ECE 3318
Applied Electricity and Magnetism

Spring 2023

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Notes 2
Charge

Notes prepared by the EM Group
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Statics

Definition: No time variation. In terms of frequency, $f = 0$ [Hz]

The electromagnetic field splits into two independent parts:

Electrostatics: $(q, \underline{E})$ *charge produces electric field*

Magnetostatics: $(I, \underline{B})$ *current produces magnetic field*

The static approximation is usually accurate for $d << \lambda_0$
(d is the dimension of the circuit or device).

$$\lambda_0 = c / f \quad \text{(free - space wavelength)}$$

Example:
The electric field from a 60 [Hz] power line mainly comes from the charge on it.
The magnetic field from a 60 [Hz] power line mainly comes from the current on it.

Statics (cont.)

Example: $f = 60$ [Hz]

Note: This is an exact (defined) value since 1983.

$$\lambda_0 = c / f$$

$$c = 2.99792458 \times 10^8 \; [\text{m/s}]$$

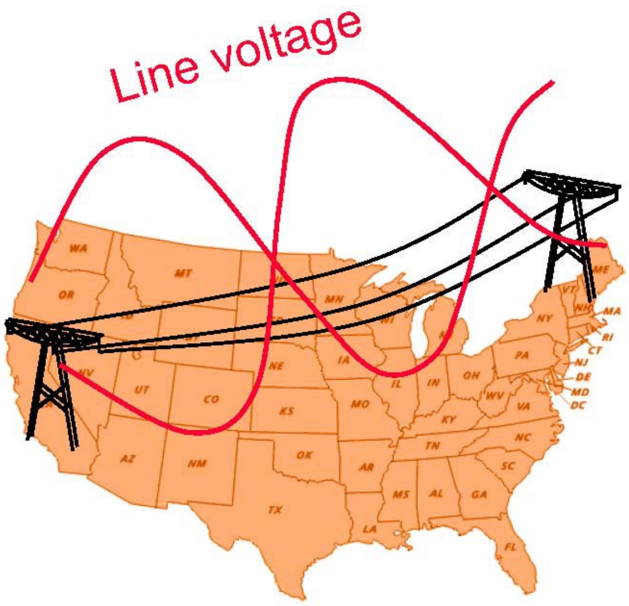
$$f = 60 \; [\text{Hz}]$$

This gives:

$$\lambda_0 = 4.9965 \times 10^6 \; [\text{m}]$$

$$= 4{,}996.5 \; [\text{km}]$$

$$= 3.097.8 \; [\text{miles}]$$



Clearly, most circuits fall into the static-approximation category at 60 [Hz]!

Statics (cont.)

The following are special cases of electromagnetics at low frequency:

- Circuit theory (e.g., ECE 2201, ECE 2202)
- Electronics
- Power engineering
- Magnetics (design of motors, generators, transformers, etc.)

Examples of high-frequency systems that are <u>not</u> modeled by statics:

- Antennas
- Transmission lines
- Microwaves
- Optics

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Statics (cont.)

Examples of low-frequency systems

Power buses in a substation

A transformer in a substation

Overhead high-voltage power lines

Generators at Hoover dam

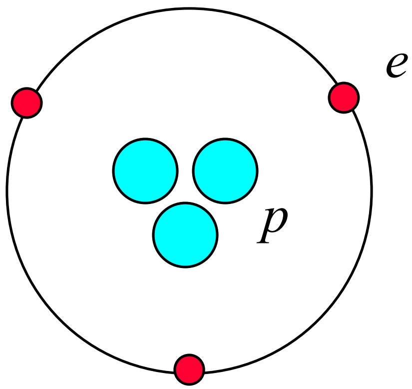
Charge

Proton: $q = e \equiv 1.602176634 \times 10^{-19}\ [\mathrm{C}]$

Electron: $q = -e = -1.602176634 \times 10^{-19}\ [\mathrm{C}]$

Ben Franklin chose the convention of using positive and negative charges.

$$1[\mathrm{C}] \approx \left(1/1.602 \times 10^{-19}\right) \text{ protons} = 6.242 \times 10^{18} \text{ protons}$$



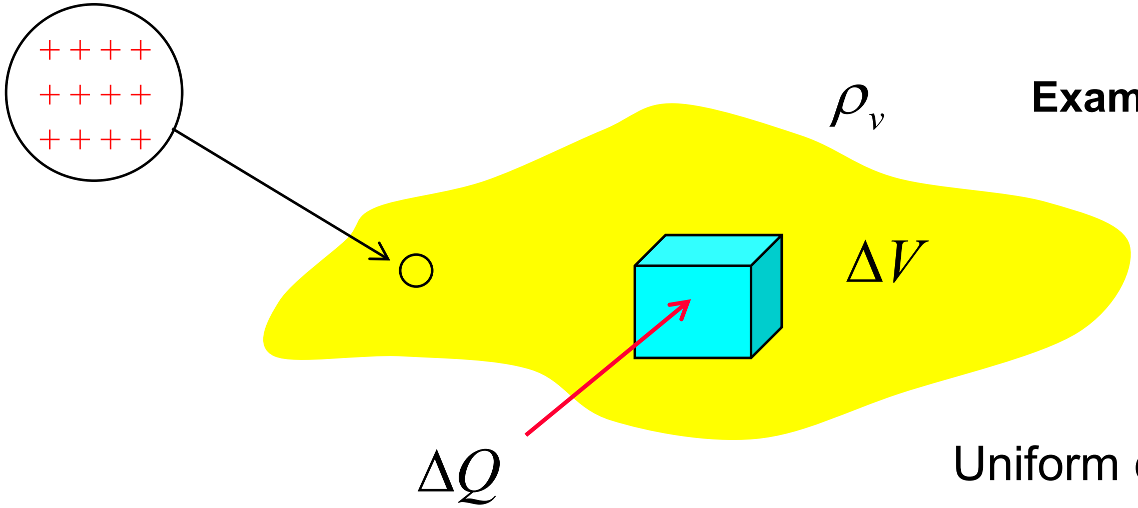
Atom

Ben Franklin

Charge Density

1) Volume charge density ρ_v [C/m^3]

a) Uniform (homogeneous) volume charge density



Example: protons floating in space.

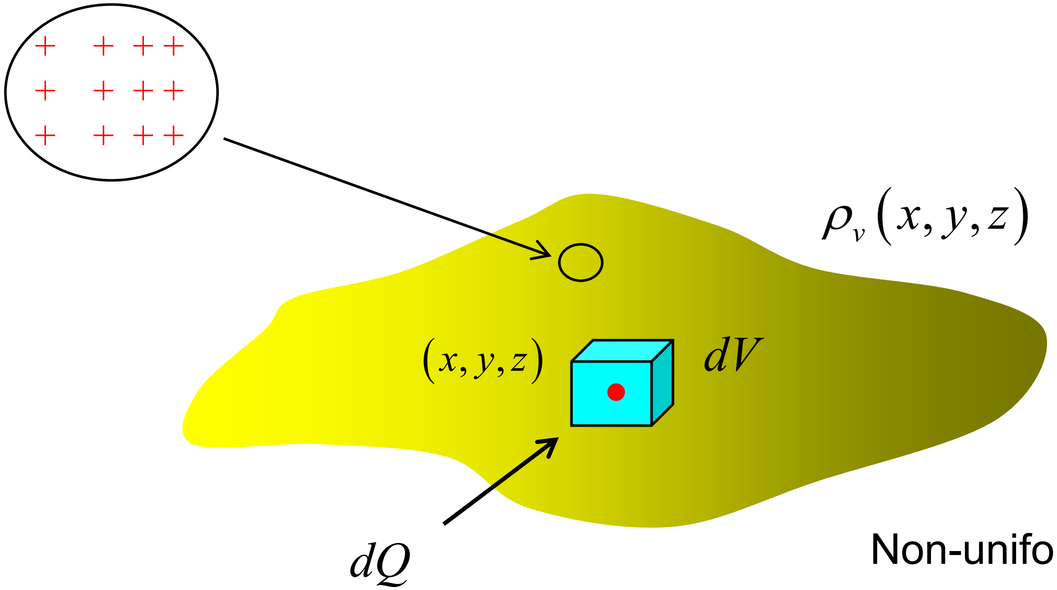
Uniform cloud of charge density

$$\rho_v = \frac{\Delta Q}{\Delta V} \quad [\text{C/m}^3]$$

Charge Density (cont.)

b) Non-uniform (inhomogeneous) volume charge density

$$\rho_v(x, y, z) = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$



Non-uniform cloud of charge density

Example: Protons are closer together as we move to the right.

Charge Density (cont.)

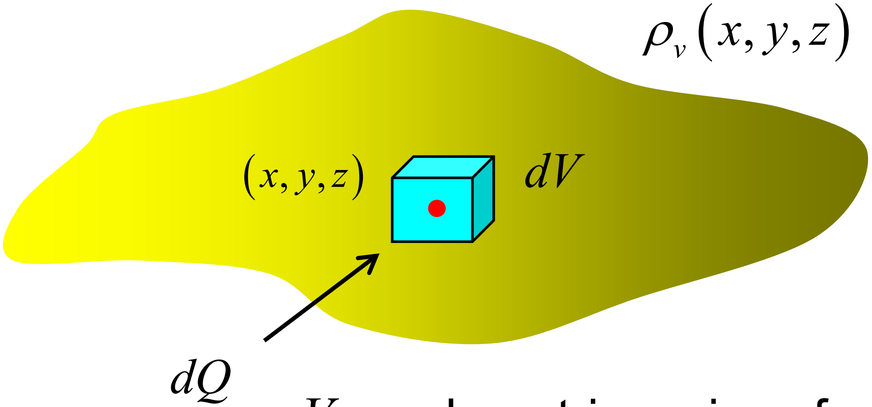
$$\rho_v(x,y,z) = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

$$\rho_v(x,y,z) \approx \frac{\Delta Q}{\Delta V} \quad \Rightarrow \quad \Delta Q \approx \rho_v(x,y,z)\,\Delta V$$

so $\quad dQ = \rho_v(x,y,z)\,dV$

Hence

$$Q = \int_V \rho_v(x,y,z)\,dV$$

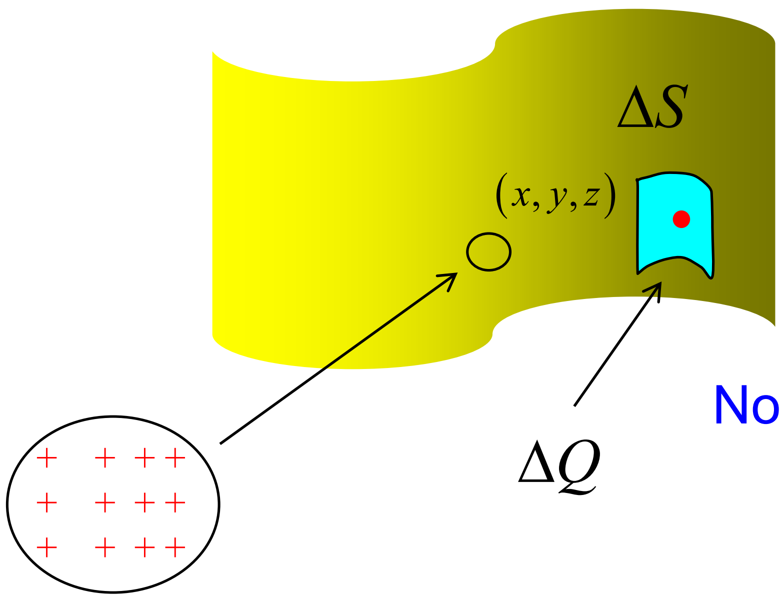


V = volumetric region of space

Charge Density (cont.)

2) Surface charge density ρ_s [C/m^2]

Example: Protons are sprayed onto a sheet of paper.



$\rho_s(x,y,z)$

Non-uniform sheet of surface charge density

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS} \quad [\text{C/m}^2]$$

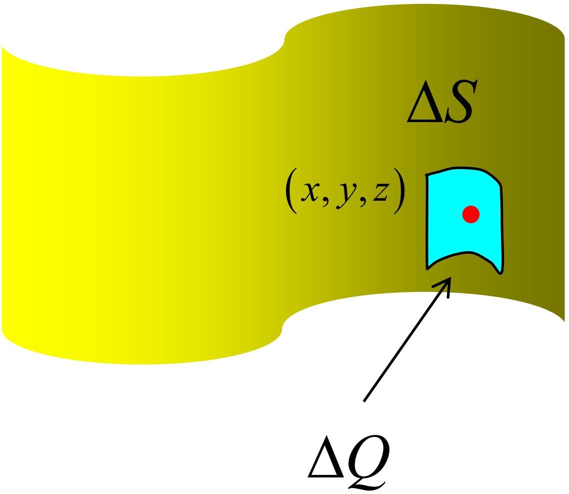
Non-uniform

$$\rho_s = \frac{\Delta Q}{\Delta S} \quad [\text{C/m}^2]$$

Uniform

Charge Density (cont.)

$S =$ surface



$\rho_s(x, y, z)$

$$\rho_s \approx \frac{\Delta Q}{\Delta S} \quad [\mathrm{C/m^2}]$$

so

$$dQ = \rho_s(x, y, z)\, dS$$

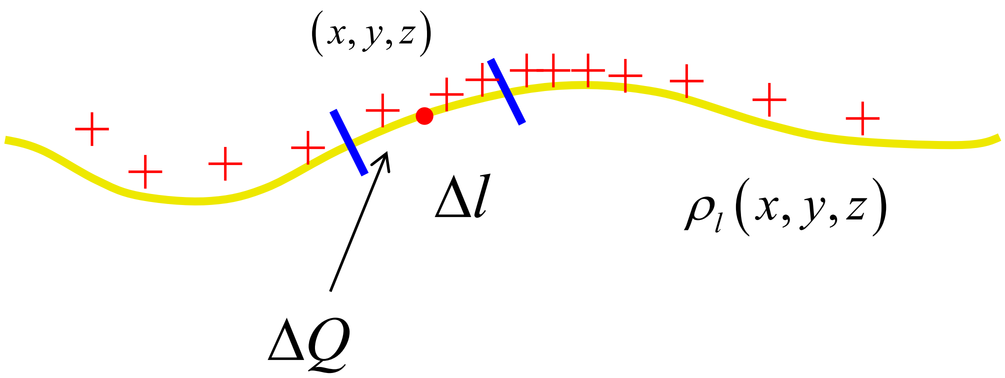
Hence

$$Q = \int_S \rho_s(x, y, z)\, dS$$

Charge Density (cont.)

3) Line charge density ρ_l [C/m]

Example: Protons are sprayed onto a thread.



Non-uniform line charge density

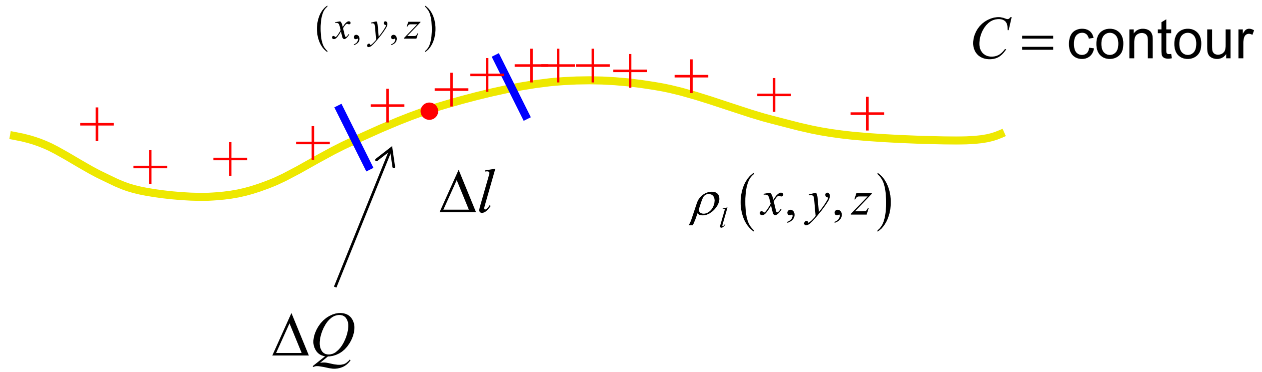
$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} = \frac{dQ}{dl} \quad [\text{C/m}]$$

Non-uniform

$$\rho_l = \frac{\Delta Q}{\Delta l} \quad [\text{C/m}]$$

Uniform

Charge Density (cont.)



$$\rho_l \approx \frac{\Delta Q}{\Delta l} \quad [\mathrm{C/m}]$$

so

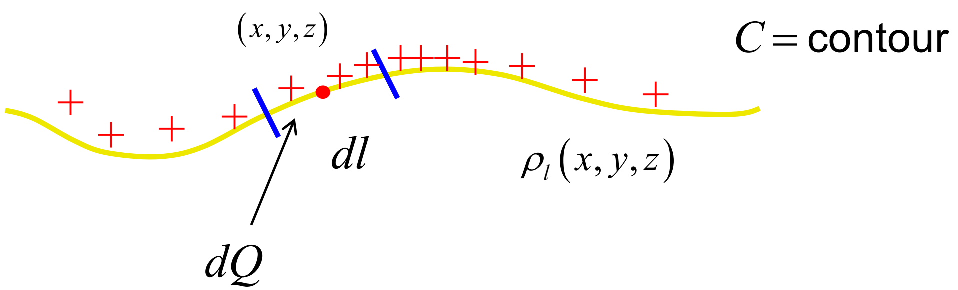
$$dQ = \rho_l(x,y,z)\,dl$$

Hence

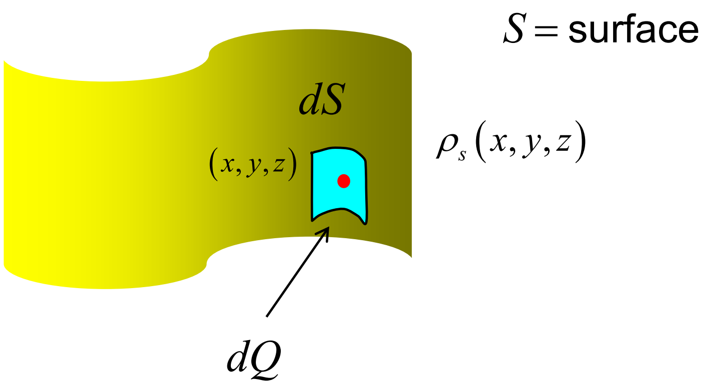
$$Q = \int_C \rho_l(x,y,z)\,dl$$

Charge Summary

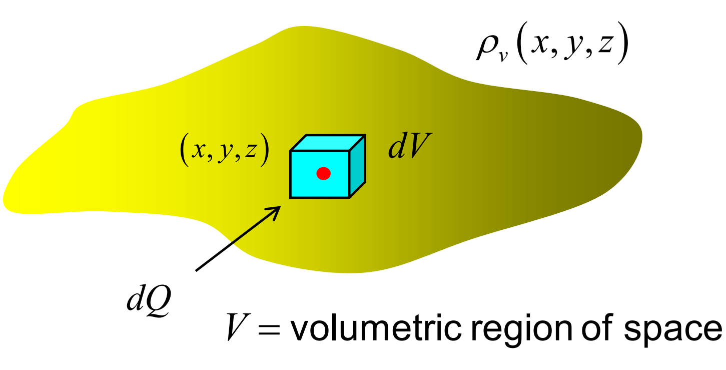
$$Q = \int_C \rho_l(x,y,z)\,dl$$



$$Q = \int_S \rho_s(x,y,z)\,dS$$



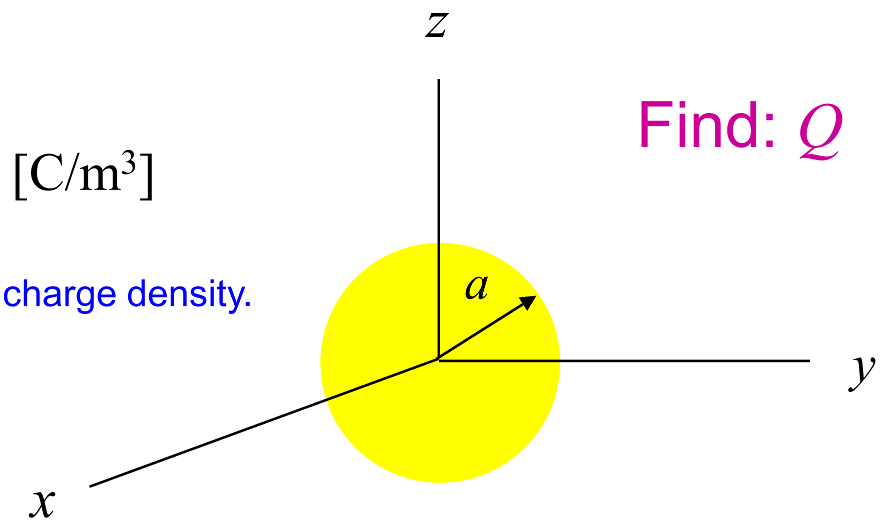
$$Q = \int_V \rho_v(x,y,z)\,dV$$



Example

$\rho_v = \rho_{v0} = 10 \ \ [\text{C/m}^3]$

Note: This is a uniform charge density.



Find: Q

$$Q = \int_V \rho_v(x, y, z)\, dV$$

$$= \int_V \rho_{v0}\, dV$$

$$= \rho_{v0} \int_V dV$$

$$= \rho_{v0} V \quad \left(V = \text{volume } [\text{m}^3]\right)$$

$$Q = 10\,\text{V}$$

$$= 10\left(\frac{4}{3}\pi a^3\right)$$

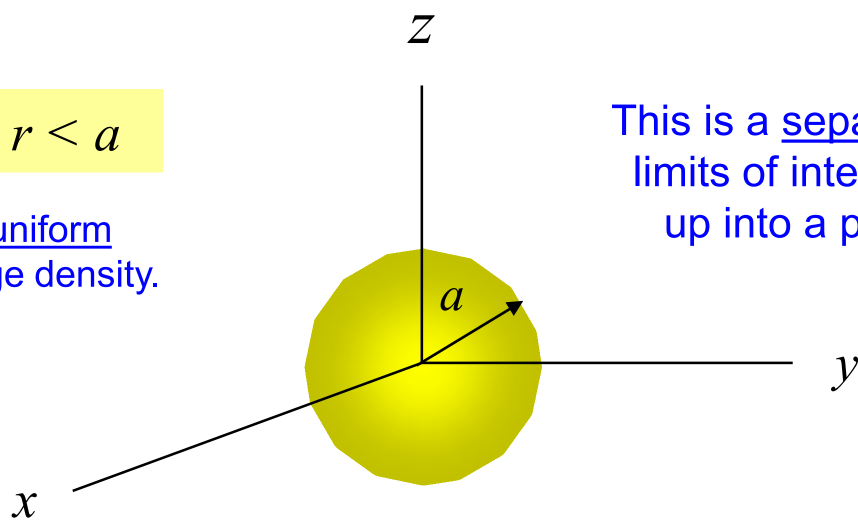
$$Q = \frac{40}{3}\pi a^3 \ \ [\text{C}]$$

Example

$\rho_v = 2r \ \ [\mathrm{C/m^3}], \quad r < a$

Note: This is a non-uniform (inhomogeneous) charge density.

Find: Q



This is a separable integrand with fixed limits of integration, so we can split it up into a product of 1-D integrals.

(Please see the next slide for a reminder about spherical coordinates.)

$$Q = \int_V \rho_v(x,y,z)\,dV$$

$$= \int_V 2r\,dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a}(2r)\,\underbrace{r^2\sin\theta\,dr\,d\theta\,d\phi}_{dV}$$

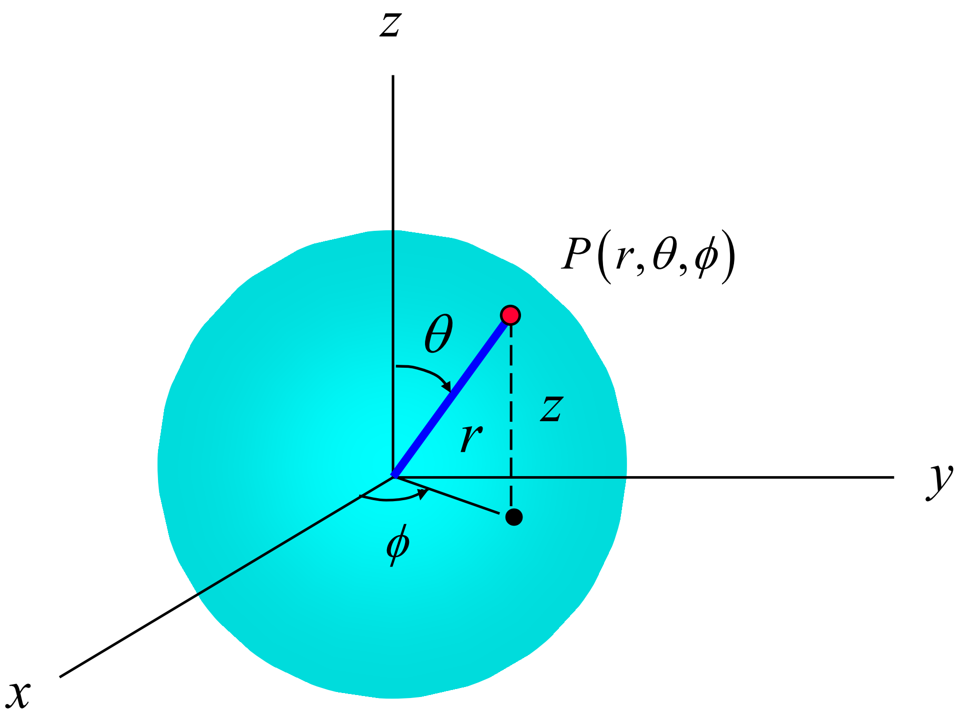
Separable integrand

$$Q = \int_0^{2\pi} d\phi \int_0^{\pi}\sin\theta\,d\theta \int_0^{a} 2r^3\,dr$$

$$= (2\pi)(2)\left(\frac{2}{4}a^4\right)$$

$$Q = 2\pi a^4 \ \ [\mathrm{C}]$$

Reminder of Spherical Coordinates

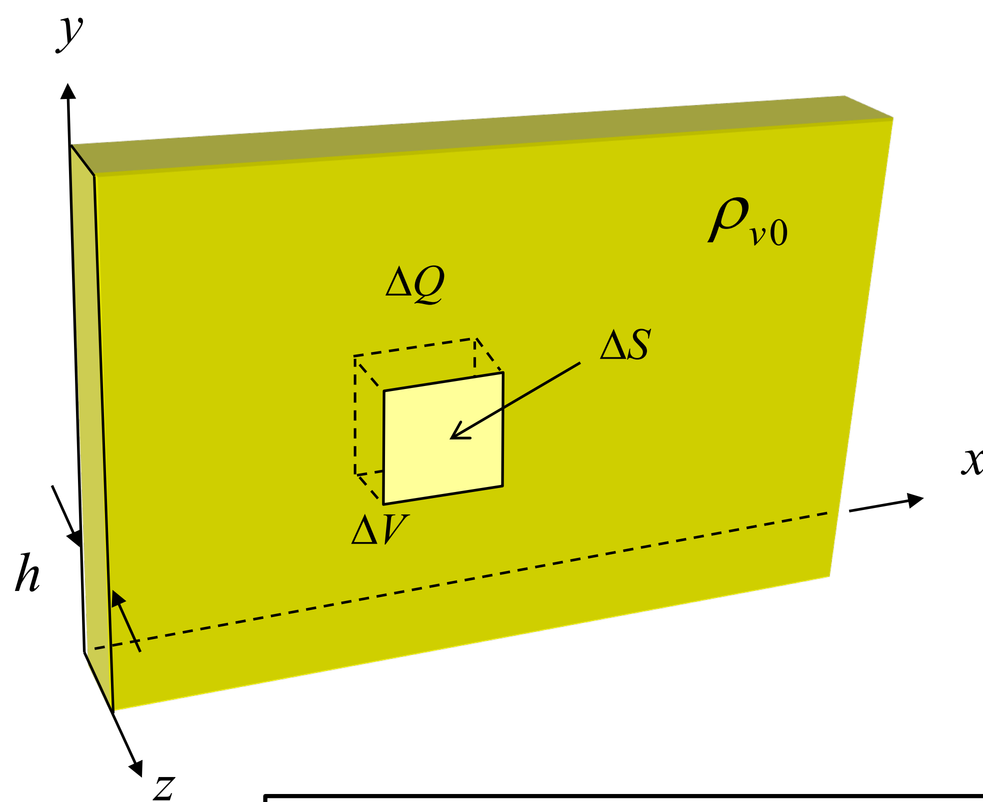


Note: $0 \le \theta \le \pi$

Example:

Find the ***Equivalent Surface Charge Density*** for a Slab of Volume Charge Density

$$\rho_v(x,y,z) = \rho_{v0} \quad \left[\text{C/m}^3\right],$$
$$0 \le z \le h$$



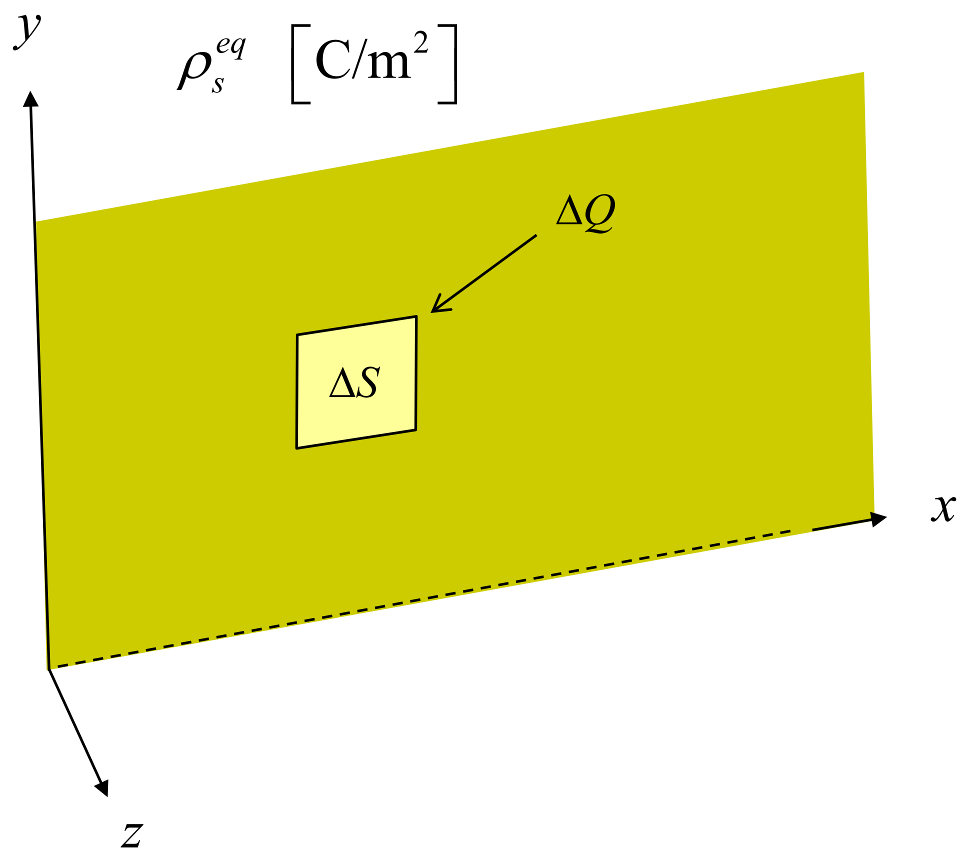
$$\Delta Q = \int_{\Delta V} \rho_v \, dV = \int_{\Delta S} \int_0^h \rho_v dz \, dS$$

Note:
Equivalent charge density is a helpful concept for finding the fields from charges, as we will see later in Gauss's law.

Example (cont.)

Equivalent surface charge density:

$$\Delta Q = \int_{\Delta S} \rho_s^{eq} \, dS$$



Example (cont.)

Compare:

$$\Delta Q = \int_{\Delta S} \left(\int_0^h \rho_v \, dz \right) dS$$

$$\Delta Q = \int_{\Delta S} \rho_s^{eq} \, dS$$

$$\Rightarrow \quad \rho_s^{eq}(x,y) = \int_0^h \rho_v(x,y,z)\, dz$$

In our case (uniform volume charge density) we have:

$$\rho_s^{eq}(x,y) = \int_0^h \rho_{v0} \, dz = \rho_{v0} h$$

so

$$\rho_{s0}^{eq} = \rho_{v0} h \quad \left[\mathrm{C/m^2} \right]$$

Example (cont.)

Analogy with mass density:

ρ_{v0} = mass density of the metal $\left[\text{kg/m}^3\right]$

$$\rho_s^{eq} = \rho_{v0} h \quad \left[\text{kg/m}^2\right]$$

(mass per unit area of the plate)