ECE 3318
Applied Electricity and Magnetism

Spring 2023

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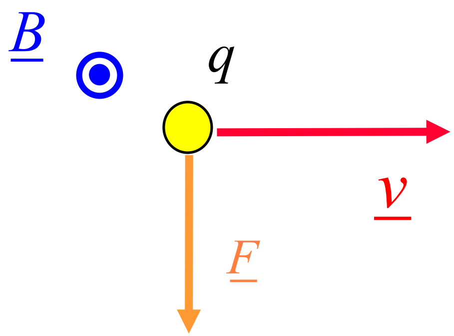
Notes 32
Magnetic Force and Torque

Force on Wire

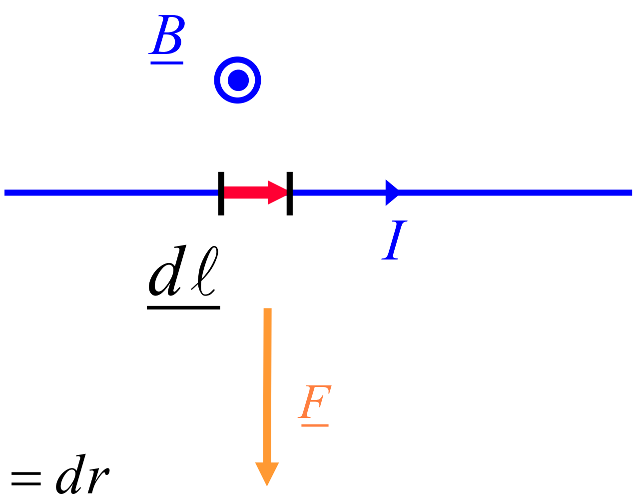
(Lorentz force law)

Single charge: $\underline{F} = q\left(\underline{v} \times \underline{B}\right)$



(derivation omitted)

Wire: $\underline{F} = \int_C I\, \underline{d\ell} \times \underline{B}$



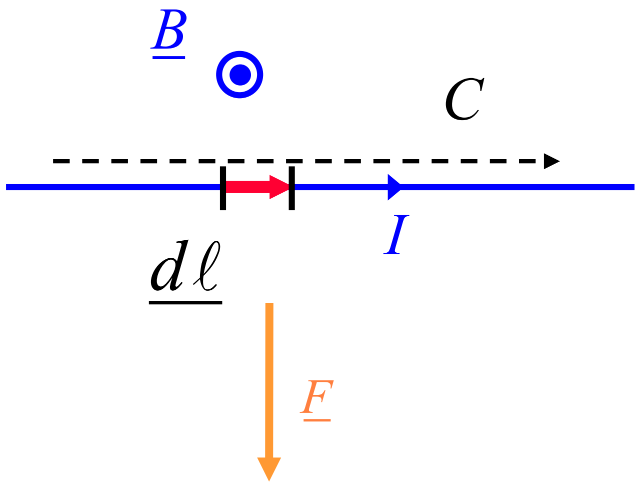
Note : $\underline{d\ell} = d\underline{r}$

Force on Wire (cont.)

Force on wire:

$$\underline{F} = \int_C I \, d\underline{\ell} \times \underline{B}$$

The contour C is in the direction of the reference direction for the current.

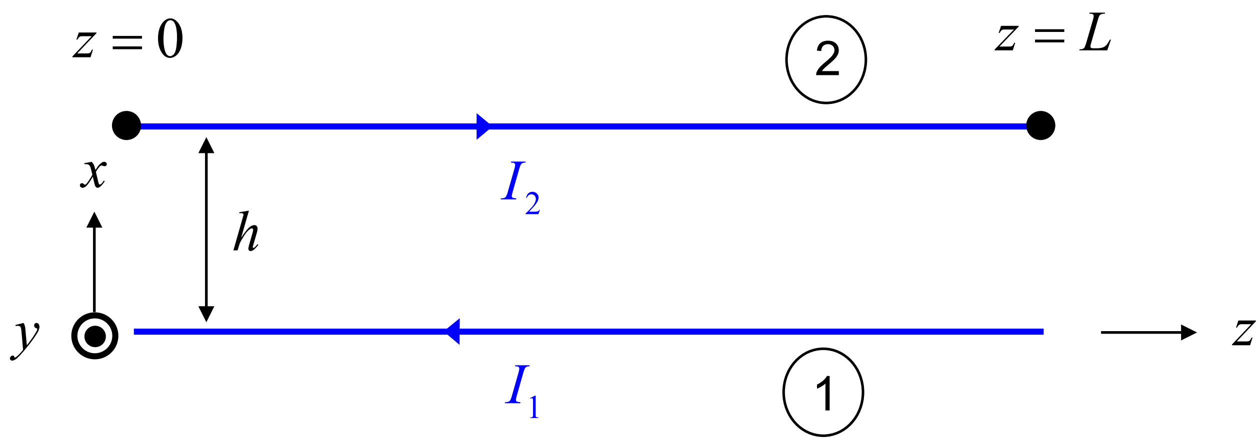


Important Point:

There is no net force on a wire due to the magnetic field produced by itself.
Therefore, the magnetic field $\underline{B}$ in the formula is taken as that due to all other currents (or magnets).

Example

Find the force $\underline{F}_2$ on wire 2

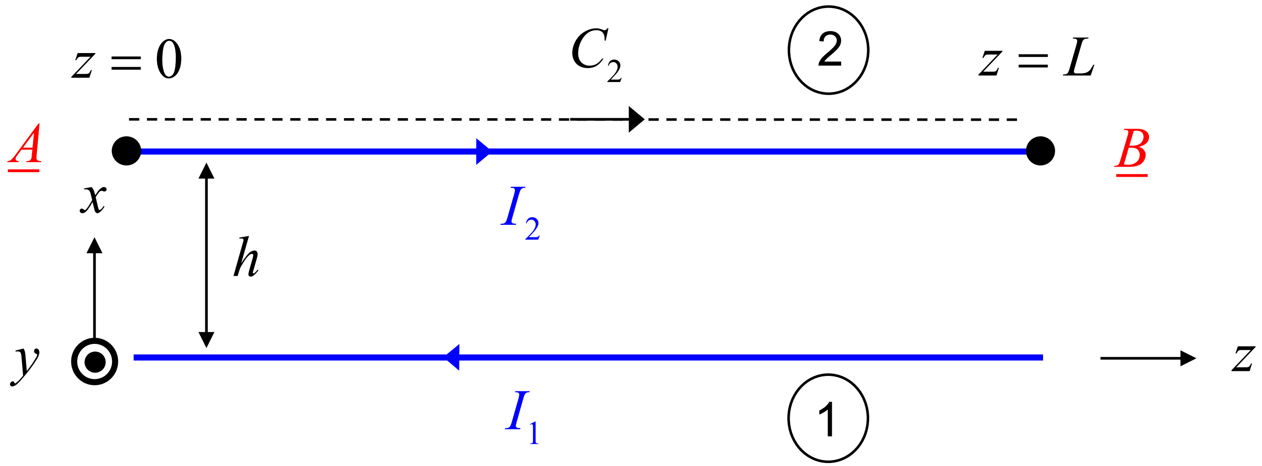


Assume that the wires are long compared to the separation.

Note:
The force on wire 2 comes from the magnetic field due to wire 1.

Example (cont.)

The contour C_2 runs from point $\underline{A}$ to point $\underline{B}$ (left to right), defined by the current reference direction on wire 2.



$$\underline{F} = \int_C I\, \underline{d\ell} \times \underline{B}$$

$$\underline{F}_2 = \int_{C_2} I_2 \left(\underline{\hat{z}}\, dz \right) \times \underline{B}_1$$

Recall:

$$\underline{H} = \underline{\hat{\phi}} \left(\frac{I}{2\pi\rho} \right)$$

(for wire on z axis)

$$\underline{F}_2 = \int_{C_2} I_2 \left(\underline{\hat{z}}\, dz \right) \times \left(\mu_0 \left(-\underline{\hat{y}} \right) \frac{I_1}{2\pi h} \right)$$

Example (cont.)

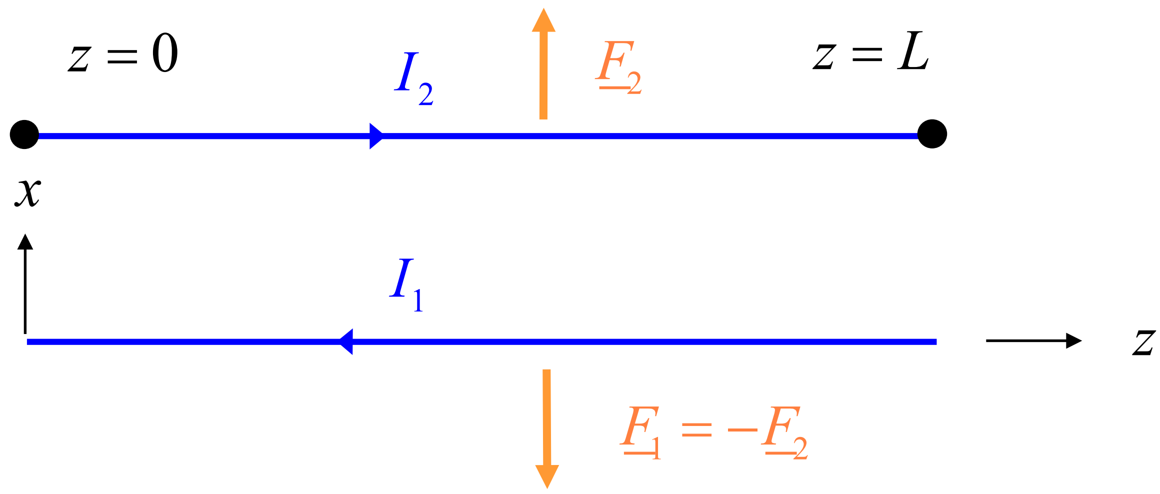
$$\underline{F}_2 = \int_{C_2} I_2 \left(\hat{\underline{z}}\, dz \right) \times \left(\mu_0 \left(-\hat{\underline{y}} \right) \frac{I_1}{2\pi h} \right)$$

$$= \frac{I_1 I_2}{2\pi h} \mu_0 \left(+\hat{\underline{x}} \right) \int_0^L dz$$

$$\underline{F}_2 = \hat{\underline{x}}\, \mu_0 \left(\frac{I_1 I_2}{2\pi h} \right) L \quad [\mathrm{N}]$$

The limits are chosen by the direction of path C_2 (the path runs from $x = 0$ to $x = L$).

(The force is a repulsive force, assuming that both currents are positive.)



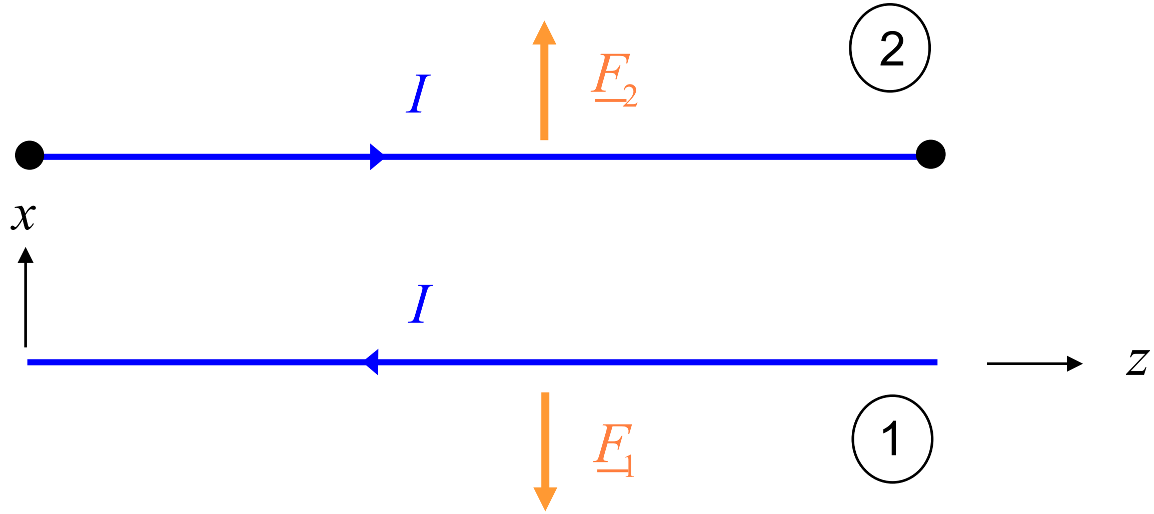
Definition of Amp

Assume $I_1 = I_2 = I$

$$\underline{F}_2 = \underline{\hat{x}}\,\mu_0 \left(\frac{I^2}{2\pi h}\right) L \quad [\mathrm{N}]$$

Note:

$\underline{F}_1 = -\underline{F}_2$



Definition of Amp (prior to May 20, 2019):

$I = 1.0\ [\mathrm{A}]:\quad F_{lx2} = 2\times 10^{-7}\ [\mathrm{N/m}]$ when $h = 1.0 [\mathrm{m}]$

F_{lx2} = force per unit length in x direction on wire 2

Definition of Amp (cont.)

Force per unit length:

$$F_{x2}^{l} = \mu_0 \left(\frac{I^2}{2\pi h} \right) \ \text{[N/m]}$$

From the Amp definition:

$$2 \times 10^{-7} = \mu_0 \left(\frac{(1.0)^2}{2\pi (1.0)} \right)$$

$$\Rightarrow \quad \mu_0 = 4\pi \times 10^{-7} \text{[H/m]}$$

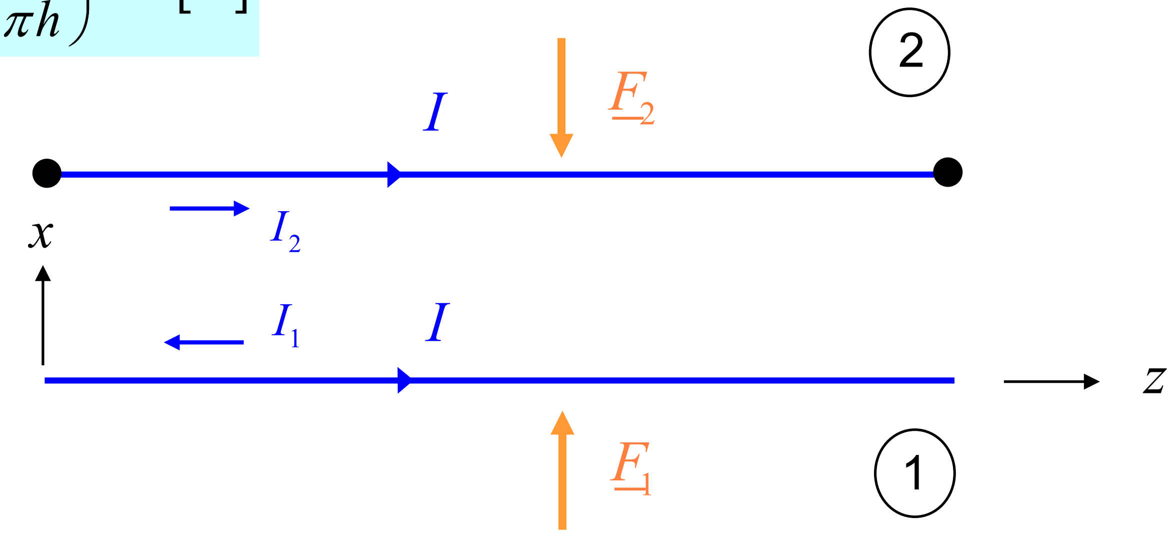
An exact constant!
(prior to May 20, 2019)

Example

In a power substation, the current in two parallel buses is 1 [kA]. The buses are 1 [m] apart.

What is the force per unit length between the two buses?

$$\underline{F}_2 = \underline{\hat{x}}\,\mu_0 \left(\frac{I_1 I_2}{2\pi h} \right) L \quad [\mathrm{N}]$$



$$I_2 = I$$

$$I_1 = -I$$

$$\underline{F}_2^l = -\underline{\hat{x}}\,\mu_0 \left(\frac{I^2}{2\pi h} \right) \quad [\mathrm{N/m}]$$ (attractive force)

Example (cont.)

$$\underline{F}_2^l = -\hat{\underline{x}}\left(4\pi \times 10^{-7}\right)\left(\frac{\left(1\times 10^3\right)^2}{2\pi\left(1.0\right)}\right)$$

$$\underline{F}_2^l = -\hat{\underline{x}}\left(0.20\right)\ \text{[N/m]}$$

or

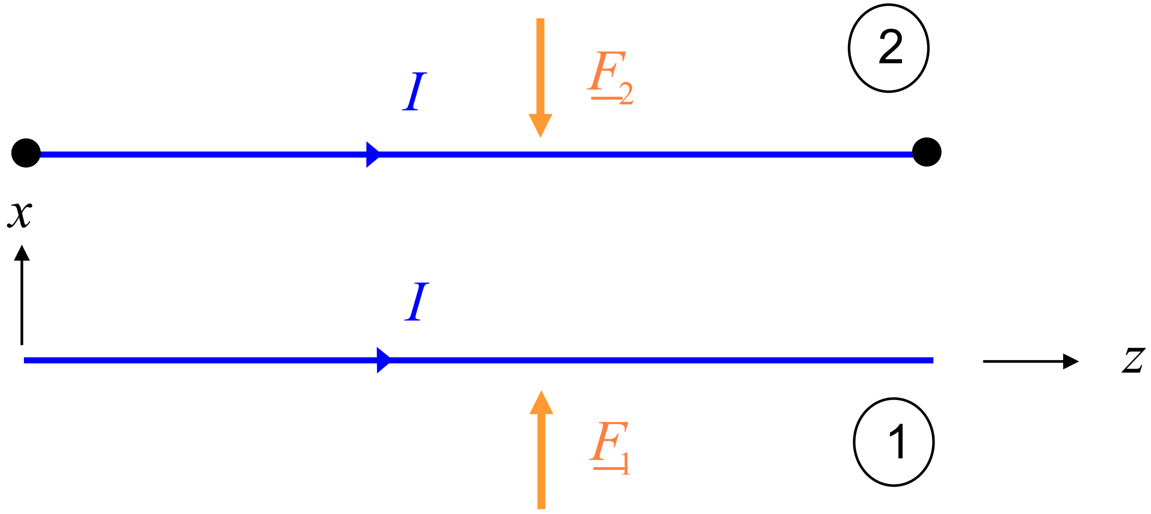
$$\underline{F}_2^l = -\hat{\underline{x}}\left(0.045\right)\ \text{[lbs/m]}$$

Note: 1 [kG] $\rightarrow$ 9.8 [N] = 2.2 [lbs]

Example

During a lighting strike, the current in two parallel wires inside the wall of a house reaches 10 [kA]. The wires are 1 [cm] apart.

What is the force per unit length between the two wires?



$$\underline{F}_2^l = -\underline{\hat{x}}\,\mu_0\left(\frac{I^2}{2\pi h}\right)\ [\mathrm{N/m}]$$ (attractive force)

Example (cont.)

$$\underline{F}_2^l = -\underline{\hat{x}}\left(4\pi \times 10^{-7}\right)\left(\frac{\left(10 \times 10^3\right)^2}{2\pi\left(0.01\right)}\right)$$

$$\underline{F}_2^l = -\underline{\hat{x}}\left(2000\right) \text{ [N/m]}$$

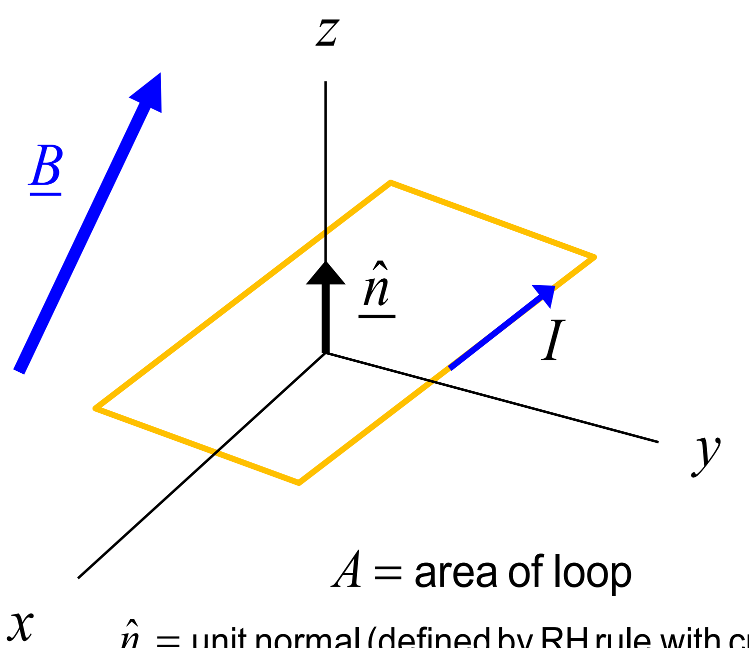
or

$$\underline{F}_2^l = -\underline{\hat{x}}\left(449.0\right) \text{ [lbs/m]}$$

Note: 1 [kG] $\rightarrow$ 9.8 [N] = 2.2 [lbs]

Torque on Current Loop

A planar current loop of wire with an <u>arbitrary shape</u> carries a current I.



A = area of loop

$\underline{\hat{n}}$ = unit normal (defined by RH rule with current)

Define *magnetic dipole moment* of loop:

$$\underline{m} \equiv \underline{\hat{n}}\left(AI\right)$$

Note:
There is no net <u>force</u> on the loop if the magnetic field is a constant.
But there is a <u>torque</u>.

$$\underline{F} = I\left(\oint_C \underline{d\ell}\right) \times \underline{B} = \underline{0}$$

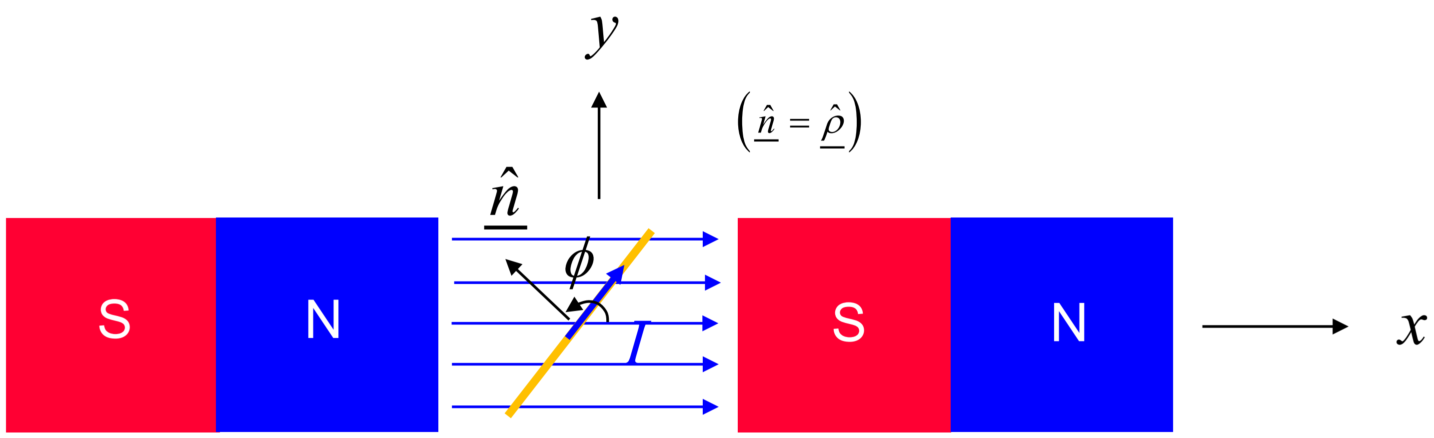
$\underline{B}$ = magnetic flux density vector (assumed constant over the loop)

Torque vector on loop: $\underline{T} = \underline{m} \times \underline{B}$ (Please see the textbooks for a derivation.)

Note: Put in a factor of N for an N-turn loop.

DC Motor

A loop rotating in a DC magnetic field is shown below.



$\underline{B} = \underline{\hat{x}} B_0$

$$\underline{T} = \underline{m} \times \underline{B} = (\underline{\hat{n}} AI) \times \underline{B} = AI(\underline{\hat{x}} \cos\phi + \underline{\hat{y}} \sin\phi) \times (\underline{\hat{x}} B_0)$$

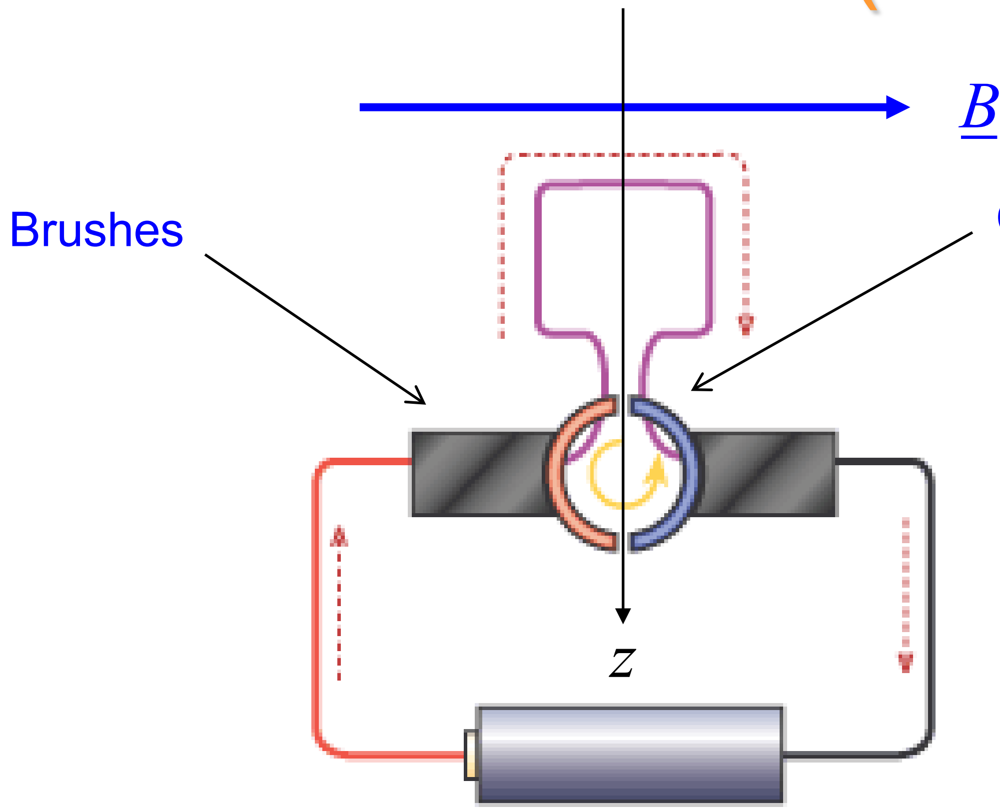
$$\underline{T} = -\underline{\hat{z}}(AIB_0)\sin\phi$$

The average value is zero!

A *commutator* is needed to reverse the current every 180° and make the torque in same direction.

http://en.wikipedia.org/wiki/DC_motor

DC Motor (cont.)



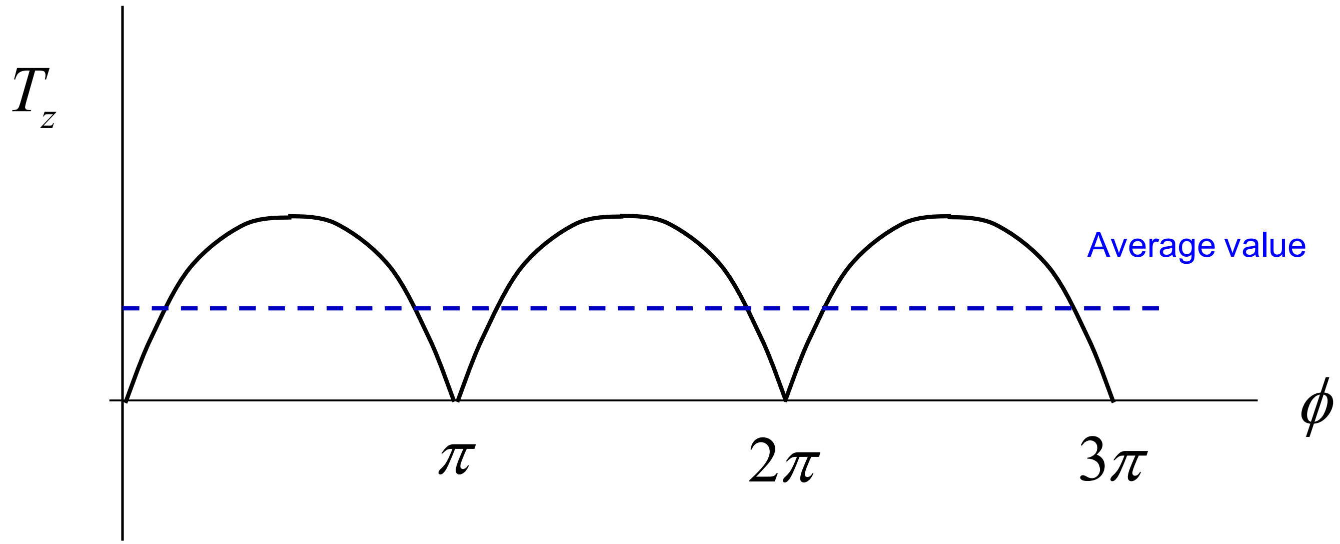
Note:
The rings are positioned so that the torque changes sign (the current in the loop reverses) at the point where the torque is zero.

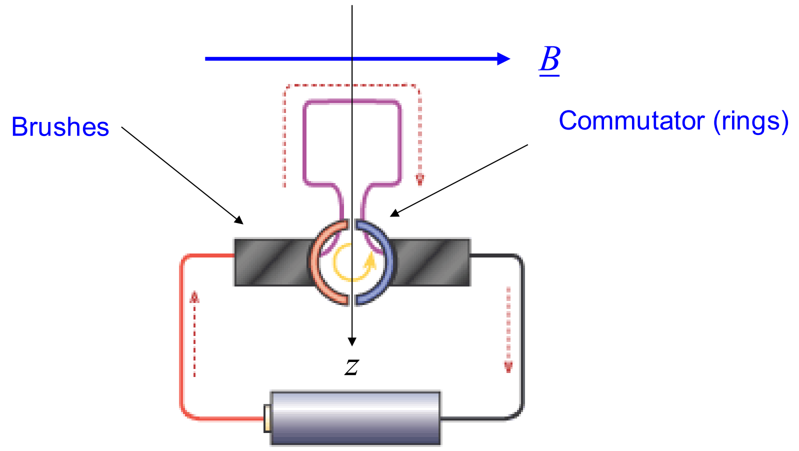
The *commutator* reverses the loop current every 180° of rotation.
(It keeps the current flowing clockwise in the picture above.)

$$\underline{T} = \underline{\hat{z}}\left(AIB_0\right)\left|\sin\phi\right|$$

http://en.wikipedia.org/wiki/Commutator_(electric)

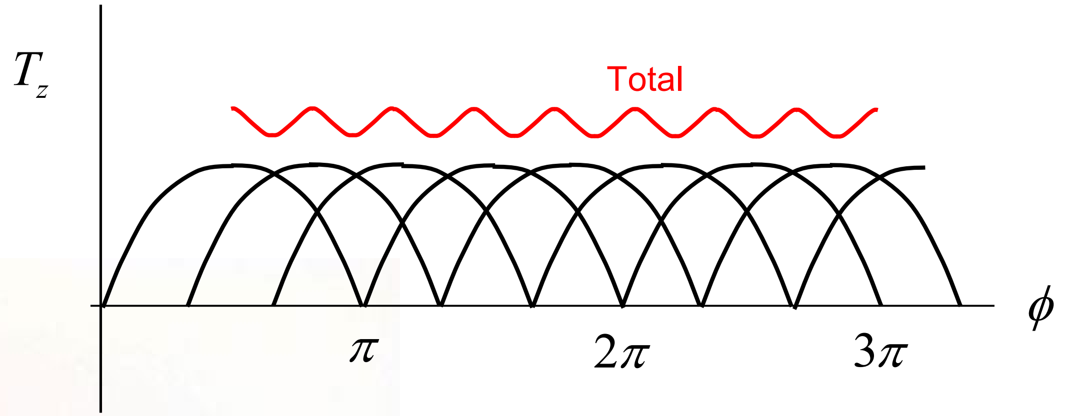
DC Motor (cont.)





DC Motor (cont.)

In practice, there are multiple loops and commutator segments. The torque is thus more constant as the armature turns.



http://www.mmsonline.com/columns/gaging-commutators

Example

In a DC motor, the armature consists of $N = 10{,}000$ turns (loops) of wire, of length $L = 0.1$ [m] in length (parallel to the z axis). The magnetic flux density produced by the stator is $B = 0.5$ [T]. The radius of the armature is $R = 0.05$ [m].

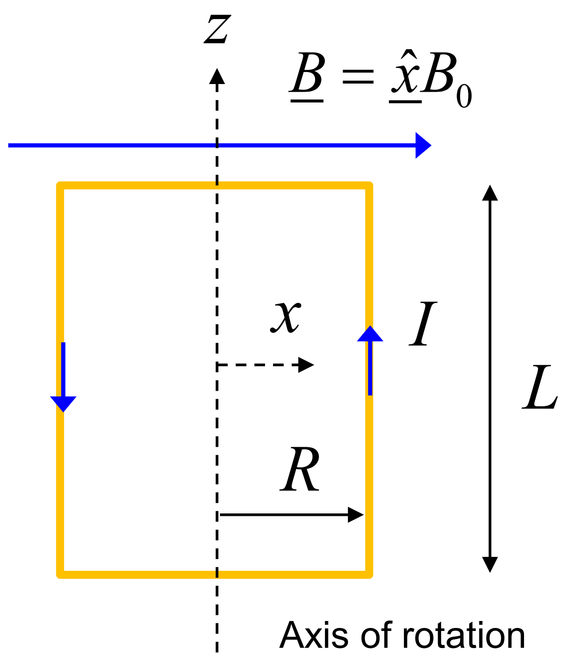
Find the maximum torque on the armature. The current through the motor is 3 [A].

Assume that the magnetic field is constant and perpendicular to the loop axis (i.e., we are at the point of maximum torque in the rotation cycle).

$$\underline{T} = N\left[\underline{m} \times \left(\underline{\hat{x}} B_0\right)\right]$$

$$\underline{m} = -\underline{\hat{y}}\left(AI\right)N$$

$$A = L\left(2R\right)$$



$N = 10^4, \;\; B_0 = 0.5[T], \;\; I = 3\,[\mathrm{A}], \;\; L = 0.1\,[\mathrm{m}], \;\; R = 0.05\,[\mathrm{m}]$

Example (cont.)

We then have

$$\underline{T} = \underline{\hat{z}}T_z = \underline{\hat{z}}\left(NAIB_0\right)$$

$$A = L\left(2R\right)$$

Hence

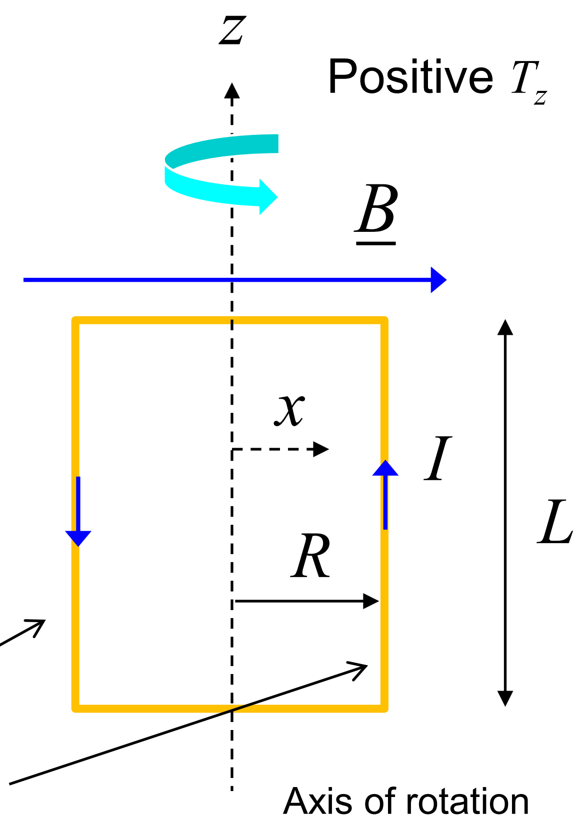
$$T_z = 150 \text{ [Nm]}$$

$$\left(110.5 \text{ [foot lb]}\right)$$

$$\underline{F} = \int_C I\, \underline{d\ell} \times \underline{B}$$

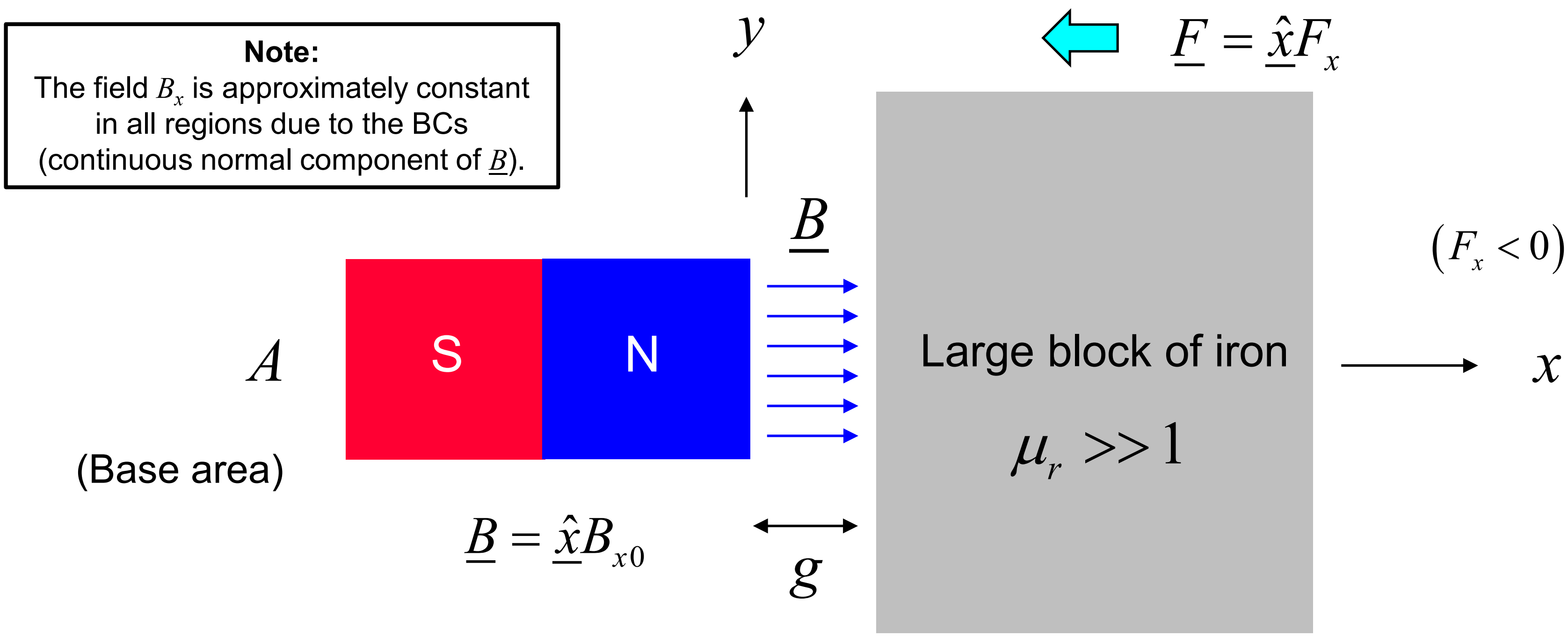
Force is pointing up on left side

Force is pointing down on right side



Note: The top and bottom parts of the loop do not contribute to the torque.

Force from a Magnet

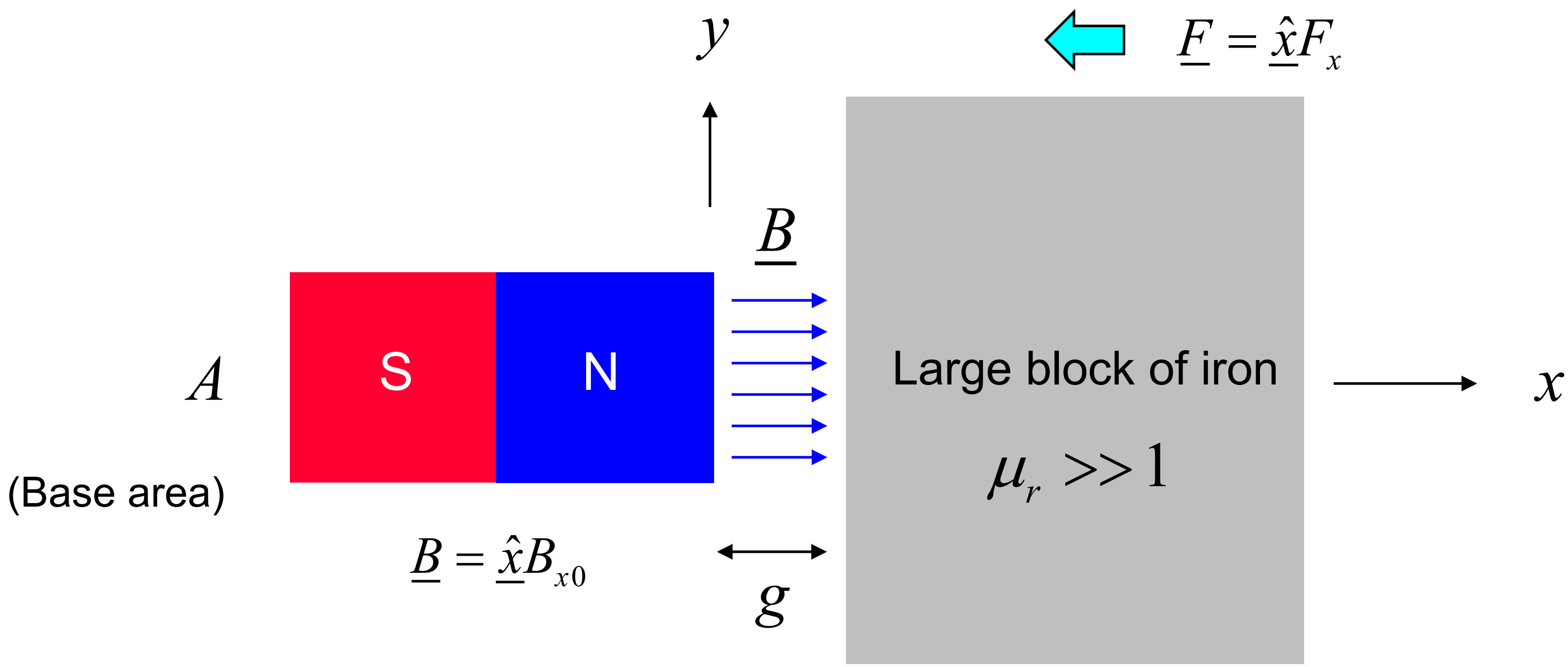


Assume that most of the stored energy is inside the <u>gap</u> region (where there is air, and $H_x = B_x / \mu$ is the strongest).

$$U_H = \frac{1}{2} \int_V \underline{B} \cdot \underline{H} \, dV \approx \frac{1}{2\mu_0} \int_{V_{gap}} B_x^2 \, dV \approx \frac{1}{2\mu_0} (Ag) B_{x0}^2 = g \left(\frac{A}{2\mu_0} B_{x0}^2 \right)$$

The magnetic field is assumed to be constant inside the air gap region, since the gap is small.

Force from a Magnet (cont.)



Principle of "virtual work":

$$dU_H = (-F_x)\,dg$$

so

$$F_x = -\frac{dU_H}{dg}$$

Note:
The force $-\underline{F}$ is the force we would exert on the block of iron to keep it fixed in position.

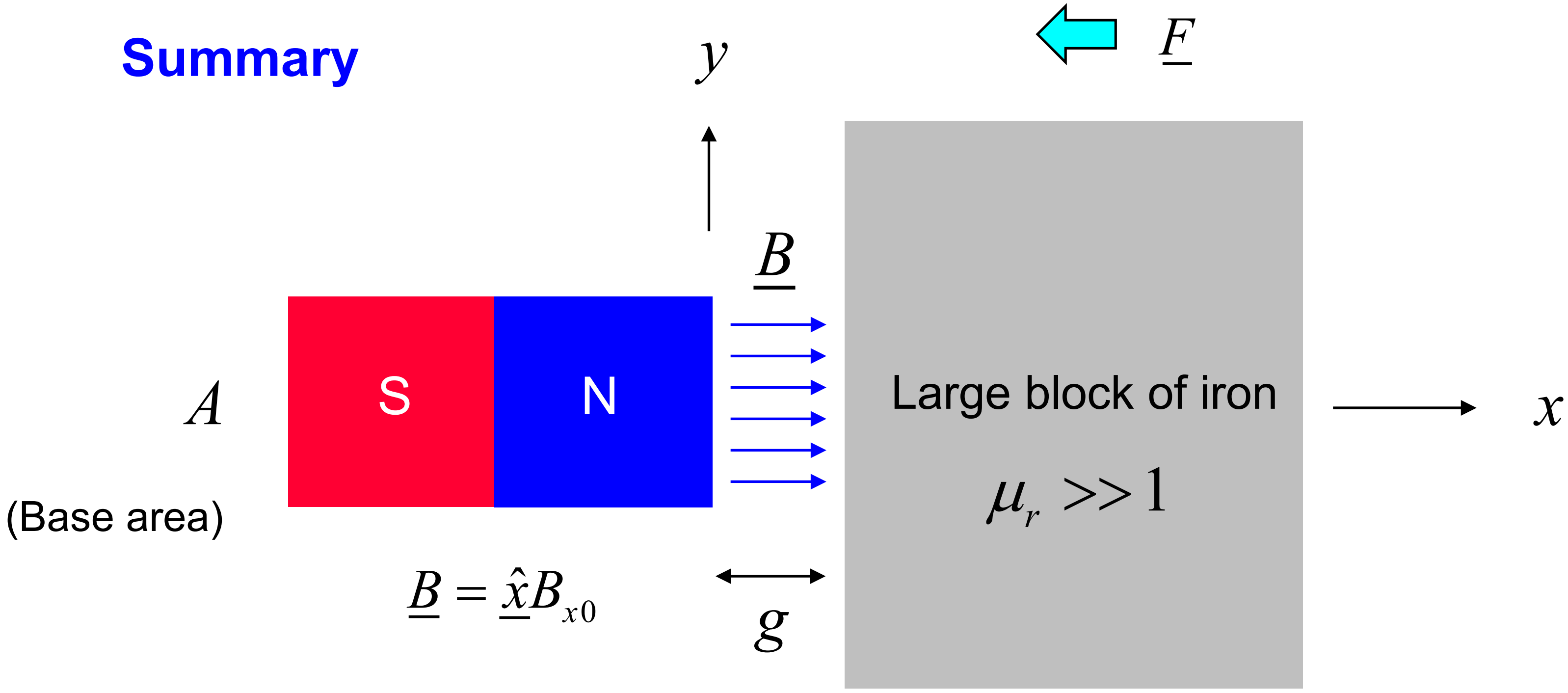
$$U_H = g\left(\frac{A}{2\mu_0} B_{x0}^2\right)$$

We then have:

$$F_x = -\frac{A}{2\mu_0} B_{x0}^2$$

Force from a Magnet (cont.)

Summary



$$\underline{F} = -\underline{\hat{x}} \frac{A}{2\mu_0} B_{x0}^2$$