ECE 3318
Applied Electricity and Magnetism

Spring 2023

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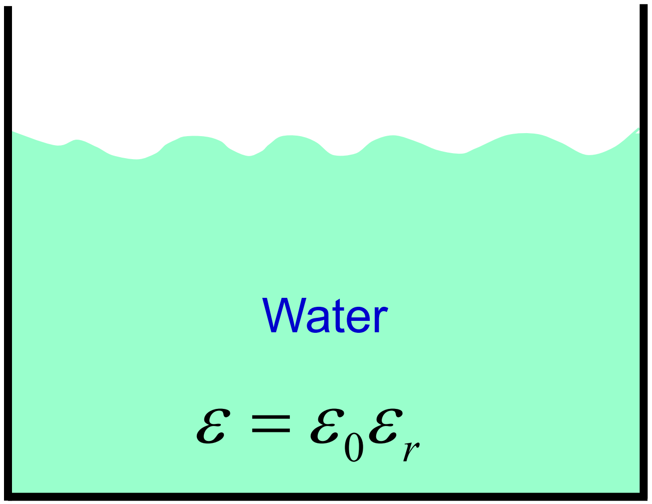
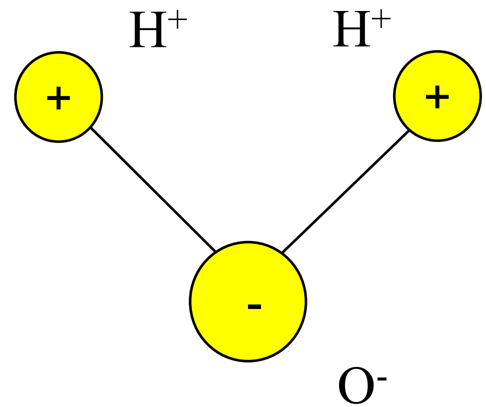
Dept. of ECE

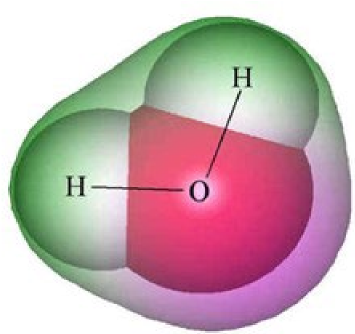
Notes 20
Dielectrics

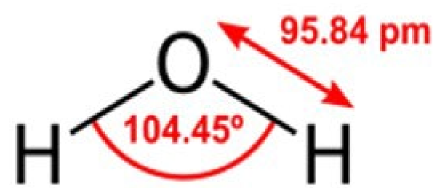
Dielectrics



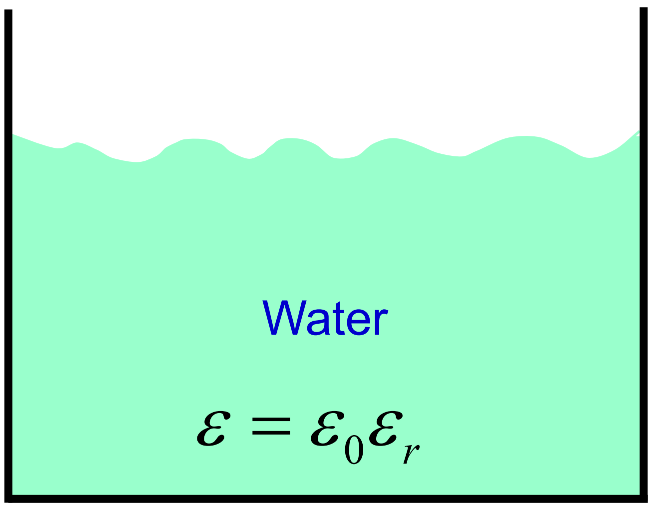
Single H_2O molecule:

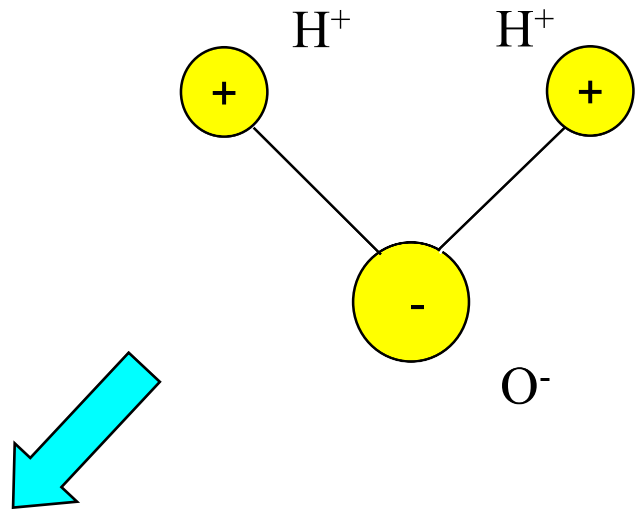




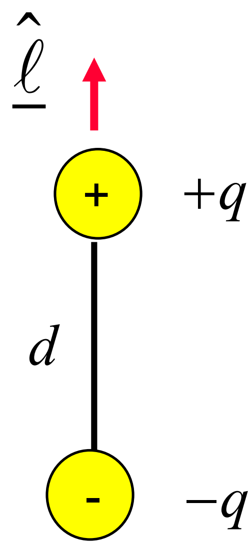


Dielectrics (cont.)





"Dipole" model:

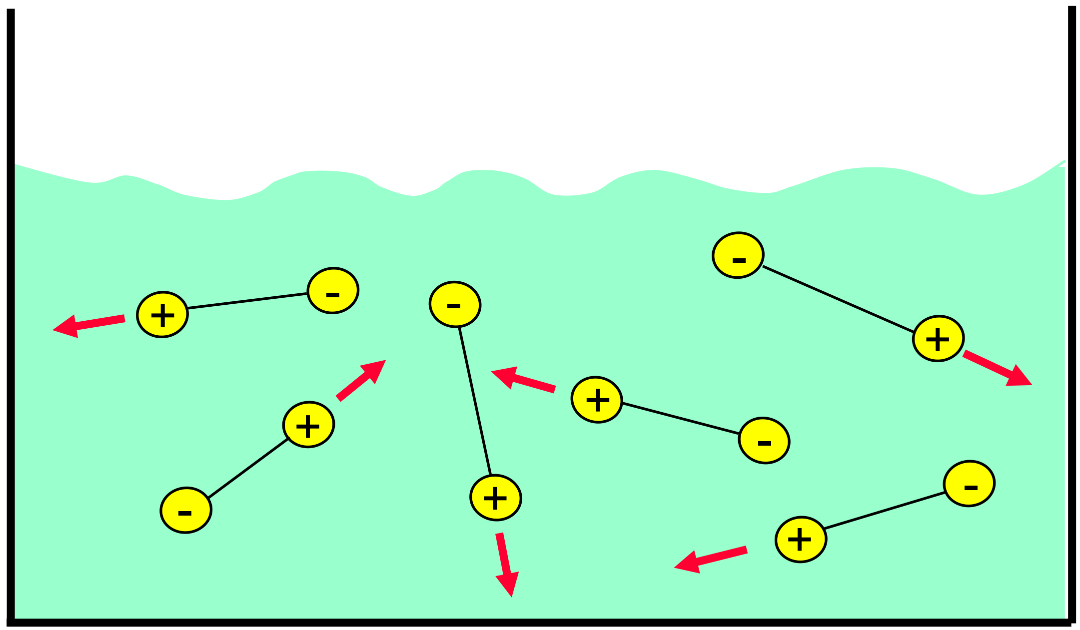


"Vector dipole moment" of molecule $\underline{p}$:

$$\underline{p} = p\,\underline{\hat{\ell}}$$

$$p = qd$$

Dielectrics (cont.)

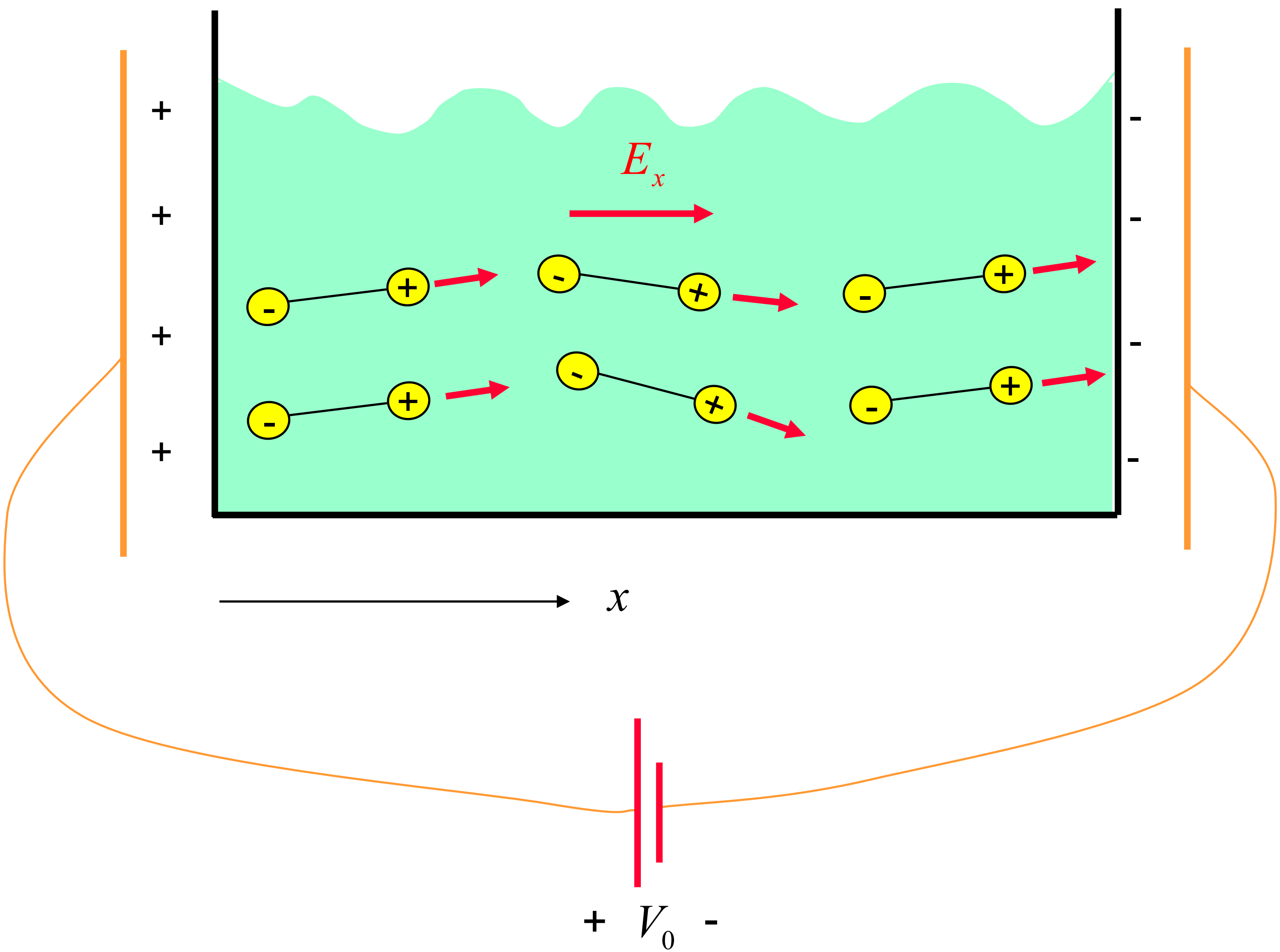


The dipoles representing the water molecules are normally pointing in random directions.

Note: The molecules are not floating in water – they make up the water.

Dielectrics (cont.)

The dipoles partially align under an applied electric field.

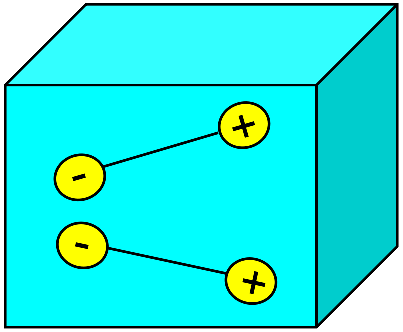


Dielectrics (cont.)

Define: $\underline{P}$ = total dipole moment/volume

N molecules (dipoles) inside ΔV

$$\underline{P} \equiv \frac{1}{\Delta V} \sum_{\Delta V} \underline{p}_i$$



ΔV

We can also write this as

$$\underline{P} = \left(\frac{N}{\Delta V}\right)\left(\frac{1}{N} \sum_{\Delta V} \underline{p}_i\right)$$

N_v = number of molecules per unit volume

$$N_v = N / \Delta V$$

$$\underline{P} = N_v \, \underline{p}^{ave}$$

$\underline{p}^{ave}$ = average vector dipole moment

Define the electric flux density vector $\underline{D}$:

$$\underline{D} \equiv \varepsilon_0 \underline{E} + \underline{P}$$

Dielectrics (cont.)

Linear material: $\underline{P} = \varepsilon_0 \chi_e \underline{E}$

Note: $\chi_e > 0$ for most materials

The term χ_e is called the "electric susceptibility."

$$\underline{D} = \varepsilon_0 \underline{E} + \varepsilon_0 \chi_e \underline{E}$$
$$= \varepsilon_0 \left(1 + \chi_e\right) \underline{E}$$

Note:
A negative value of χ_e would mean that the dipoles align against the field.

Define: $\varepsilon_r \equiv 1 + \chi_e$

Then we have $\underline{D} = \varepsilon_0 \varepsilon_r \underline{E}$

or $\underline{D} = \varepsilon \underline{E}$ where $\varepsilon = \varepsilon_0 \varepsilon_r$

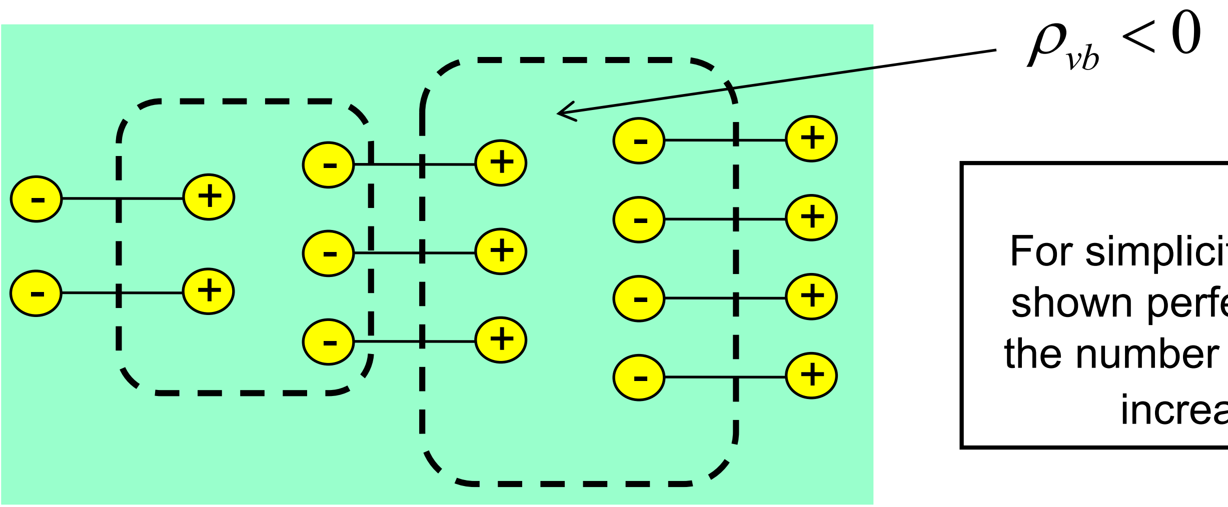
Typical Linear Materials

Teflon	$\varepsilon_r = 2.2$	
Water	$\varepsilon_r = 81$	(a very polar molecule, fairly free to rotate)
Styrofoam	$\varepsilon_r = 1.03$	
Quartz	$\varepsilon_r = 5$	

Note: $\varepsilon_r > 1$ for most materials: $\varepsilon_r \equiv 1 + \chi_e, \quad \chi_e > 0$

Dielectrics: Bound Charge

Assume (hypothetically) that P_x increases as x increases inside the material:



Note:
For simplicity, the dipoles are shown perfectly aligned, with the number of aligned dipoles increasing with x.

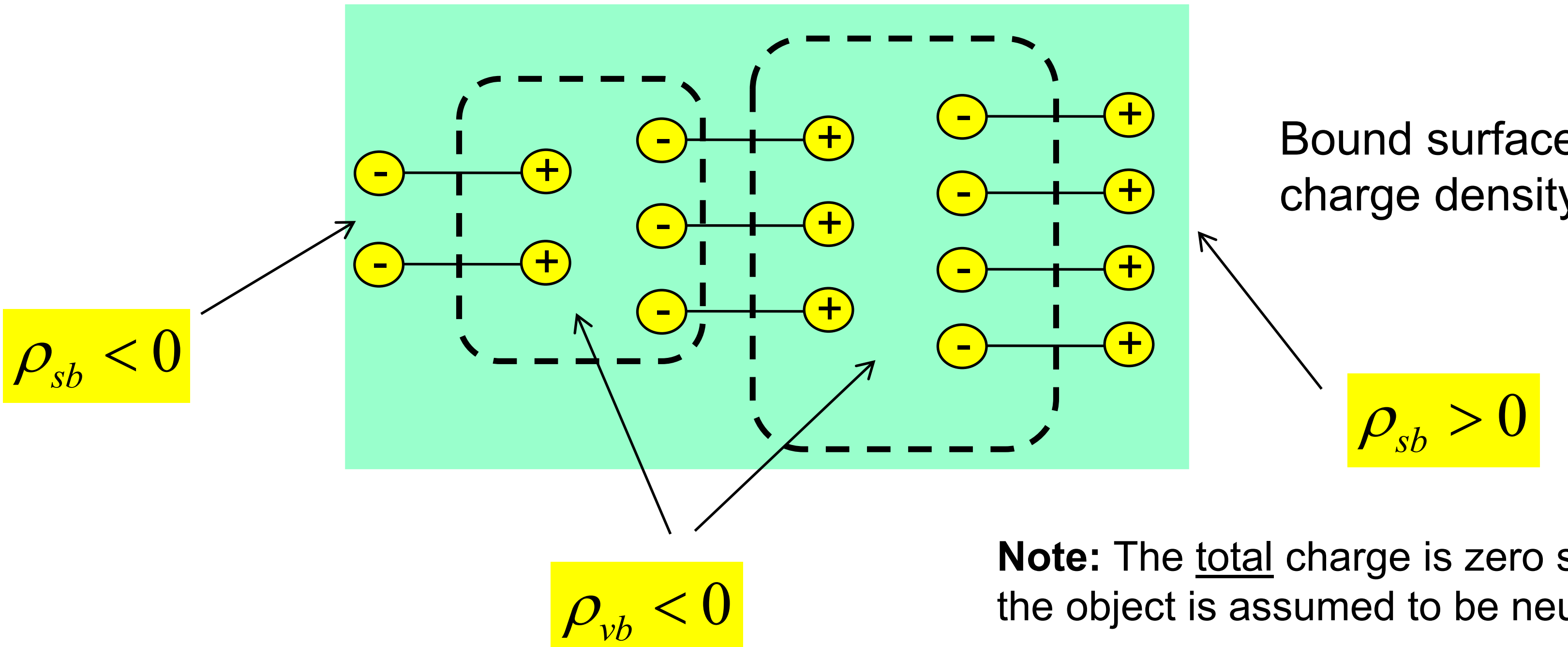
A net volume charge density is created inside the material, called the "bound charge density" or "polarization charge density."

$\rho_{vb} =$ "bound charge density": it is caused by the molecules rotating.

$\rho_v =$ "free charge density": this is charge that <u>you</u> place inside the material. You can freely place it wherever you want.

Dielectrics: Bound Charge (cont.)

Bound charge densities



Note: The <u>total</u> charge is zero since the object is assumed to be neutral.

Total volume charge density inside the material:

$$\rho_v^{total} = \rho_v + \rho_{vb}$$

This total charge density may be viewed as being in free space

(since there is no material left after the molecules are removed).

Dielectrics: Bound Charge (cont.)

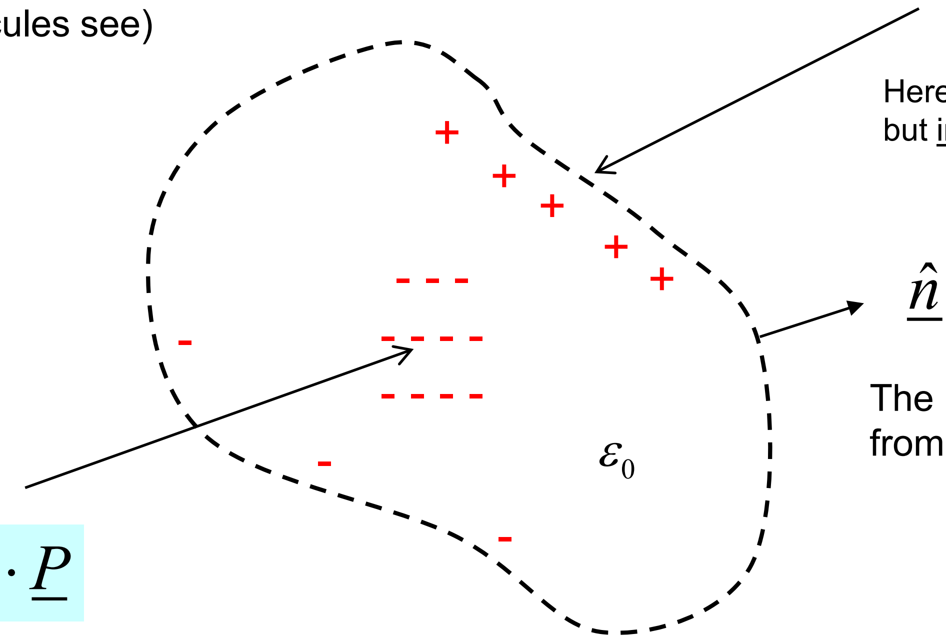
Formulas for the two types of bound charge density

$$\underline{P} = \varepsilon_0 \chi_e \underline{E} = \varepsilon_0 \left(\varepsilon_r - 1\right) \underline{E}$$

$\underline{E}$ = electric field <u>inside</u> the object
(what the molecules see)

$$\rho_{sb} = \underline{P} \cdot \underline{\hat{n}}$$

Here $\underline{P}$ comes from $\underline{E}$ at the surface, but <u>inside</u> the object.



The unit normal points outward from the dielectric.

$$\rho_{vb} = -\nabla \cdot \underline{P}$$

Here $\underline{P}$ comes from $\underline{E}$ inside the object.

A derivation of these formulas is given in the Appendix.

Dielectrics: Gauss's Law

Inside a material:

$$\nabla \cdot \left(\varepsilon_0 \underline{E} \right) = \rho_v^{total} = \rho_v + \rho_{vb}$$

The above equation is valid inside of a material, even though it involves the permittivity of free space. But we prefer to have only the <u>free charge density</u> in the equation, since this is what is known.

The goal is to calculate (and hopefully eliminate) the bound-charge density term on the right-hand side.

Dielectrics: Gauss's Law (cont.)

The free-space form of Gauss's law is

$$\nabla \cdot \left(\varepsilon_0 \underline{E} \right) = \rho_v^{total} = \rho_v + \rho_{vb} = \rho_v - \nabla \cdot \underline{P}$$

or

$$\nabla \cdot \left(\varepsilon_0 \underline{E} + \underline{P} \right) = \rho_v$$

Hence

$$\boxed{\nabla \cdot \underline{D} = \rho_v}$$

This is the usual Gauss's law: This is why the definition of $\underline{D}$ is so convenient!

Dielectrics: Gauss's Law (Summary)

$$\nabla \cdot \underline{D} = \rho_v$$

$$\Downarrow$$

$$\oint_S \underline{D} \cdot \underline{\hat{n}}\, dS = Q_{encl}$$

$$\underline{D} = \varepsilon \underline{E}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

Important conclusion:

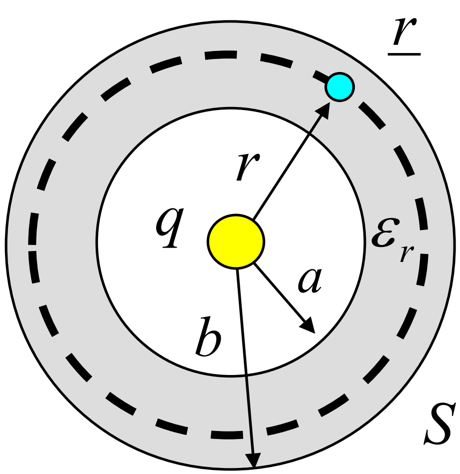
Gauss' law works the same way inside a dielectric as it does in vacuum, with only the <u>free charge density</u> (i.e., the charge that is actually placed inside the material) being used on the right-hand side.

We simply use ε instead of ε_0 !

Example

Point charge inside dielectric shell

Find $\underline{D}$, $\underline{E}$

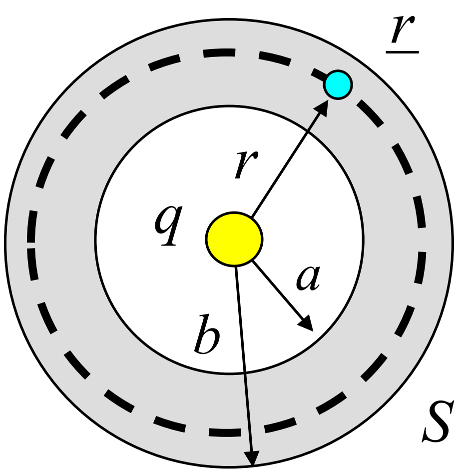


Dielectric spherical shell with ε_r

Gauss' law: $$\oint_S \underline{D} \cdot \underline{\hat{n}}\, dS = Q_{encl} = q$$

(The point charge q is the only free charge in the problem.)

Example (cont.)



$$D_r\left(4\pi r^2\right) = q$$

Hence

$$\underline{D} = \underline{\hat{r}}\left(\frac{q}{4\pi r^2}\right) \quad \left[\mathrm{C/m^2}\right]$$

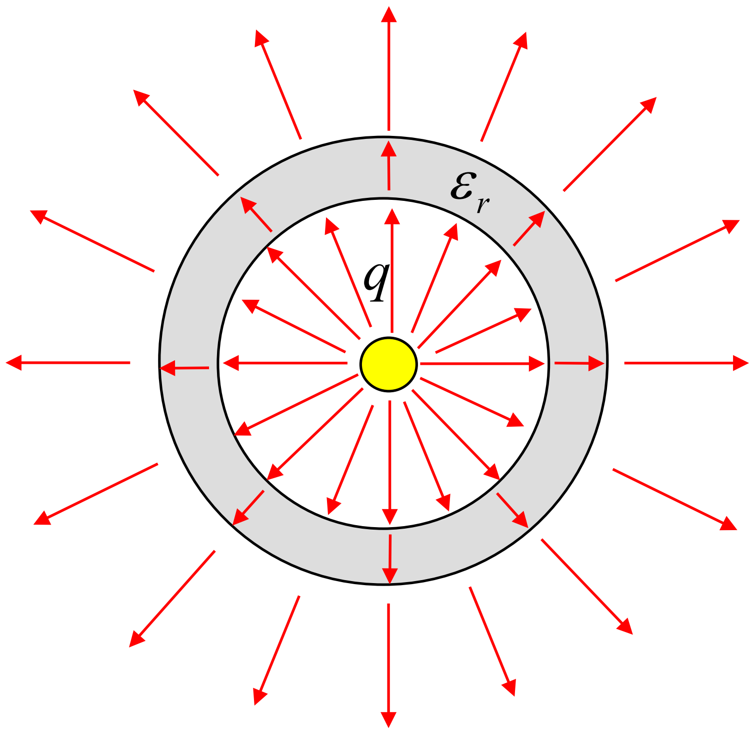
We then have

$$\underline{E} = \underline{\hat{r}}\left(\frac{q}{4\pi\varepsilon_0 r^2}\right) \quad [\mathrm{V/m}] \qquad r < a,\ r > b$$

$$\underline{E} = \underline{\hat{r}}\left(\frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\right) \quad [\mathrm{V/m}] \qquad a < r < b$$

Example (cont.)

Flux Plot



Note that there are less flux lines inside the dielectric region (assuming that the flux lines represent the electric field).

Exotic Materials

Plasmas

Plasmas have a relative permittivity that is less than one
(and can even be negative)

Lossless plasma:

$$\varepsilon = \varepsilon_0 \left[1 - \left(\frac{\omega_p}{\omega} \right)^2 \right]$$

$\omega_p =$ plasma resonance frequency

(derived in ECE 6340)

Lossy plasma:

$$\varepsilon = \varepsilon_0 \left[1 - \frac{\omega_p^2}{\omega(\omega - j\upsilon)} \right]$$

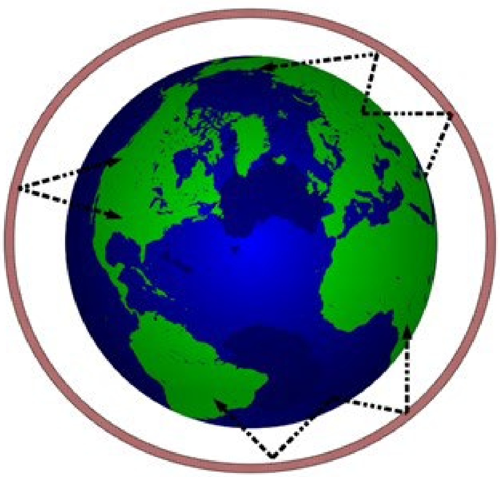
$\upsilon =$ plasma collision frequency (loss term)

Exotic Materials (cont.)

Plasmas

Low frequencies ($f < 30$ MHz) will reflect off the ionosphere.

The ionosphere has a relative permittivity that is less than one, so the waves will bend (refract) “away from the normal” and travel back down to the earth, bouncing off of the earth.



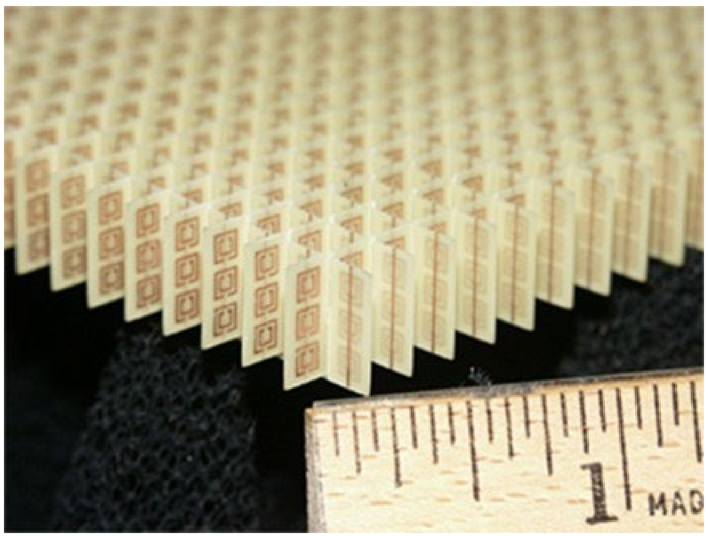
Shortwave radio signals propagate around the earth by “skipping” off the ionosphere.

https://en.wikipedia.org/wiki/Skywave

Exotic Materials (cont.)

Artificial "metamaterials" that have been designed that have exotic permittivity and/or permeability performance.

http://en.wikipedia.org/wiki/Mhttp://en.wikipedia.org/wiki/Metamaterialetamaterial

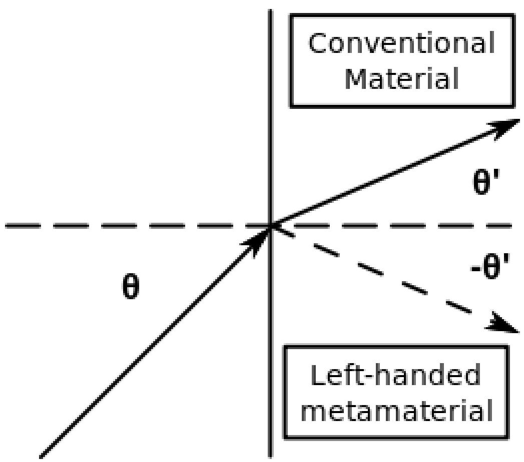


"Negative index" metamaterial array configuration, which was constructed of copper split-ring resonators and wires mounted on interlocking sheets of fiberglass circuit board. The total array consists of 3 by 20×20 unit cells with overall dimensions of 10×100×100 mm.

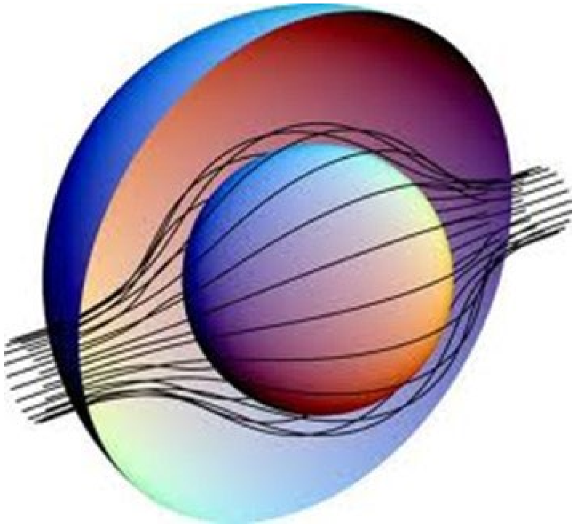
$$\varepsilon_r < 0$$

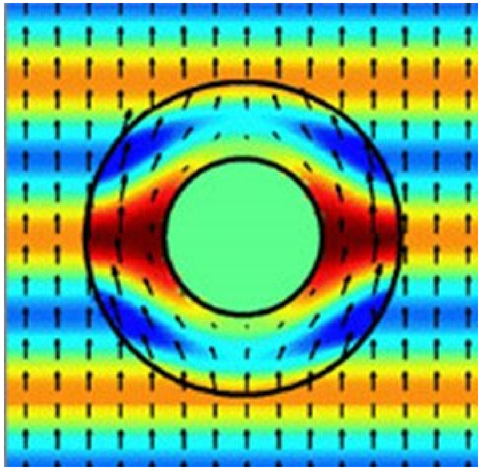
$$\mu_r < 0$$

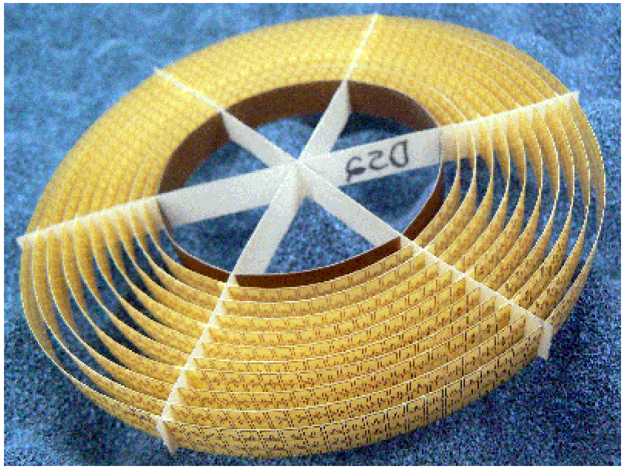
(over a certain bandwidth of operation)



Exotic Materials (cont.)







The Duke cloaking device masks an object from one wavelength at microwaves.

Image from Dr. David R. Smith.

Cloaking of objects is one area of research in metamaterials.

Appendix

In this appendix we derive the formulas for the bound change densities:

$$\rho_{vb} = -\nabla \cdot \underline{P}$$

$$\rho_{sb} = \underline{P} \cdot \underline{\hat{n}}$$

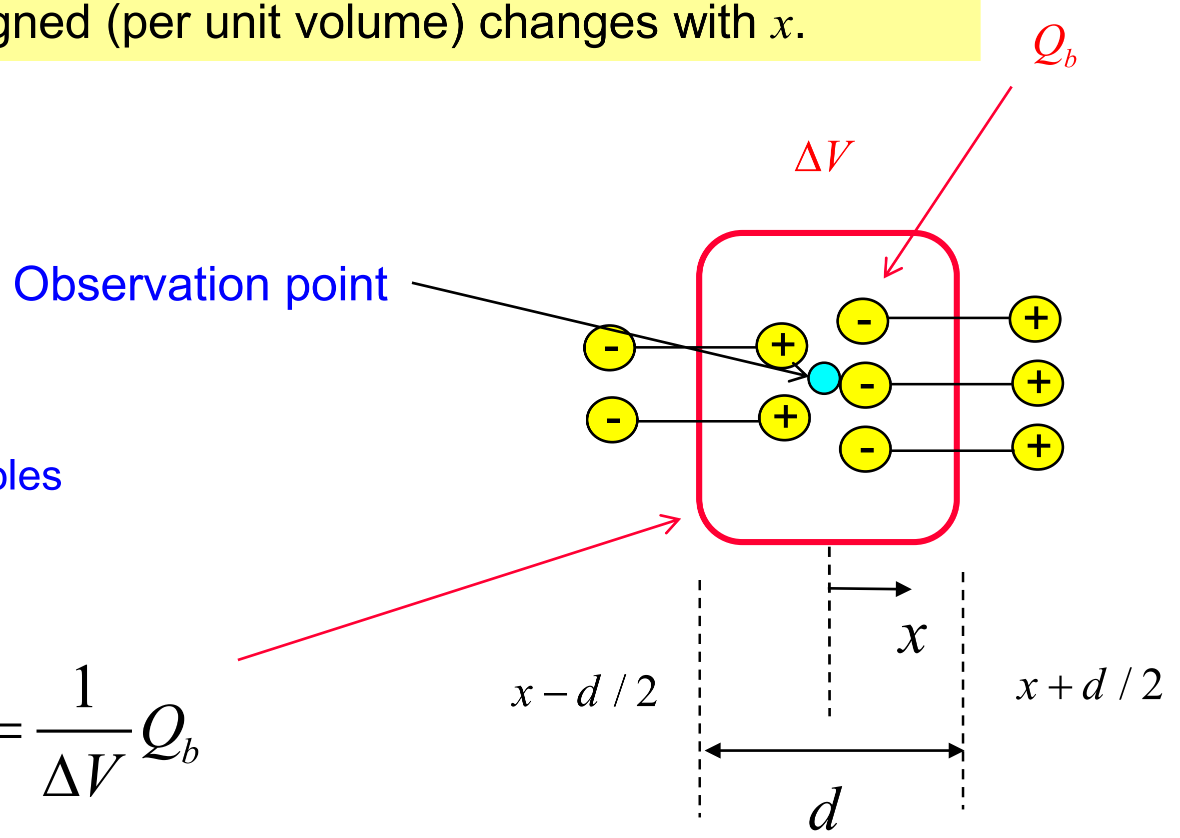
where

$$\underline{\hat{n}} = \underline{\hat{n}}_b = \text{outward normal to the dielectric boundary}$$

Appendix (cont.)

Model:

Dipoles are aligned in the x direction end-to-end. The number of dipoles that are aligned (per unit volume) changes with x.



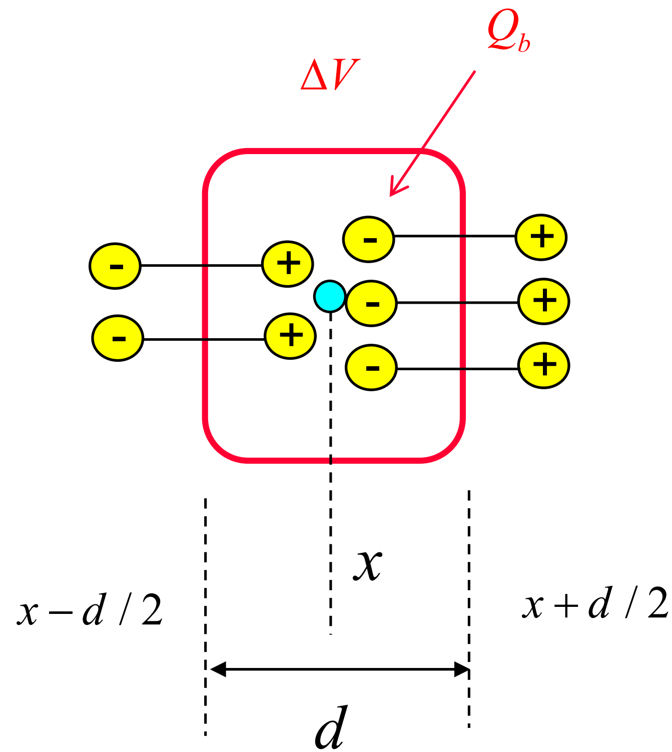
N_v = # of aligned dipoles per unit volume

$$\rho_{vb} = \frac{1}{\Delta V} Q_b$$

Appendix (cont.)

$$\rho_{vb} = \frac{1}{\Delta V} Q_b$$

$$Q_b = -q\, N_v\Big|_{x+\frac{d}{2}} \Delta V + q\, N_v\Big|_{x-\frac{d}{2}} \Delta V$$



Hence

$$\rho_{vb} = -q\left[N_v\Big|_{x+\frac{d}{2}} - N_v\Big|_{x-\frac{d}{2}} \right]$$

$$= -q \Delta N_v$$

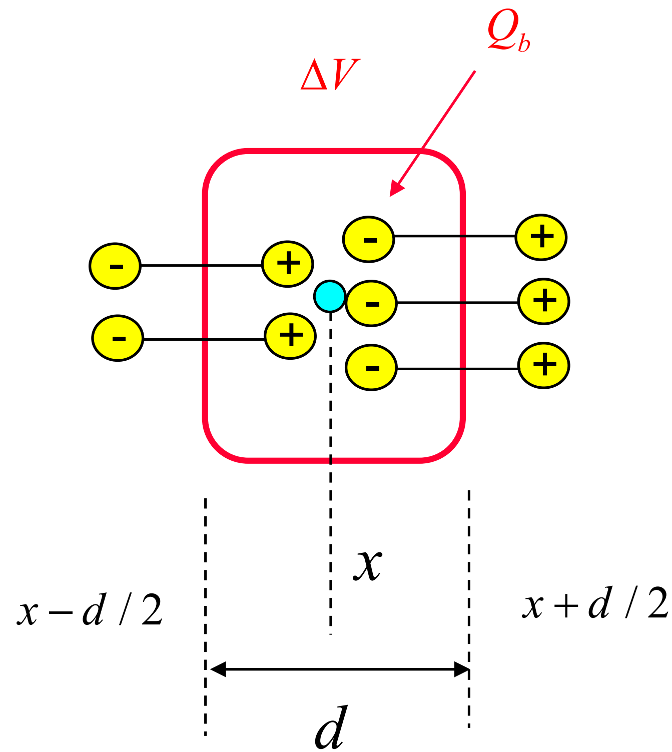
N_v = # of aligned dipoles per unit volume

Appendix (cont.)

$$\rho_{vb} = -q\Delta N_v$$

$$P_x = pN_v$$

$$\Rightarrow \Delta P_x = p\,\Delta N_v$$



Hence

$$\rho_{vb} = -q\left(\frac{1}{p}\Delta P_x\right)$$

$$= -q\left(\frac{1}{qd}\Delta P_x\right)$$

$$= -\frac{1}{d}\Delta P_x$$

$$= -\frac{\Delta P_x}{\Delta x}$$

N_v = # of aligned dipoles per unit volume

or

$$\rho_{vb} = -\frac{dP_x}{dx}$$

Appendix (cont.)

For dipole aligned in the x direction,

$$\rho_{vb} = -\frac{dP_x}{dx}$$

In general,

$$\rho_{vb} = -\frac{dP_x}{dx} - \frac{dP_y}{dy} - \frac{dP_z}{dz}$$

or

$$\boxed{\rho_{vb} = -\nabla \cdot \underline{P}}$$

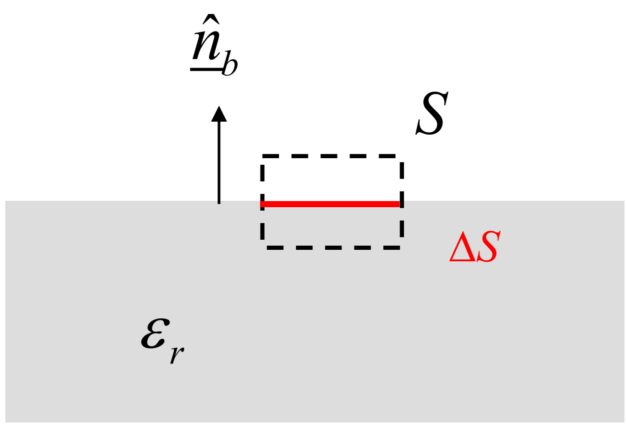
Appendix (cont.)

After applying the divergence theorem, we have the integral form

$$-\oint_S \underline{P} \cdot \underline{\hat{n}} = Q_{encl}^{b}$$

Applying this to a shallow pillbox surface at a dielectric boundary, we have

$$\left(\underline{P}^{diel} - \underline{P}^{air}\right) \cdot \underline{\hat{n}}_b \, \Delta S = Q_{encl}^{b} = \rho_{sb} \, \Delta S$$



Denoting

$$\underline{P} = \underline{P}^{diel}$$

we have

$$\underline{P} \cdot \underline{\hat{n}}_b = \rho_{sb}$$