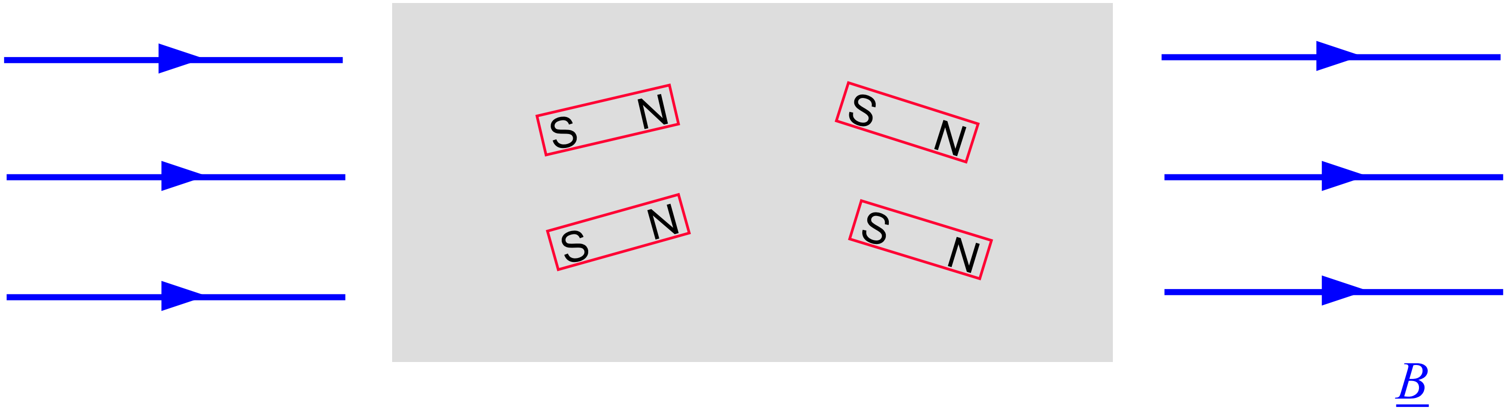
ECE 3318
Applied Electricity and Magnetism

Spring 2023

Prof. David R. Jackson
Dept. of ECE

Notes 30
Magnetic Materials and Stored Energy

Magnetic Materials



Because of *electron spin*, atoms tend to acts as little current loops, and hence as electromagnetics, or bar magnets.

When a magnetic field is applied, the little atomic magnets tend to line up. This effect is what causes the material to have a relative permeability μ_r.

$$\mu = \mu_0 \mu_r$$

Magnetic Materials (cont.)

$$\mu = \mu_0 \mu_r$$

Type of Material	Property
Nonmagnetic	$\mu_r = 1$
Diamagnetic	$\mu_r < 1$
Paramagnetic	$\mu_r > 1$
Ferrimagnetic	$\mu_r >> 1$
Ferromagnetic	$\mu_r >> 1$

Please see the textbooks to learn more about magnetic materials and permeability.

For more information:

http://en.wikipedia.org/wiki/Magnetism

Magnetic Materials

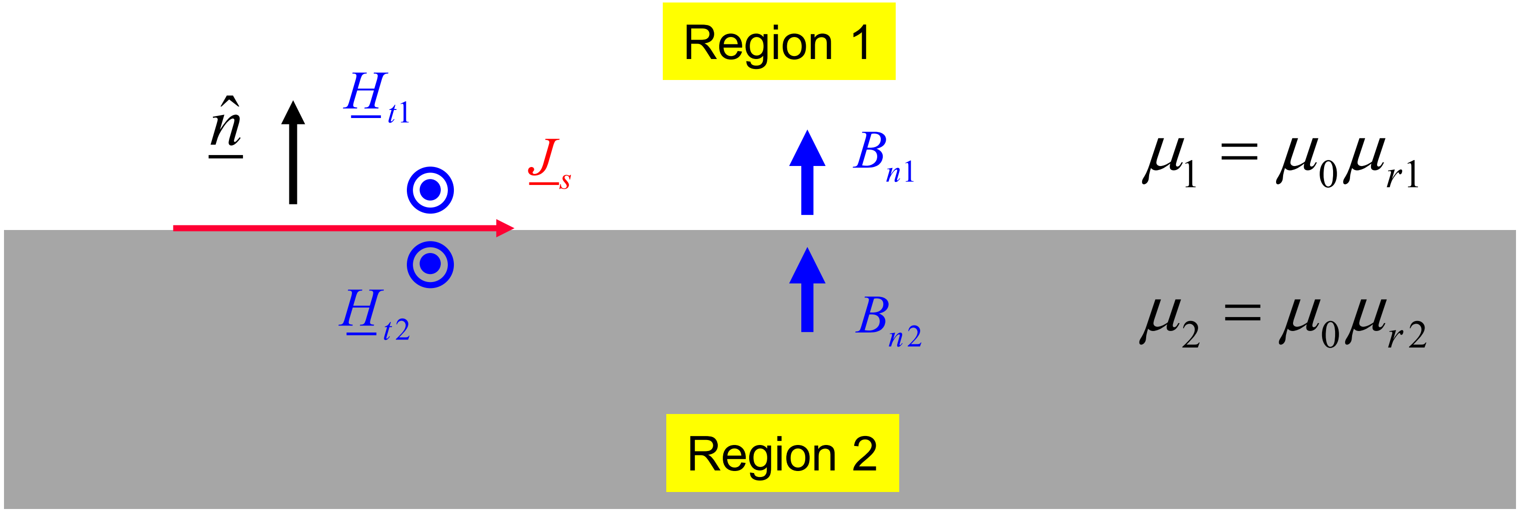
Material	Relative Permeability μ_r
Vacuum	1
Air	1.0000004
Water	0.999992
Copper	0.999994
Aluminum	1.00002
Silver	0.99998
Nickel	600
Iron	5000
Carbon Steel	100
Transformer Steel	2000
Mumetal	50,000
Supermalloy	1,000,000

Note: Values can often vary depending on purity and processing.

http://en.wikipedia.org/wiki/Permeability_(electromagnetism)

Boundary Conditions

(Please see the textbooks for a derivation.)

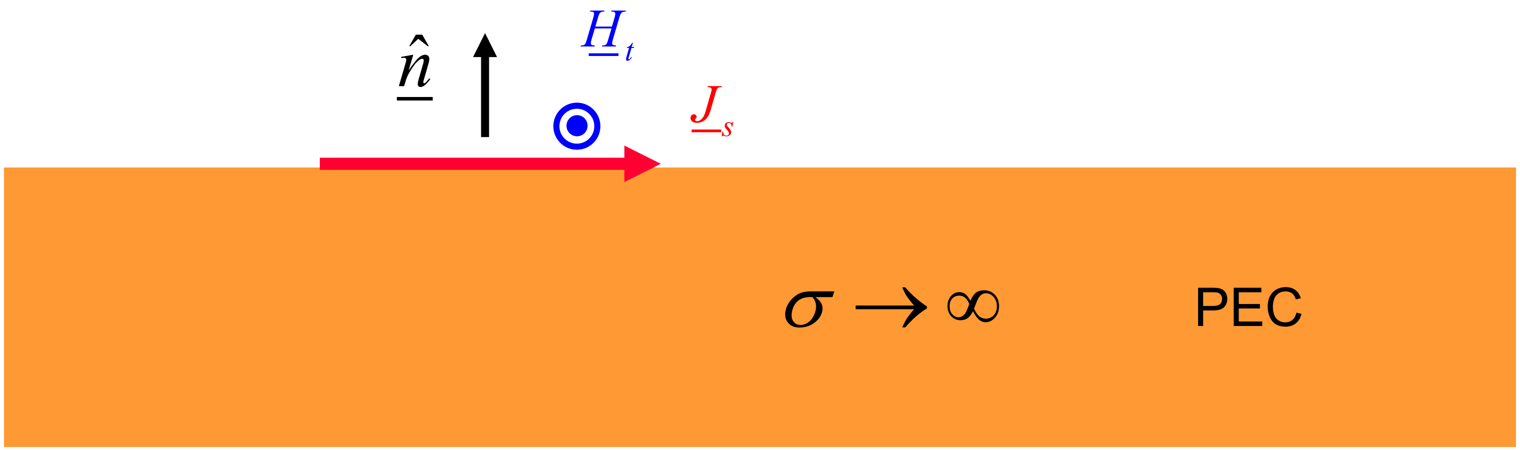


The unit normal vector points towards region 1.

$$\underline{\hat{n}} \times \left(\underline{H}_{t1} - \underline{H}_{t2} \right) = \underline{J}_s \qquad B_{n1} = B_{n2}$$

Note: If there is no surface current: $\underline{H}_{t1} = \underline{H}_{t2}$

Boundary Conditions (cont.)

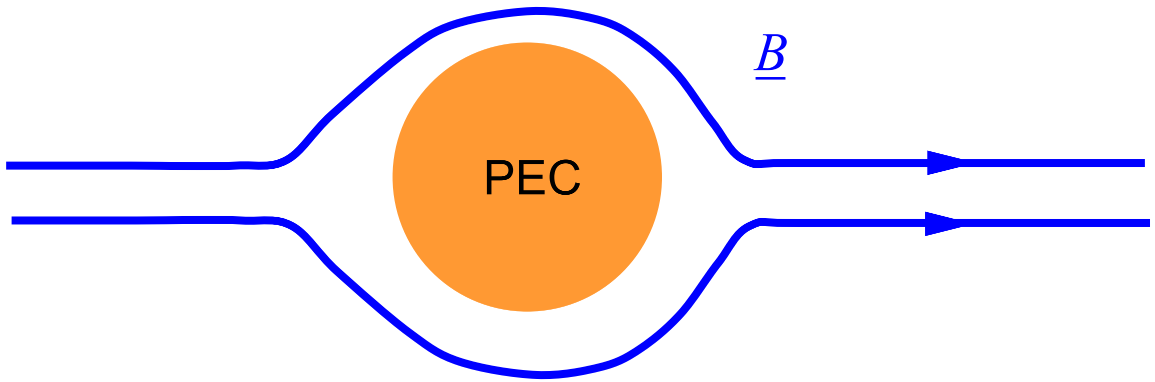


Assume zero magnetic field inside the PEC

$$\underline{\hat{n}} \times \underline{H}_t = \underline{J}_s \qquad B_n = 0$$

Note:
For a practical conductor, these BCs will be accurate as long as the conductivity is high enough so that the skin depth is small.

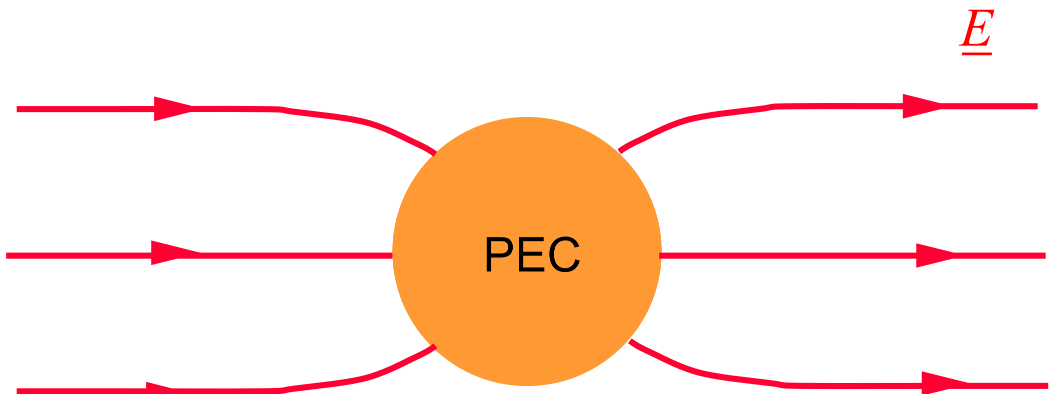
Boundary Conditions (cont.)



$$B_n = 0$$

Magnetic field lines must bend around a perfect electric conductor (PEC).

This is the opposite behavior of electric field lines.



$$\underline{E}_t = 0$$

Magnetic Stored Energy

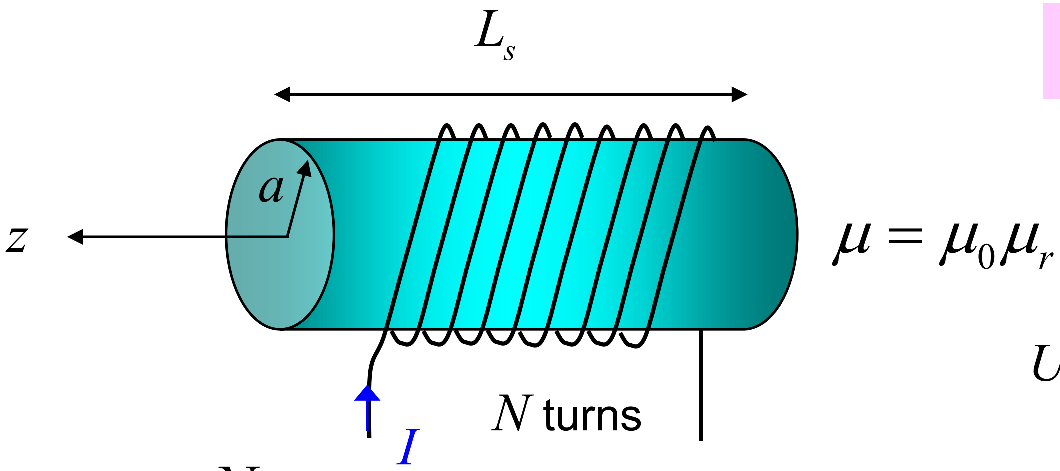
$$U_H = \int_V \frac{1}{2} \underline{B} \cdot \underline{H} \, dV$$

We also have $\underline{B} = \mu \underline{H} = \mu_0 \mu_r \underline{H}$

Hence we can write

$$U_H = \int_V \frac{1}{2} \mu_0 \mu_r \left| \underline{H} \right|^2 dV$$

Example



Find U_H inside solenoid

Assume infinite solenoid approximation.

$$n = \frac{N}{L_s}$$

$$\underline{H} = \hat{\underline{z}}\left(nI\right), \quad \rho < a$$
$$= \underline{0}, \quad \rho > a$$

$$U_H = \int_V \frac{1}{2}\mu_0 \mu_r \left(H_z\right)^2 dV$$

$$\int_V \frac{1}{2}\mu_0 \mu_r \left(nI\right)^2 dV$$

$$= \left(\pi a^2 L_s\right)\left[\frac{1}{2}\mu_0 \mu_r \left(nI\right)^2\right]$$

$$= \frac{1}{2}\pi a^2 L_s \mu_0 \mu_r I^2 \left(\frac{N}{L_s}\right)^2$$

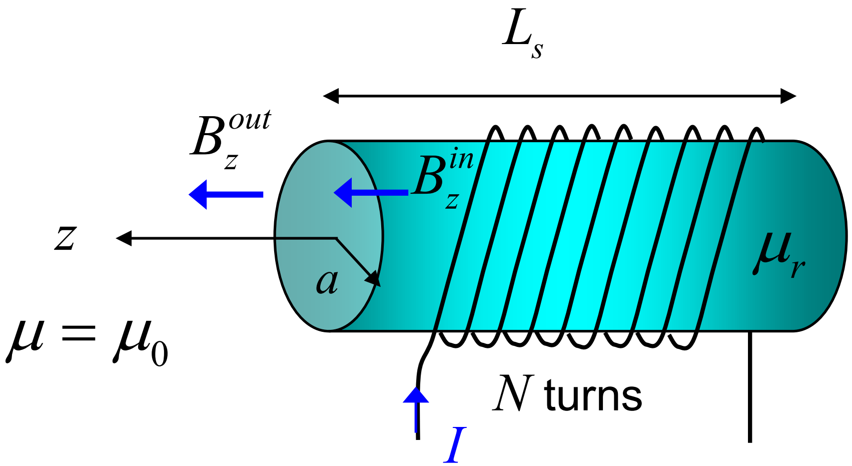
$$U_H = \frac{1}{2}\mu_0 \mu_r \pi a^2 \left(\frac{N^2}{L_s}\right) I^2 \ \ [\mathrm{J}]$$

Finite-Length Solenoid

In a practical <u>finite-length</u> solenoid, the magnetic flux density immediately outside the solenoid is usually what is important.

From BCs:

$$B_z^{in} = B_z^{out}$$



The solenoid is long but finite.

$$\underline{H} \approx \underline{\hat{z}}\left(nI\right), \text{ inside}$$

$$\underline{B} \approx \mu_0 \mu_r \, \underline{\hat{z}}\left(nI\right), \text{ inside}$$

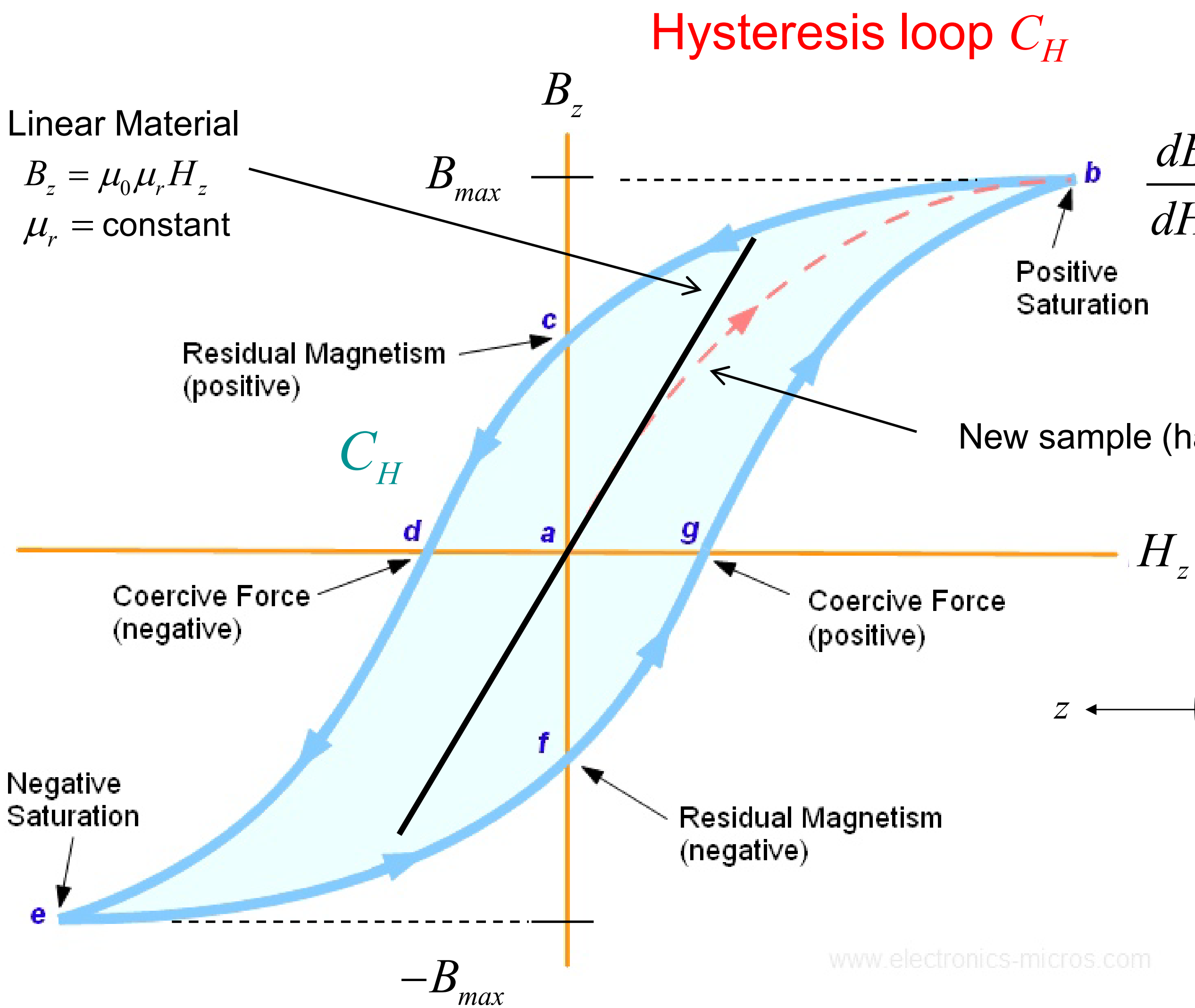
(from BCs)

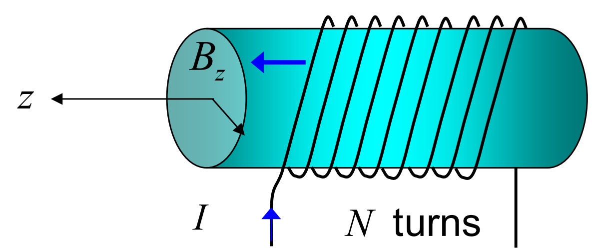
$$B_z^{out} \approx \mu_0 \mu_r \left(nI\right)$$

Note:
It is the strength of the $\underline{B}$ field immediately outside the solenoid that determines the lifting strength of the magnet.
A larger value of μ_r makes better electromagnet!

Hysteresis

Hysteresis is a nonlinear effect that many magnetic materials exhibit.

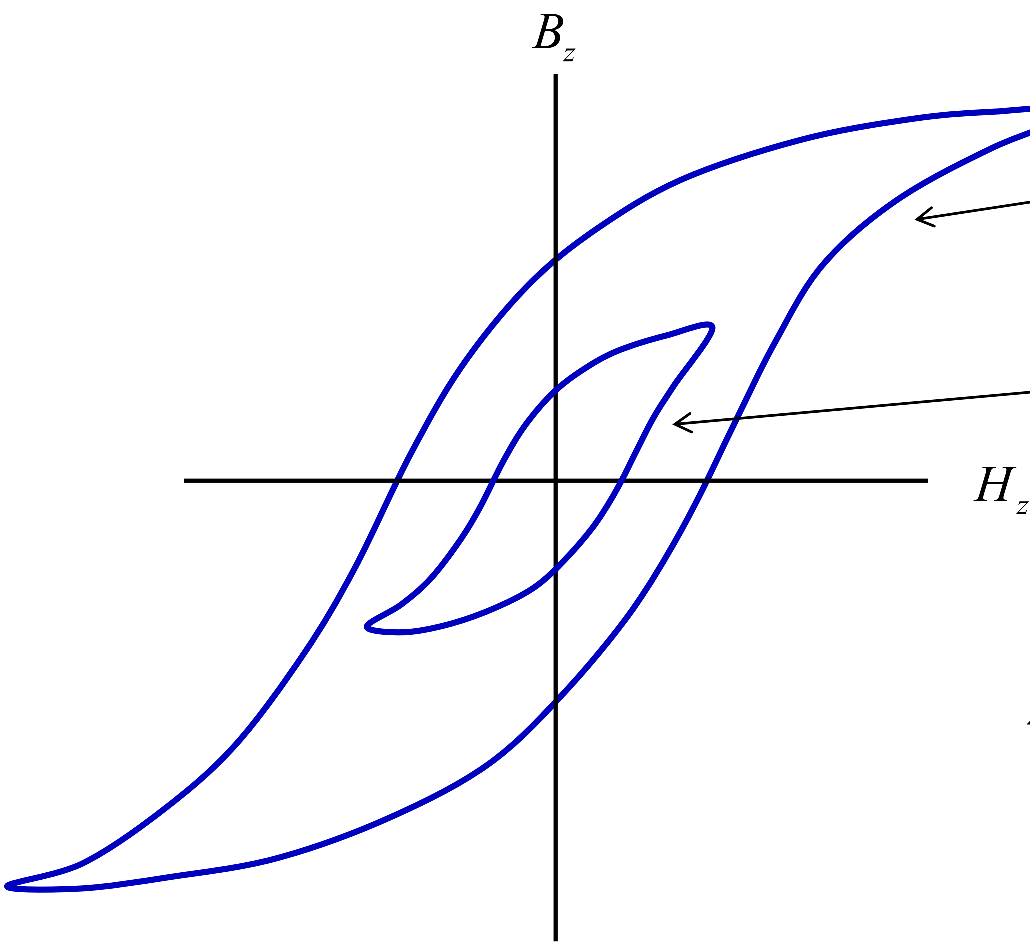


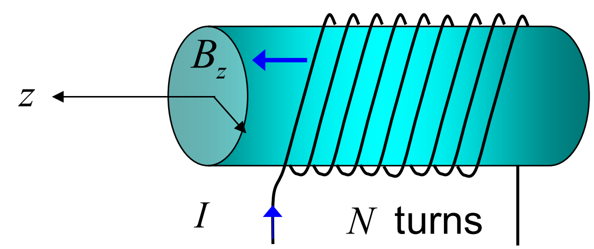


$$H_z = nI, \quad \rho < a$$

Hysteresis (cont.)

Note: The size of the hysteresis loop depends on the maximum current in the coil.

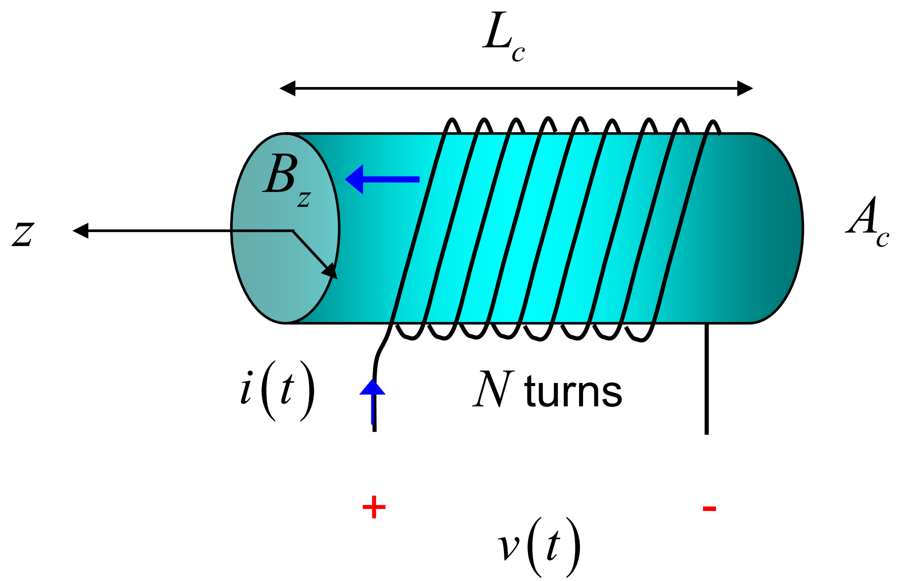




$$H_z = nI, \quad \rho < a$$

Hysteresis (cont.)

Hysteresis causes power loss for an AC magnetic field in a nonlinear material (such as in a transformer core).



f = frequency [Hz]

Hysteresis (cont.)

Power going into coil:

$$p(t) = v(t)i(t)$$
$$= N\frac{d\psi}{dt}i(t)$$
$$= NA_c\frac{dB_z}{dt}i(t)$$
$$= NA_c\frac{dB_z}{dt}i(t)\frac{n}{n}$$
$$= NA_c\frac{dB_z}{dt}H_z\frac{1}{n}$$
$$= (nL_c)A_c\frac{dB_z}{dt}H_z\frac{1}{n}$$
$$= (A_cL_c)\frac{dB_z}{dt}H_z$$
$$= V_c\frac{dB_z}{dt}H_z$$

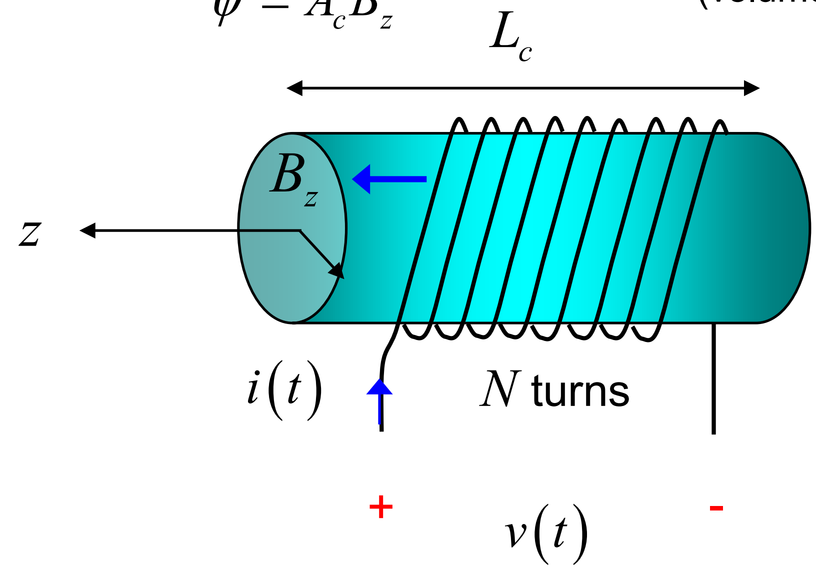
$\psi = \psi_z$

(right-hand rule for inductor flux)

$V_c = A_cL_c$

(volume of core)

$\psi = A_cB_z$



A_c

(area of core)

$H_z = n\,i(t), \quad \rho < a$

Hysteresis (cont.)

Energy going into the coil in <u>one cycle</u> of the waveform (period T):

$$W = \int_0^T p(t)\,dt$$

$$= V_c \int_0^T \frac{dB_z}{dt} H_z\,dt$$

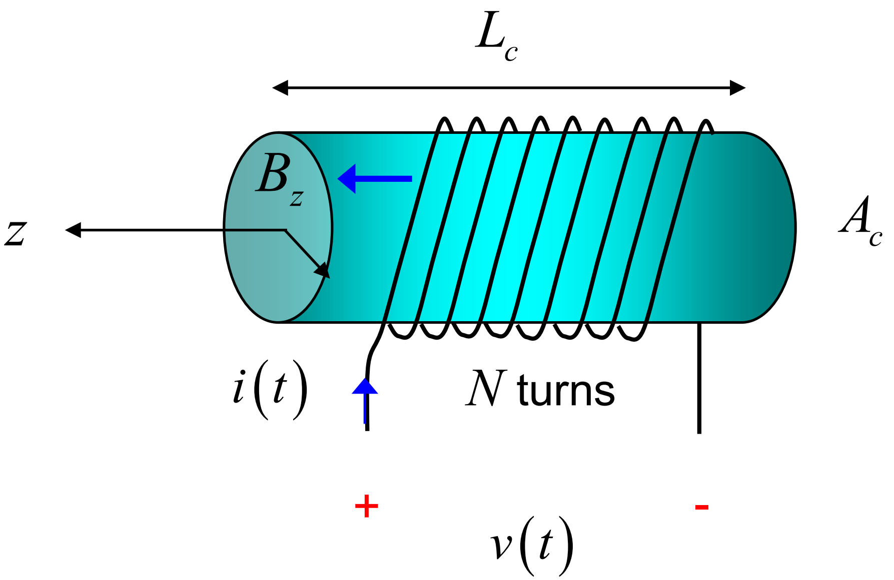
$$= V_c \int_{C_H} H_z dB_z$$

$$= V_c \int_{-B_{max}}^{B_{max}} H_z^{right}\,dB_z + V_c \int_{B_{max}}^{-B_{max}} H_z^{left}\,dB_z$$

$$= V_c \int_{-B_{max}}^{B_{max}} \left(H_z^{right} - H_z^{left}\right) dB_z$$

$$\Rightarrow \quad W = V_c A_h$$

A_h = area inside hysteresis curve



Note:

$$\int_a^b \left(y_2(x) - y_1(x)\right) dx = \text{area between curves}$$

The energy is not zero (there is loss)!

Hysteresis (cont.)

The average power loss (in watts) due to hysteresis is:

$$P_{hys}^{ave} = \frac{W}{T} = Wf = \left(V_c A_f\right) f$$

We thus have

$$P_{hys}^{ave} = A_h f \, V_c \quad [\mathrm{W}]$$

Note:
There would be no hysteresis loss if the core material was <u>linear</u> ($A_h = 0$).

where

A_h = area inside hysteresis curve $[\mathrm{T\ A/m}]$

f = frequency $[\mathrm{Hz}]$

V_c = volume of core $[\mathrm{m}^3]$