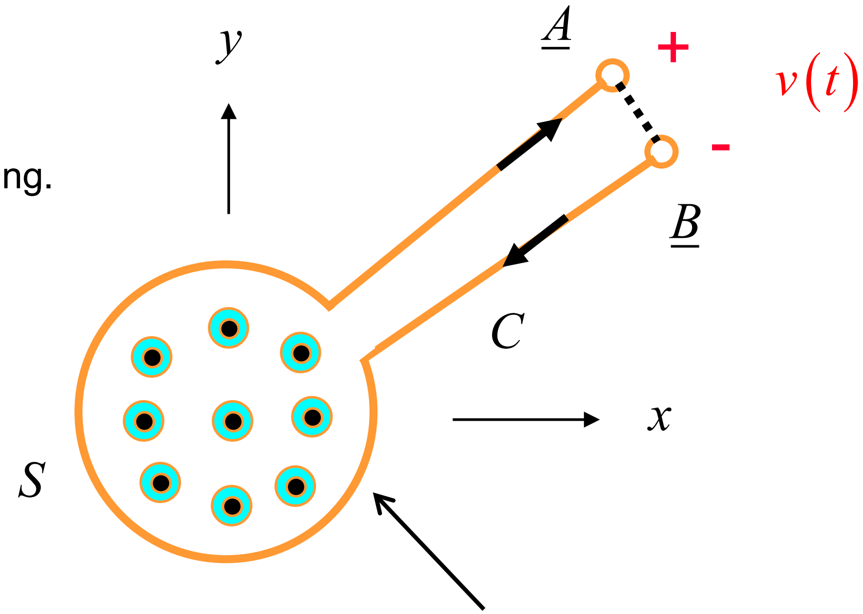


Magnetic field $\underline{B}$ (B_z is <u>changing</u> with time)

Faraday's Law for a Loop (cont.)

$$v = v_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot d\underline{r} = \oint_C \underline{E} \cdot d\underline{r}$$

Note: A lower case v denotes that it is time-varying.



Also

$$\oint_C \underline{E} \cdot d\underline{r} = -\frac{d\psi}{dt}$$

$$(\underline{\hat{n}} = -\underline{\hat{z}})$$

Note:
The voltage drop along the PEC wire (from $\underline{B}$ to $\underline{A}$ inside the wire) is zero.

So we have

$$v = -\frac{d\psi}{dt} \qquad \psi = \int_S (-B_z)\, dS = -\psi_z$$

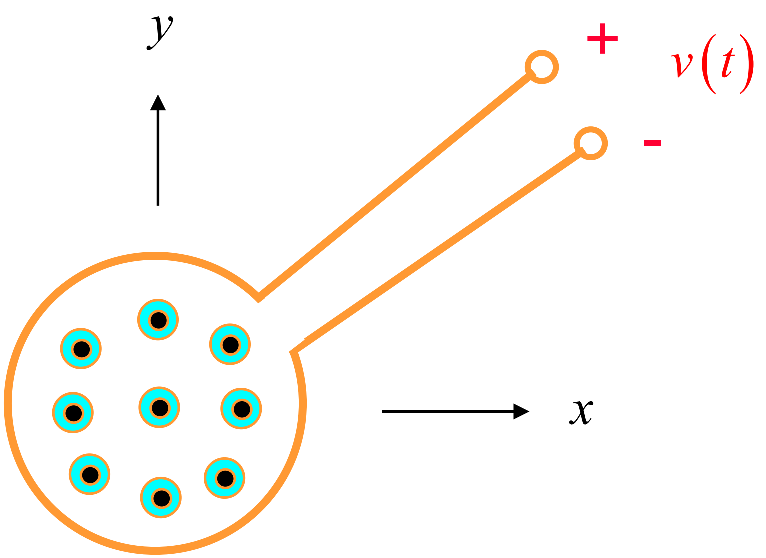
$$\Rightarrow \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot d\underline{r} = \oint_C \underline{E} \cdot d\underline{r}$$

Faraday's Law for a Loop (cont.)

Hence:

$$v(t) = \frac{d\psi_z}{dt}$$

$$\psi_z \equiv \int_S B_z \, dS$$



Faraday's Law for a Loop (cont.)

Assume a uniform magnetic field for simplicity.

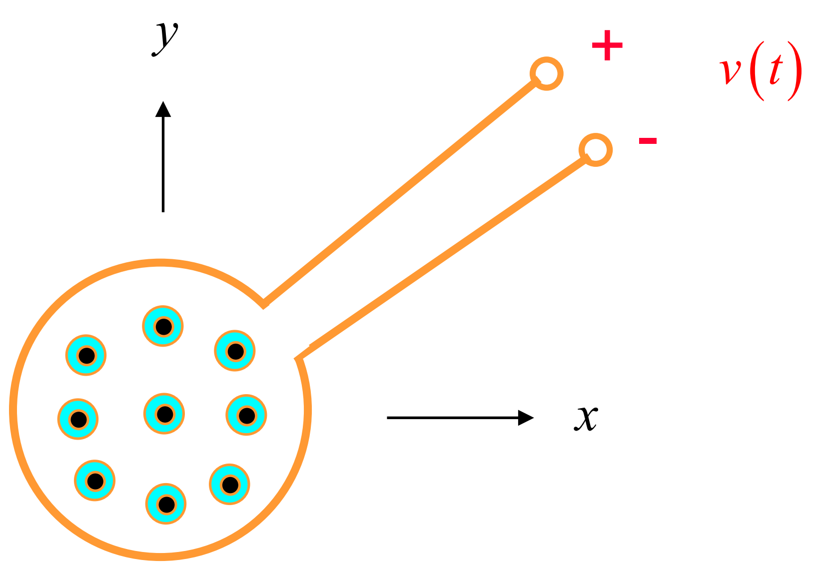
(At least it is uniform over the loop area.)

Then we have:

$$\psi_z = B_z A$$

so

$$v(t) = A\frac{dB_z}{dt}$$



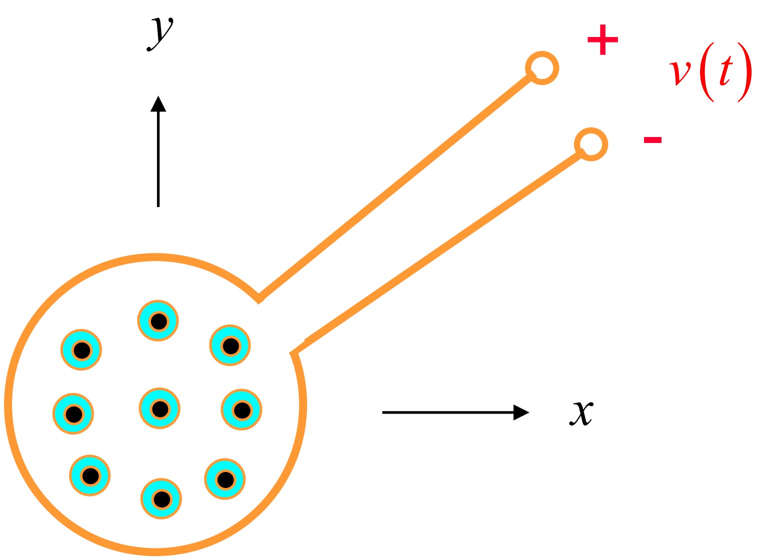
A = area of loop

Faraday's Law for a Loop (cont.)

Summary

$$v(t) = \frac{d\psi_z}{dt}$$ General form

$$v(t) = A\frac{dB_z}{dt}$$ Uniform field



A = area of loop

$$\psi_z \equiv \int_S B_z dS$$

ψ_z = magnetic flux through loop in z direction

Lenz's Law

This is a simple rule to tell us the polarity of the output voltage (without having to do any calculation).

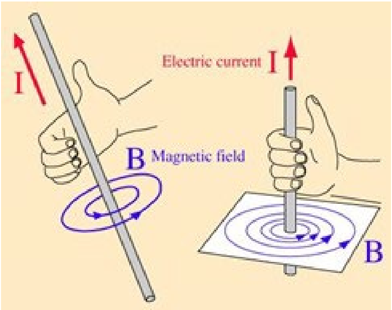
We visualize a high-impedance resistor R added to the circuit:

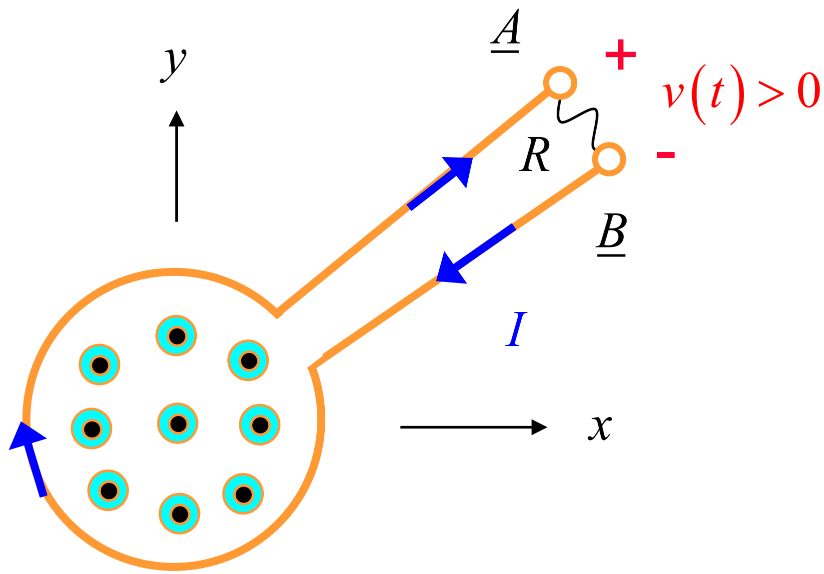
The output voltage polarity corresponds to a current flow that opposes the change in the flux in the loop.

In this example, a clockwise current is set up, since this opposes the change in flux through the loop. The clockwise current then corresponds to the output voltage polarity shown.

Note:
A right-hand rule tells us the direction of the magnetic field due to a wire carrying a current.

(A wire carrying a current in the z direction produces a magnetic field in the positive ϕ direction.)





A = area of loop

B_z is increasing with time.

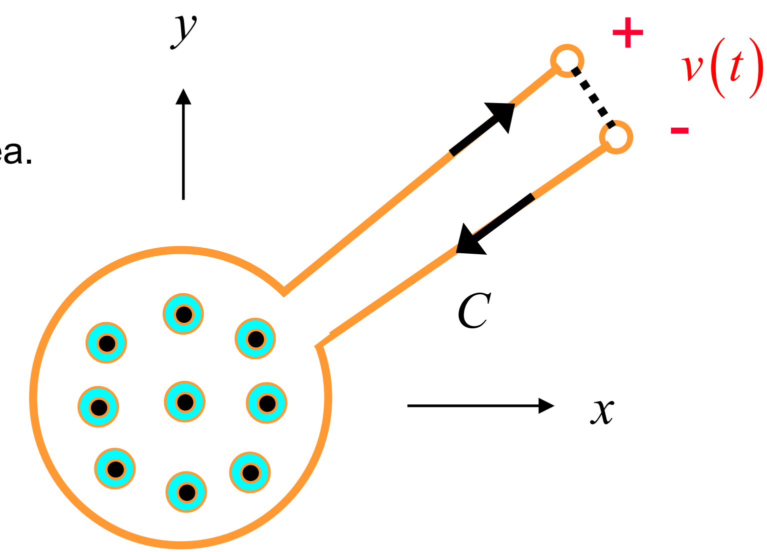
$$v(t) = A\frac{dB_z}{dt} > 0$$

Example: Magnetic Field Probe

A small loop can be used to measure the magnetic field (for AC).

$$v(t) = A\frac{\partial B_z}{\partial t}$$

Note: The magnetic field is assume constant over the loop area.



Assume

$$B_z = B_0 \cos(\omega t + \phi)$$

$$\omega = 2\pi f$$

Then we have

$$v(t) = -A\omega B_0 \sin(\omega t + \phi)$$

A = area of loop

$$A = \pi a^2$$

At a given frequency, the output voltage is proportional to the strength of the magnetic field.

Applications of Faraday's Law

Faraday's law explains:

- How AC generators work
- How transformers work

$$v(t) = \frac{d\psi_z}{dt}$$

→ Output voltage of generator

→ Output voltage on secondary of transformer

Note: For N turns in a loop we have $$v(t) = N\frac{d\psi_z}{dt}$$

The world's first electric generator!

(invented by Michael Faraday)

A magnet is slid in and out of the coil, resulting in a voltage output.

(Faraday Museum, London)

The world's first transformer!

(invented by Michael Faraday)

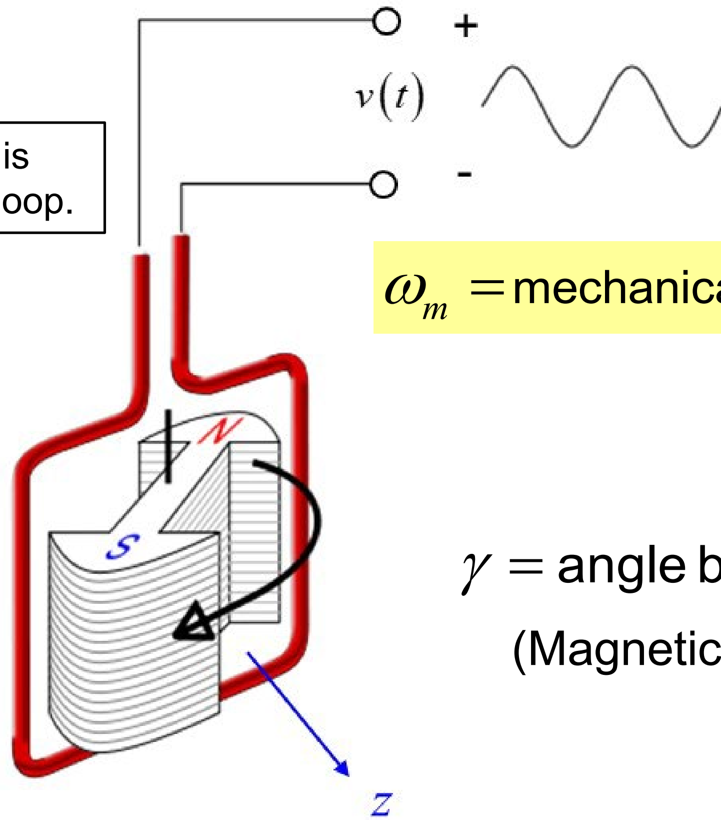
The primary and secondary coils are wound together on an iron core.

(Faraday Museum, London)

AC Generators

Diagram of a simple alternator (AC generator) with a rotating magnetic core (rotor) and stationary wire (stator), also showing the output voltage induced in the stator by the rotating magnetic field of the rotor.

Assume that B_z is constant over the loop.



Open-circuit coil

ω_m = mechanical angular velocity of magnet

$$B_z = B_0 \cos \gamma$$

γ = angle between north pole and the z axis

(Magnetic flux comes out of the north pole.)

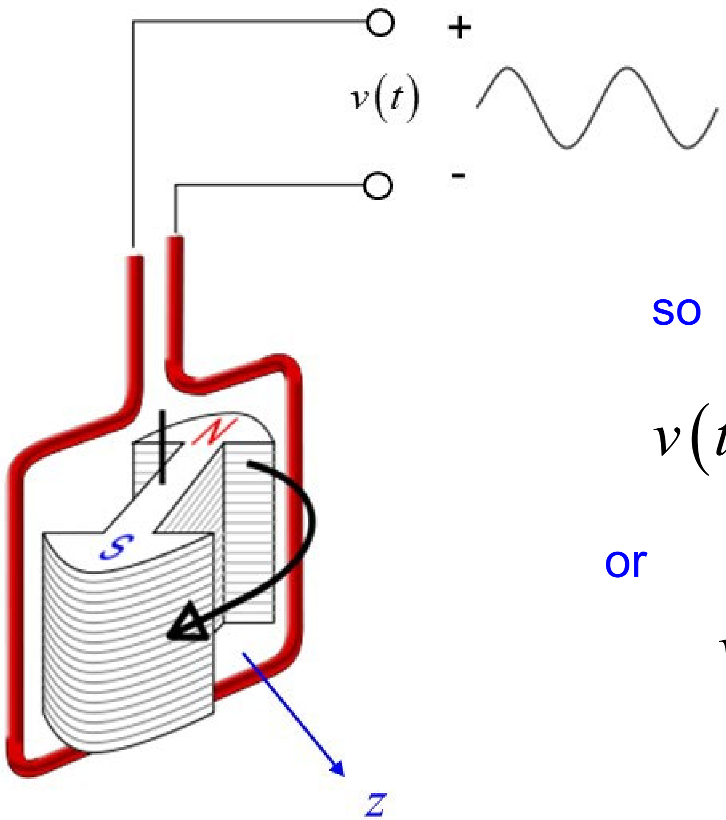
$$\gamma = \omega_m t + \gamma_0$$

A = area of loop

http://en.wikipedia.org/wiki/Alternator

AC Generators (cont.)

Open-circuit coil



A = area of loop

$$B_z = B_0 \cos(\omega_m t + \gamma_0)$$

$$v(t) = NA \frac{dB_z}{dt}$$

so

$$v(t) = (-NAB_0\omega_m)\sin(\omega_m t + \gamma_0)$$

or

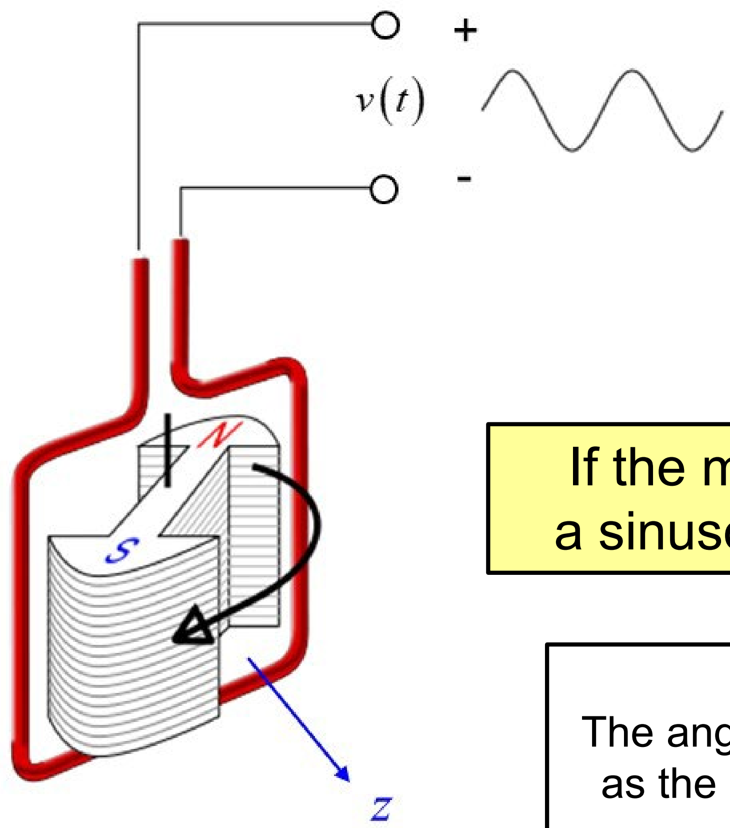
$$v(t) = V_0 \cos(\omega t + \phi)$$

$V^p = V_0 e^{j\phi}$ (phasor domain)

$$(\omega \equiv \omega_m, V_0 \equiv NAB_0\omega, \quad \phi \equiv \gamma_0 + \pi/2)$$

AC Generators (cont.)

Open-circuit coil



A = area of loop

Summary

$$v(t) = V_0 \cos(\omega t + \phi)$$

$$\omega = \omega_m$$

If the magnet rotates at a fixed speed,
a sinusoidal voltage output is produced.

Note:
The angular velocity of the magnet is the same as the radian frequency of the output voltage (for a simple two-pole magnet).

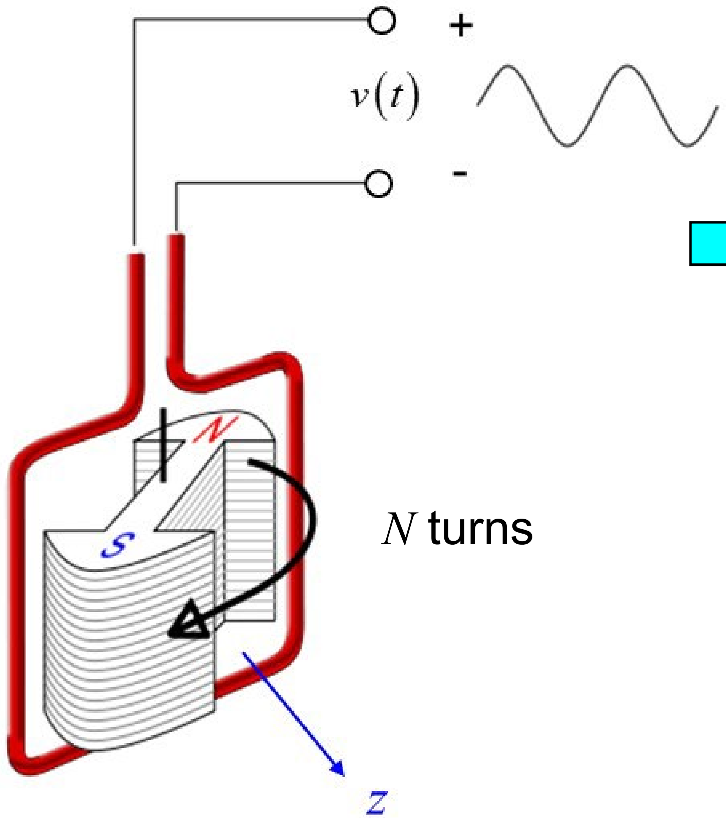
AC Generators (cont.)

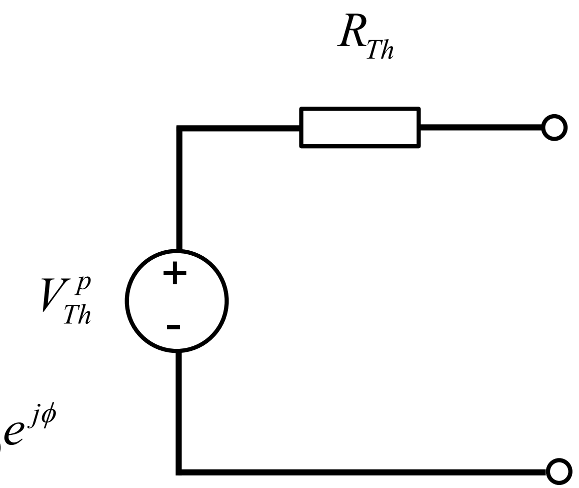
Thévenin Equivalent Circuit

$$v(t) = V_0 \cos(\omega t + \phi)$$

$$V_0 \equiv NAB_0\omega, \quad \phi \equiv \gamma_0 + \pi/2$$

Open-circuit coil





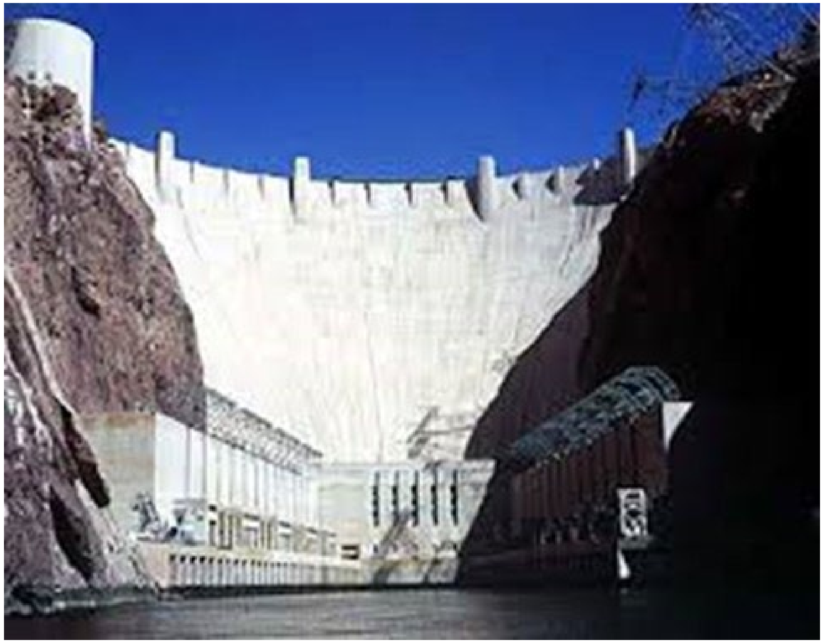
$$V_{Th}^p = V_0 e^{j\phi}$$

A = area of loop

$$V_{Th}^p = V_0\, e^{j\phi}$$

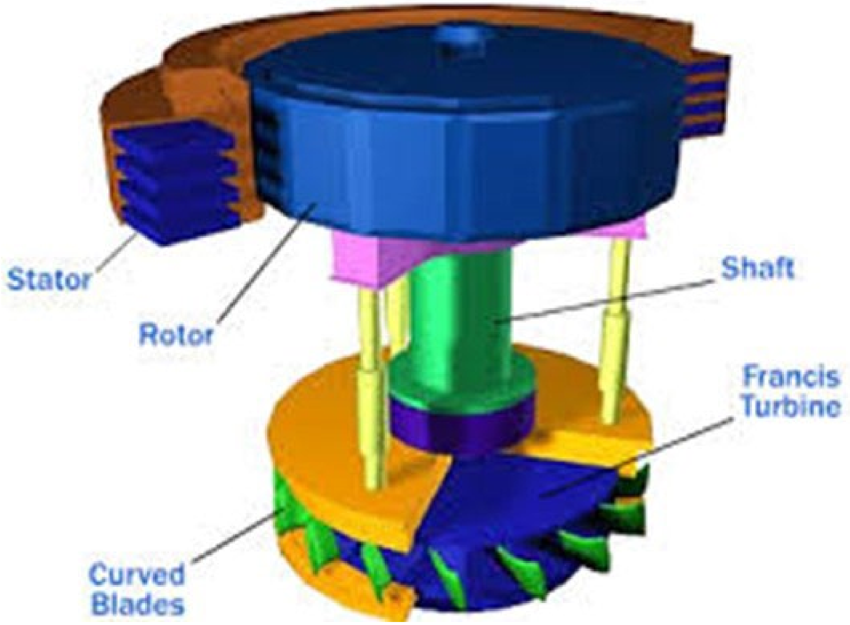
R_{Th} = resistance of wire in coil

AC Generators (cont.)



Generators at Hoover Dam

AC Generators (cont.)



Generators at Hoover Dam

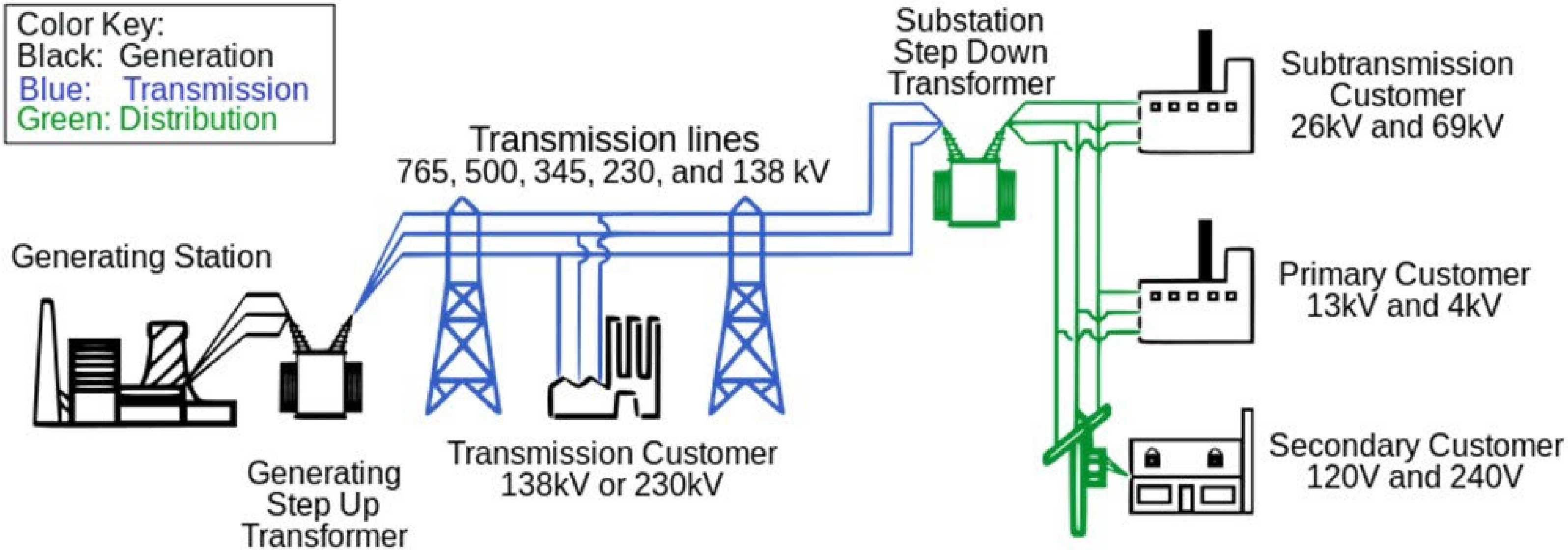
Transformers

A transformer changes an AC signal from one voltage to another.

http://en.wikipedia.org/wiki/Transformer

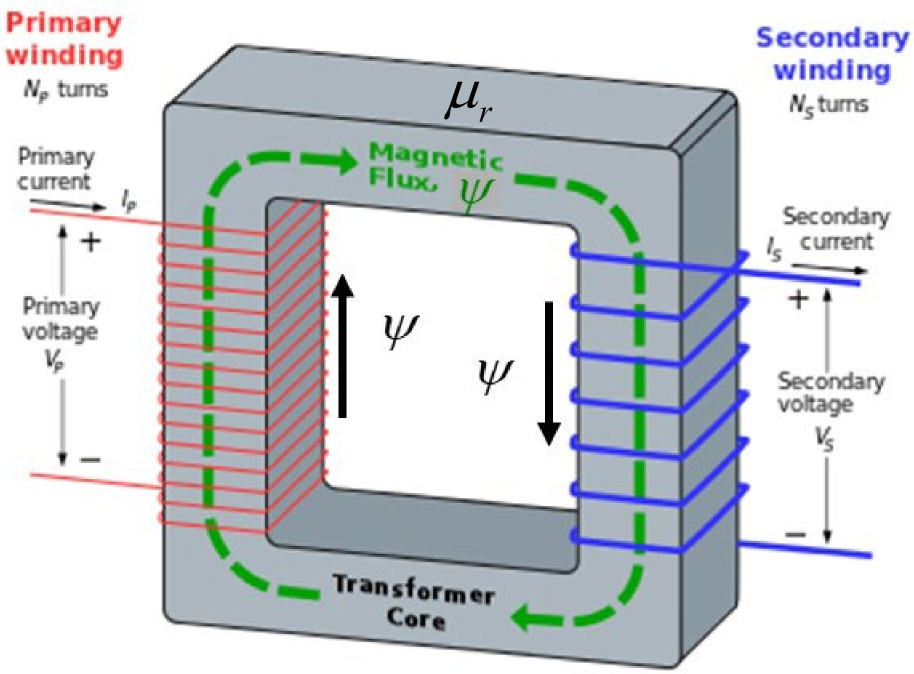
Transformers (cont.)

- High voltages are used for transmitting power over long distances (less current means less conductor loss).
- Low voltages are used inside homes for convenience and safety.



http://en.wikipedia.org/wiki/Electric_power_transmission

Transformers (cont.)



$$v_p(t) = N_p \frac{d\psi}{dt}$$

Note:
The sign is correct from Lens' law.

$$v_s(t) = N_s \frac{d\psi}{dt}$$

Note:
The sign is correct from Lens' law.

Ideal transformer (no flux leakage): $\mu_r \to \infty$

Hence

$$\frac{v_s(t)}{v_p(t)} = \frac{N_s}{N_p}$$

(time domain)

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

(phasor domain)

http://en.wikipedia.org/wiki/Transformer

Transformers (cont.)

Ideal transformer (no losses):

$$v_p(t)\,i_p(t) = v_s(t)\,i_s(t) \quad \text{(power in = power out)}$$

Hence

$$\frac{i_s(t)}{i_p(t)} = \frac{v_p(t)}{v_s(t)} = \left(\frac{v_s(t)}{v_p(t)}\right)^{-1}$$

so

$$\frac{i_s(t)}{i_p(t)} = \left(\frac{N_s}{N_p}\right)^{-1}$$

(time domain)

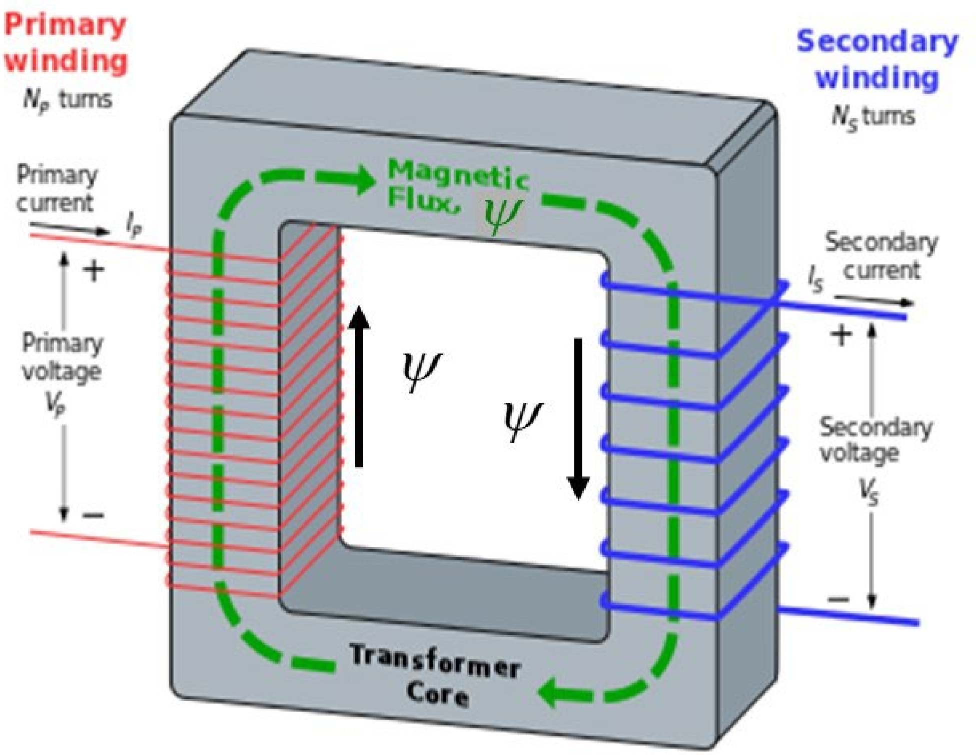
$$\frac{I_s}{I_p} = \left(\frac{N_s}{N_p}\right)^{-1}$$

(phasor domain)

Transformers (cont.)

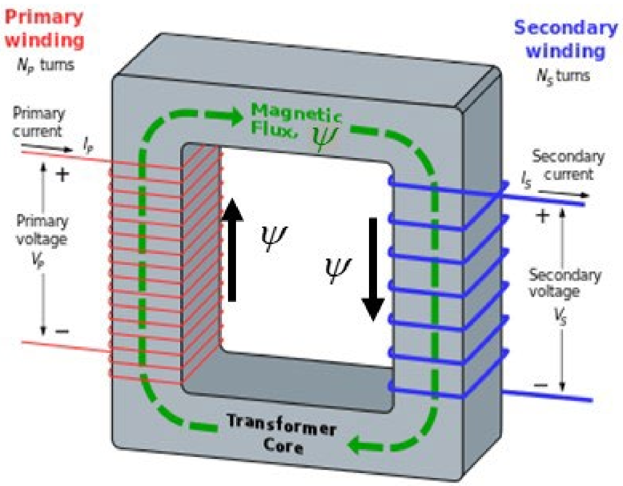
Impedance transformation (phasor domain):

$$Z_{in} \equiv \frac{V_p}{I_p} \quad , \quad Z_{out} \equiv \frac{V_s}{I_s}$$



Transformers (cont.)

$$Z_{in} \equiv \frac{V_p}{I_p} \quad , \quad Z_{out} \equiv \frac{V_s}{I_s}$$



Hence

$$\frac{Z_{in}}{Z_{out}} = \left(\frac{V_p}{I_p}\right)\frac{I_s}{V_s} = \left(\frac{V_p}{I_p}\right)\frac{I_p\left(\frac{N_s}{N_p}\right)^{-1}}{V_p\left(\frac{N_s}{N_p}\right)} = \frac{\left(\frac{N_s}{N_p}\right)^{-1}}{\left(\frac{N_s}{N_p}\right)} = \left(\frac{N_p}{N_s}\right)^2$$

so

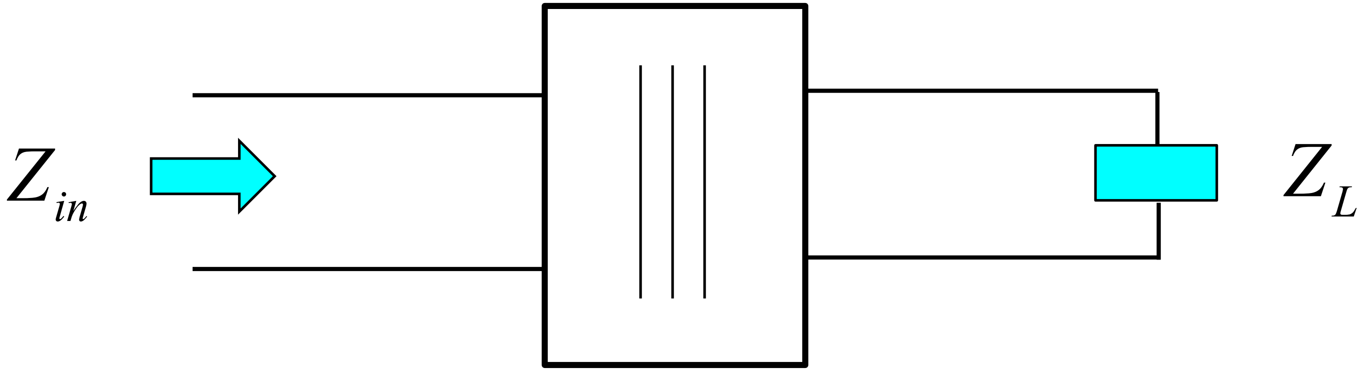
$$\frac{Z_{in}}{Z_{out}} = \left(\frac{N_p}{N_s}\right)^2$$

Transformers (cont.)

Impedance transformation

$$Z_{in} = \left(\frac{N_p}{N_s}\right)^2 Z_L$$

$N:1$ $\left(\text{This notation means } \frac{N_p}{N_s} = N\right)$

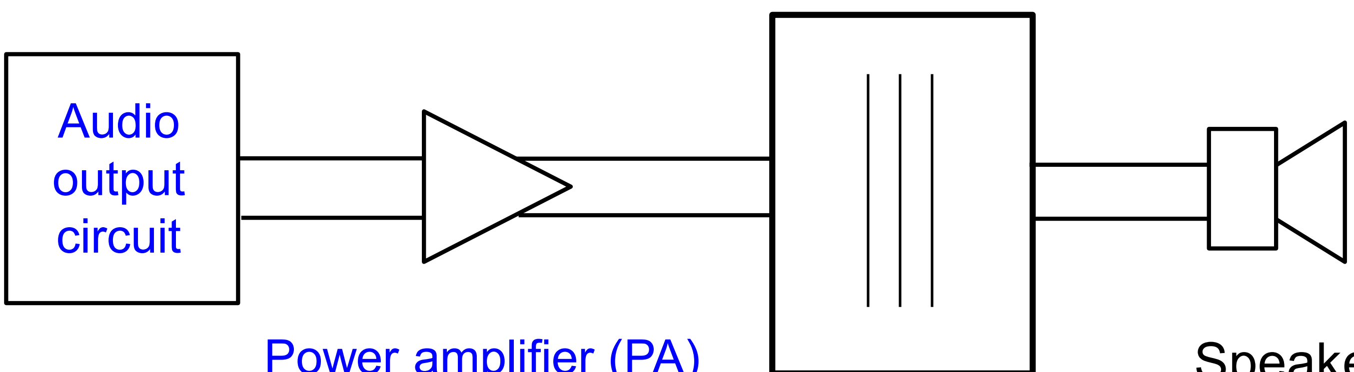


Transformers (cont.)

Example: Audio matching circuit

The PA should see a matched load (50 [Ω]) for maximum power transfer to the load (speaker).

$$N = \sqrt{\frac{50}{8}}$$

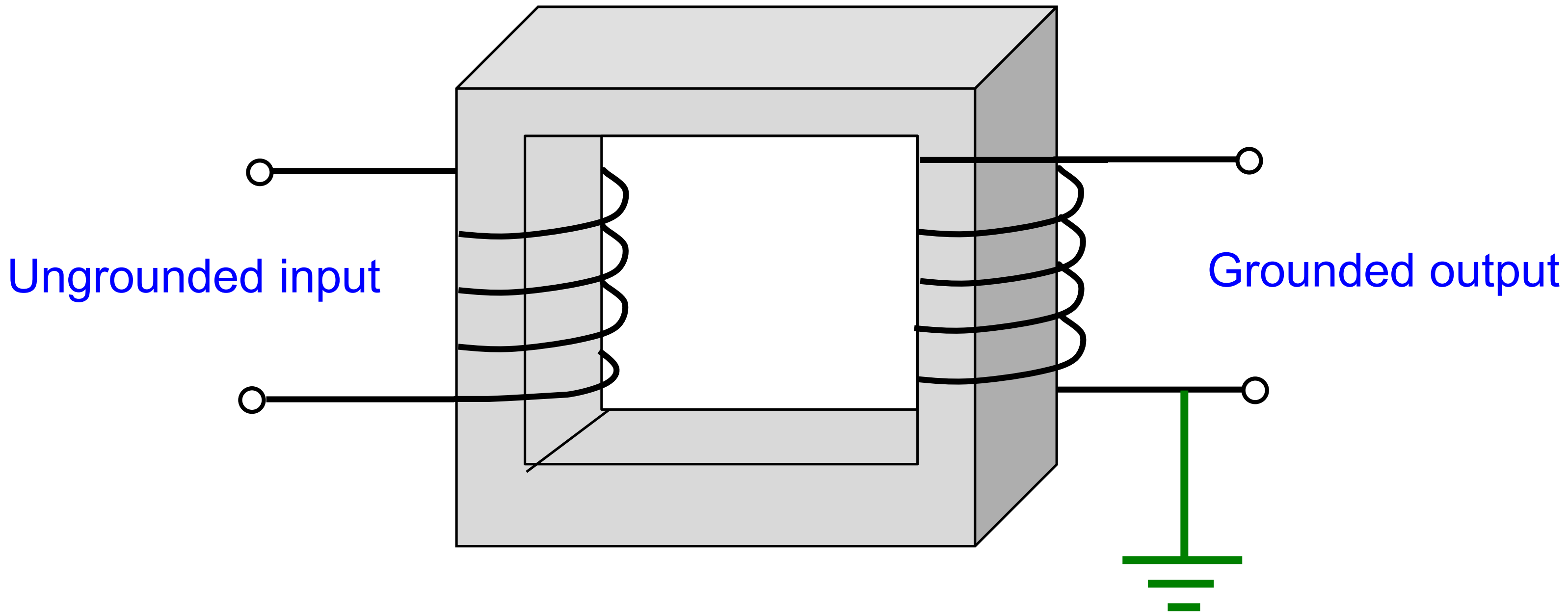


Transformers (cont.)

Isolation transformer

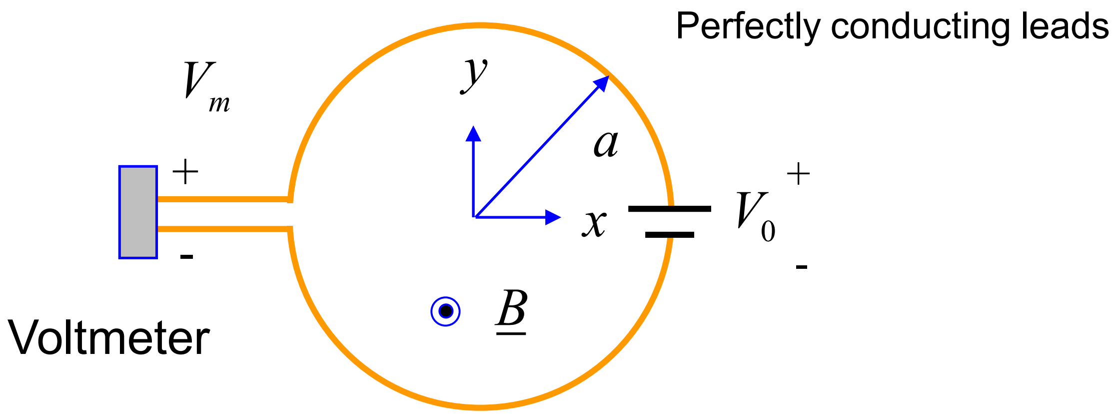
An isolation transformer is used isolate the input and output circuits (no direct electrical connection between them). It can also be used to connect a grounded circuit to an ungrounded one.

The transformer is being used as a form of "balun", which connects a "balanced circuit" (the two leads are at a symmetric +/- voltage with respect to ground) to an "unbalanced circuit" (where one lead is grounded).



Measurement Error from Magnetic Field

Find the voltage readout on the voltmeter.



Applied magnetic field: $\underline{B}(t) = \underline{\hat{z}}\cos(\omega t)$

Note:
The voltmeter is assumed to have a very high internal resistance, so that negligible current flows in the circuit. (We can neglect any magnetic field coming from the current flowing in the loop.)

Measurement Error from Magnetic Field (cont.)

Faraday's law:

$$\oint_C \underline{E} \cdot \underline{dr} = -\frac{d\psi}{dt}$$

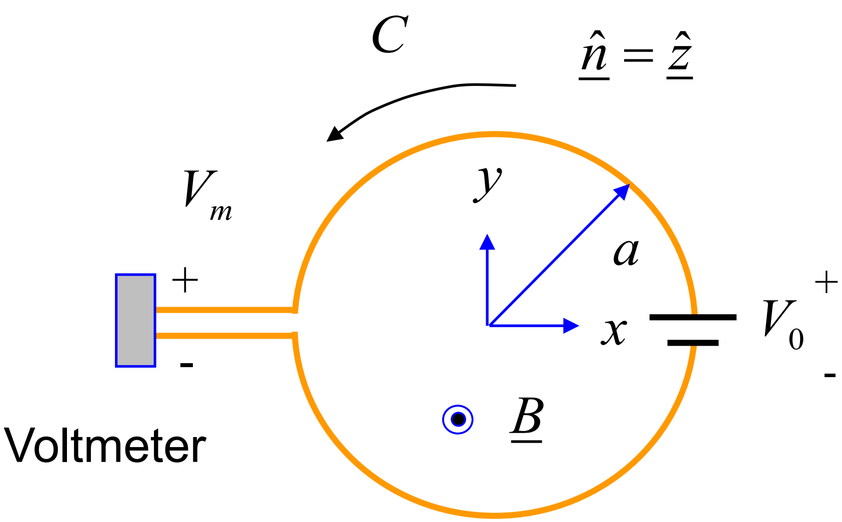
$$= -\frac{d}{dt}\int_S \underline{B} \cdot \underline{\hat{n}}\, dS$$

$$= -\frac{d}{dt}\int_S B_z\, dS$$

$$= -\frac{d}{dt}\left(B_z \pi a^2\right)$$

$$= -\frac{dB_z}{dt}\left(\pi a^2\right)$$

$$= \omega \sin \omega t \left(\pi a^2\right)$$



Perfectly conducting leads

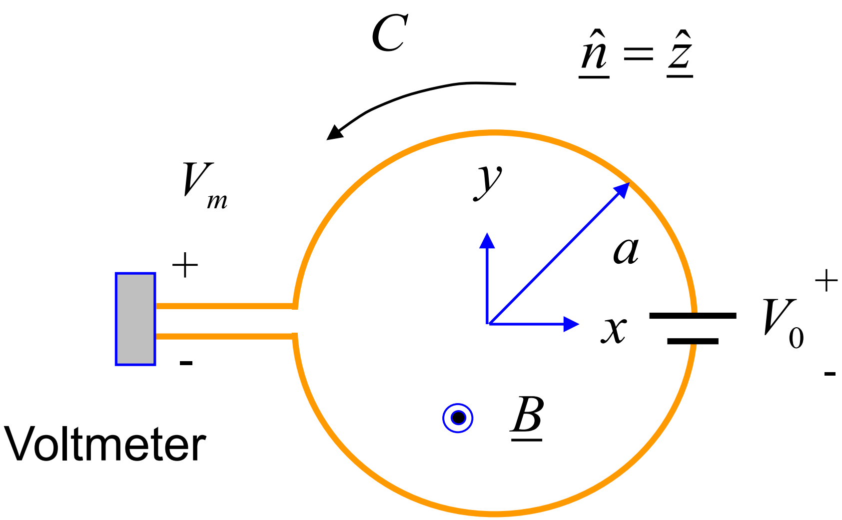
$$\underline{B}(t) = \underline{\hat{z}} \cos(\omega t)$$

$$\frac{dB_z}{dt} = -\omega \sin(\omega t)$$

Measurement Error from Magnetic Field (cont.)

From the last slide,

$$\oint_C \underline{E} \cdot \underline{dr} = \omega \sin \omega t \left(\pi a^2 \right)$$



Perfectly conducting leads

Therefore

$$V_m - V_0 = \omega \pi a^2 \sin\left(\omega t \right)$$

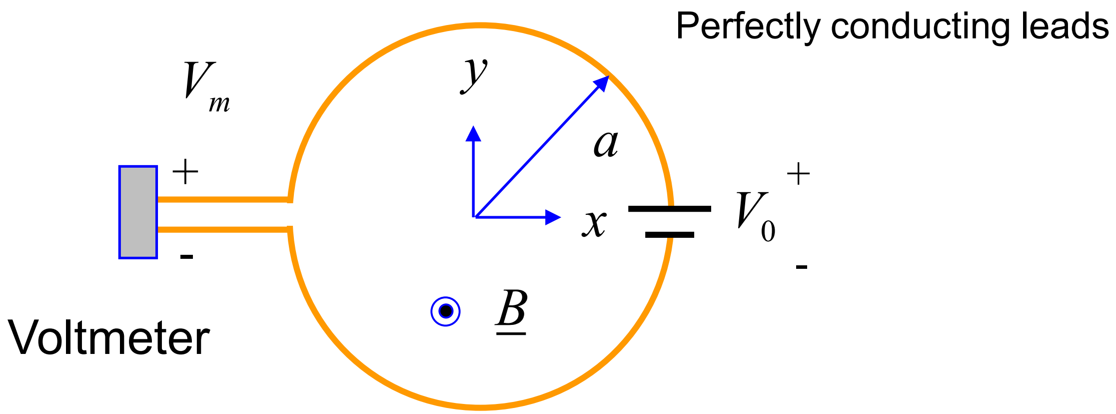
or

$$V_m = V_0 + \omega \pi a^2 \sin\left(\omega t \right)$$

Note:
There is no voltage drop along the perfectly conducting leads.

Measurement Error from Magnetic Field (cont.)

Summary



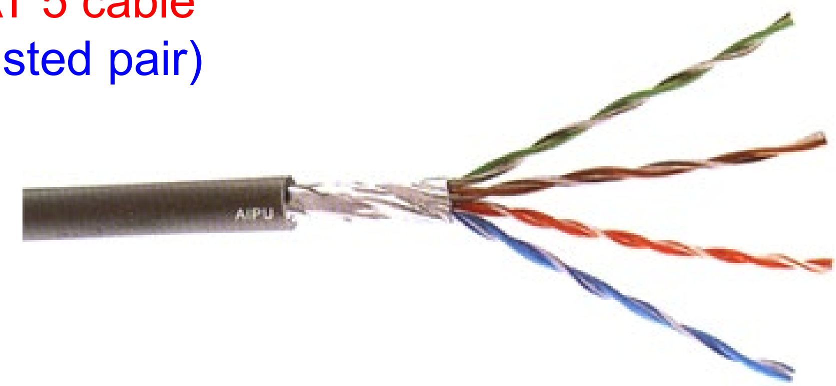
$$V_m = V_0 + \omega \pi a^2 \sin(\omega t)$$

Practical note:
In such a measurement, it is good to keep the leads close together (or even better, twist them.)

Twisted Pair Transmission Line

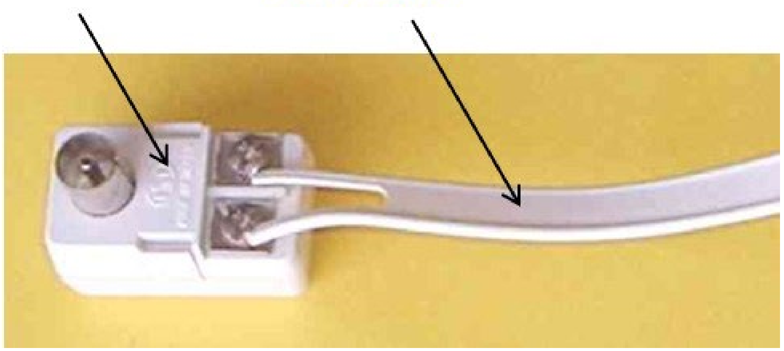
Twisted pair is used to reduced interference pickup in a transmission line, compared to “twin lead” transmission line.

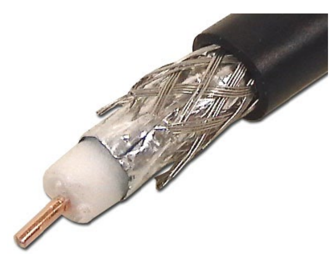
CAT 5 cable
(twisted pair)



Balun

Twin lead





Coax

Note:
Coaxial cable is perfectly shielded and has no interference.

Maxwell's Equations (Differential Form)

$$\nabla \cdot \underline{D} = \rho_v$$

Electric Gauss law

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday's law

$$\nabla \cdot \underline{B} = 0$$

Magnetic Gauss law

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$

Ampere's law

$$\underline{D} = \varepsilon_0 \underline{E} \qquad \underline{B} = \mu_0 \underline{H}$$

Constitutive equations

Maxwell's Equations (Integral Form)

$$\oint_S \underline{D} \cdot \underline{\hat{n}}\, dS = Q_{encl}$$

Electric Gauss law

$$\oint_C \underline{E} \cdot d\underline{r} = \int_S -\frac{\partial \underline{B}}{\partial t} \cdot \underline{\hat{n}}\, dS$$

Faraday's law

$$\oint_S \underline{B} \cdot \underline{\hat{n}}\, dS = 0$$

Magnetic Gauss law

$$\oint_C \underline{H} \cdot d\underline{r} = i_S + \int_S \frac{\partial \underline{D}}{\partial t} \cdot \underline{\hat{n}}\, dS$$

Ampere's law

$$i_S = \int_S \underline{J} \cdot \underline{\hat{n}}\, dS \quad (\text{current through } S)$$

Maxwell's Equations (Statics)

In <u>statics</u>, Maxwell's equations decouple into two <u>independent</u> sets.

$$\nabla \cdot \underline{D} = \rho_v$$

$$\nabla \times \underline{E} = -\cancel{\frac{\partial \underline{B}}{\partial t}}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{H} = \underline{J} + \cancel{\frac{\partial \underline{D}}{\partial t}}$$

$$\nabla \cdot \underline{D} = \rho_v$$

$$\nabla \times \underline{E} = \underline{0}$$

Electrostatics

$$\rho_v \rightarrow \underline{E}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{H} = \underline{J}$$

Magnetostatics

$$\underline{J} \rightarrow \underline{B}$$

Maxwell's Equations (Dynamics)

In dynamics, the electric and magnetic fields are coupled together.

Each one, changing with time, produces the other one.

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t}$$

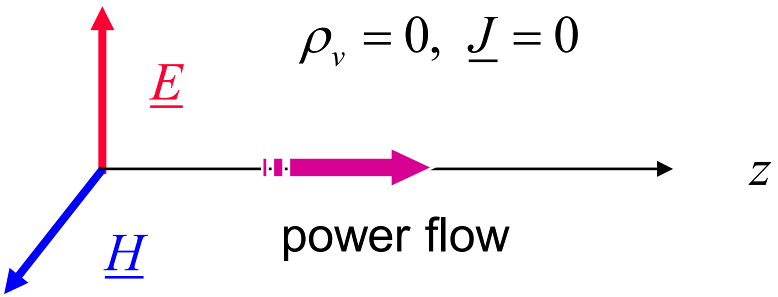
$$\underline{D} = \varepsilon_0 \underline{E}$$

$$\underline{B} = \mu_0 \underline{H}$$

Example:

A plane wave propagating through free space

$$\rho_v = 0, \ \underline{J} = 0$$



From ECE 3317:

$$\underline{E} = \underline{\hat{x}} \cos\left(\omega t - kz\right)$$

$$\underline{H} = \underline{\hat{y}} \left(\frac{1}{\eta_0}\right) \cos\left(\omega t - kz\right)$$

$$k = \omega\sqrt{\mu_0 \varepsilon_0}$$

$$\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$$