ECE 3318
Applied Electricity and Magnetism

Spring 2023

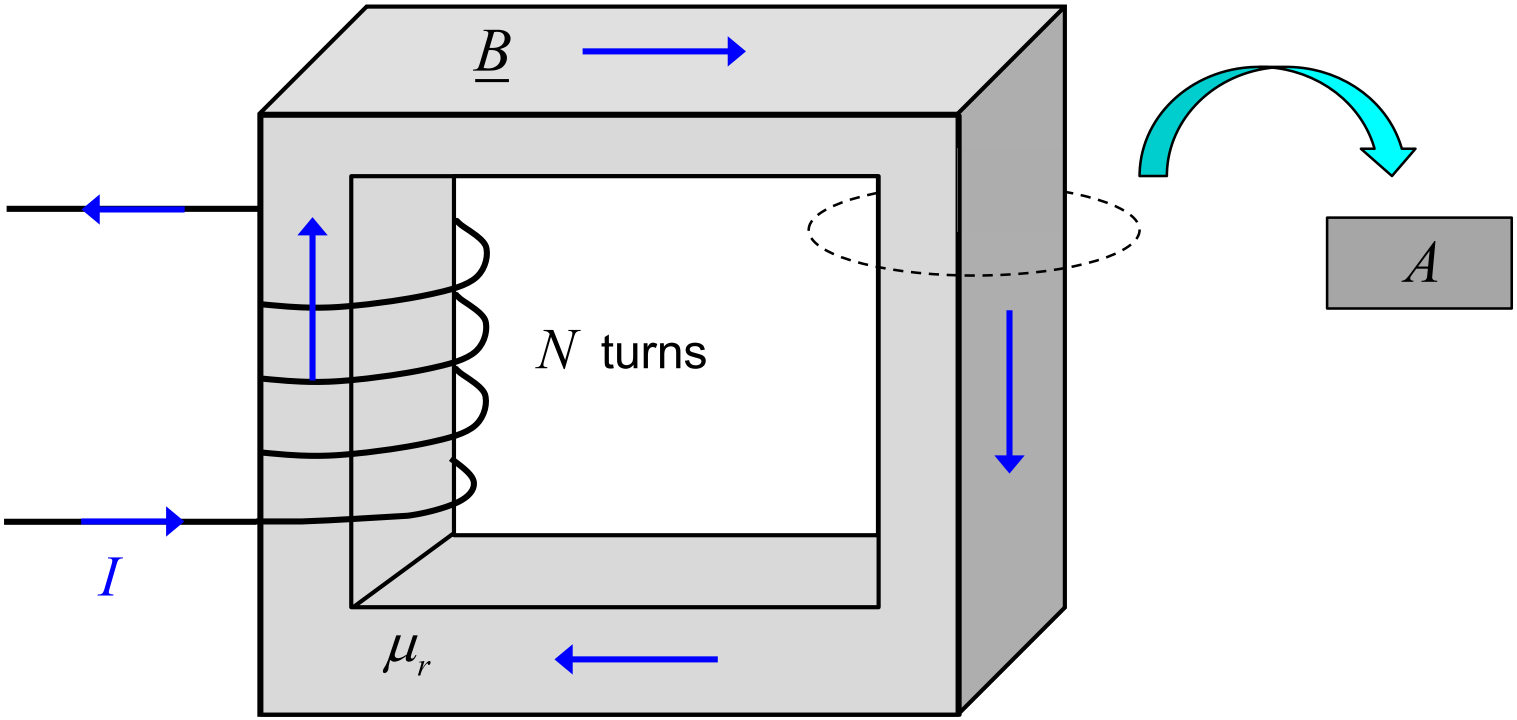
Prof. David R. Jackson
Dept. of ECE

Notes 34
Magnetic Circuits

Magnetic Circuits

Consider a transformer type of core:

$$B = \left| \underline{B} \right|$$



A = cross-sectional area of core

L = length of core (going all the way around)

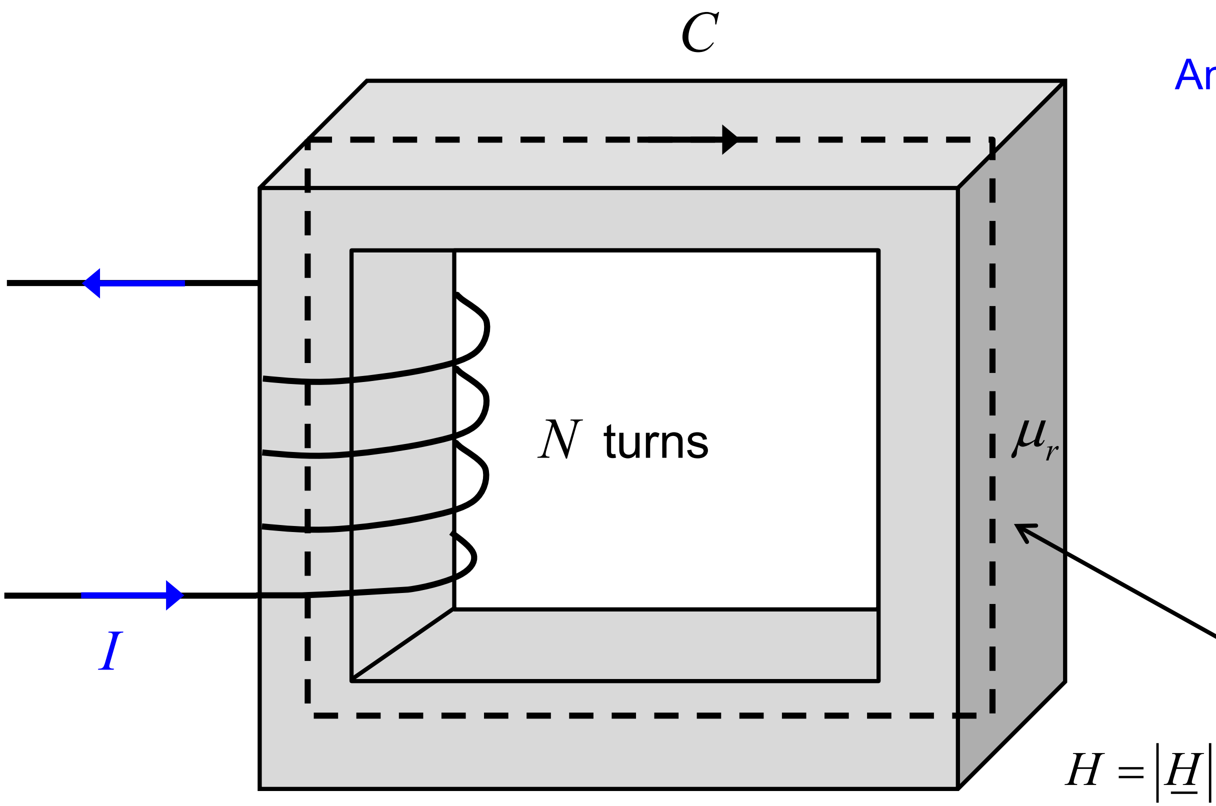
Magnetic Circuits (cont.)

Apply Ampere's law:

$$\oint_C \underline{H} \cdot d\underline{r} = I_{encl} = +NI$$

Assume that the magnetic flux in the core is <u>constant</u>.

This assumption will be accurate if the core has a high permeability (not much flux leakage from the core).



Ampere's law:

$$\oint_C \underline{H} \cdot d\underline{r} = NI$$

$$\oint_C |\underline{H}| \, dl = NI$$

$$HL = NI$$

The contour C is defined in the same direction as the magnetic field for convenience.

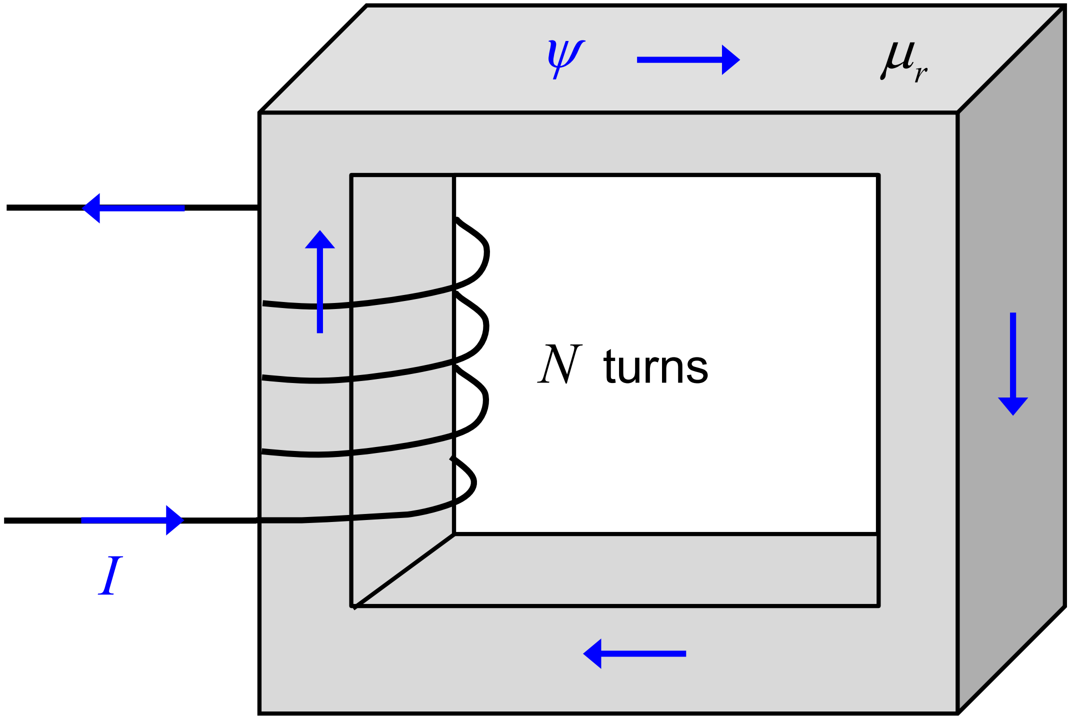
L = length of core (going all the way around)

Magnetic Circuits (cont.)

Assuming ψ is constant, the magnetic flux in the core is then:

$$\psi = BA = \left(\mu_0 \mu_r H\right) A = \mu_0 \mu_r \left(HL\right)\frac{A}{L} = HL\left(\frac{\mu_0 \mu_r A}{L}\right) = NI\left(\frac{\mu_0 \mu_r A}{L}\right)$$

Recall: $HL = NI$



Hence we have

$$\psi = \frac{NI}{\left(\dfrac{L}{\mu_0 \mu_r A}\right)}$$

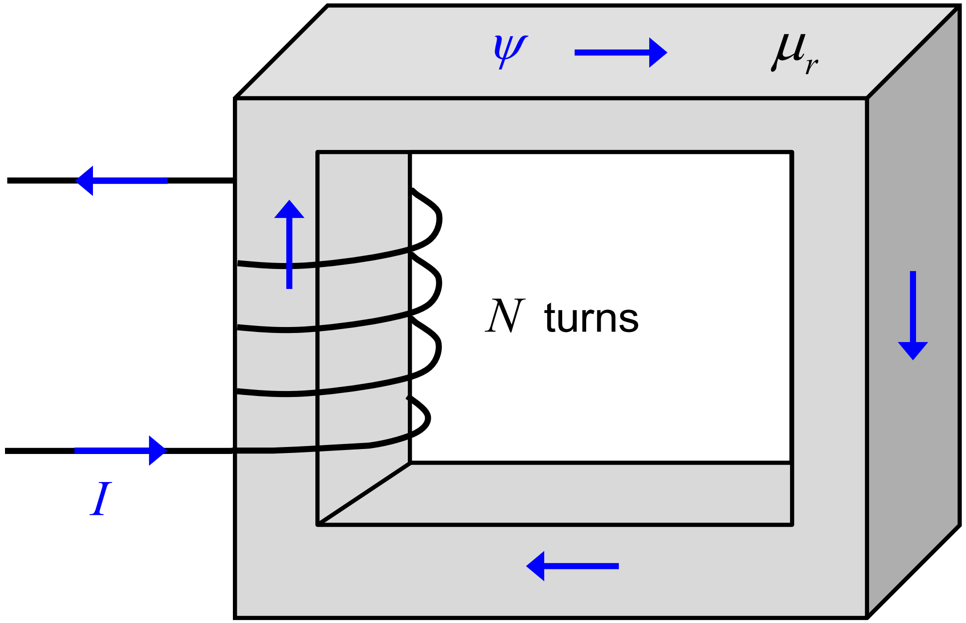
Magnetic Circuits (cont.)

Define the reluctance ("magnetic resistance") of the core:

$$R_m \equiv \frac{L}{\mu_0 \mu_r A}$$

We then have:

$$\psi = \frac{V_m^s}{R_m}$$



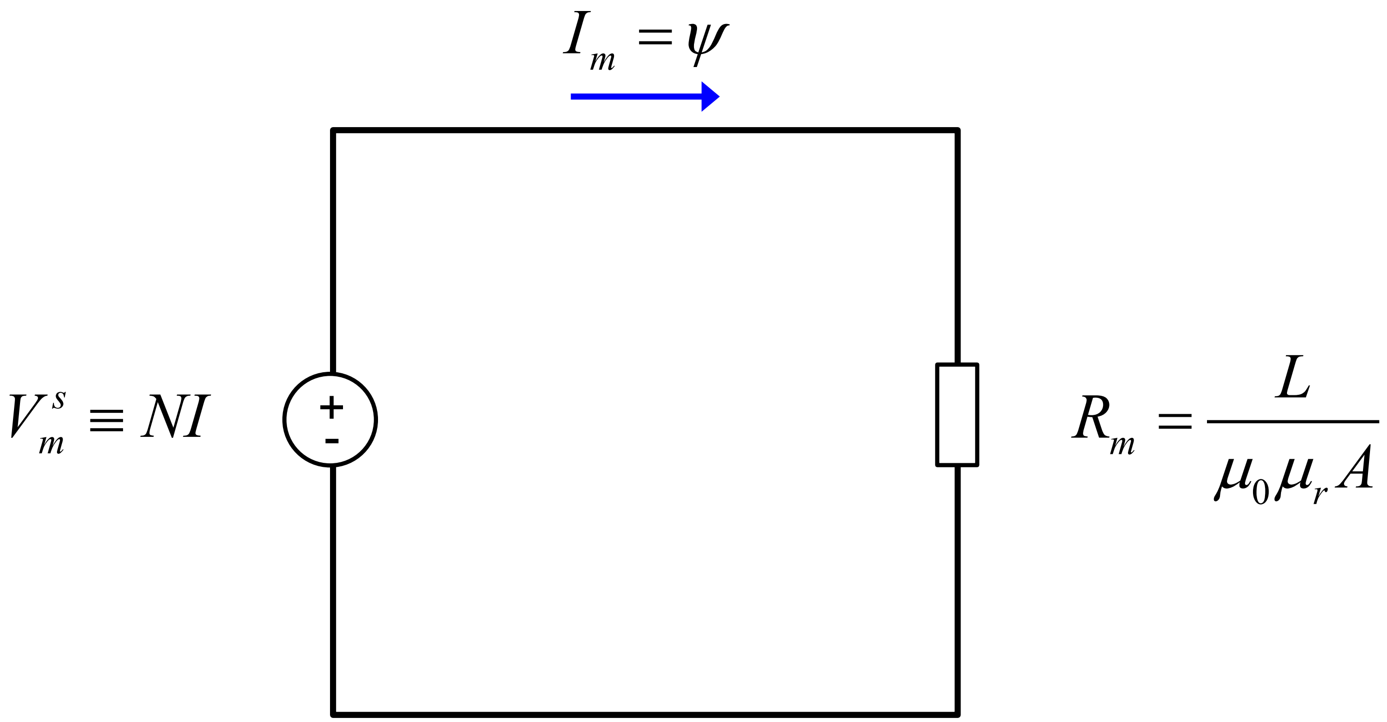
where

$$V_m^s \equiv NI$$

"magnetic voltage source"

Magnetic Circuits (cont.)

Circuit model of the system ("magnetic circuit"):



Magnetic Circuits (cont.)

Circuit analogy for the system:

$$I_m \equiv \psi$$

$$V_m^s \equiv NI$$

$$R_m \equiv \frac{L}{\mu_0 \mu_r A}$$

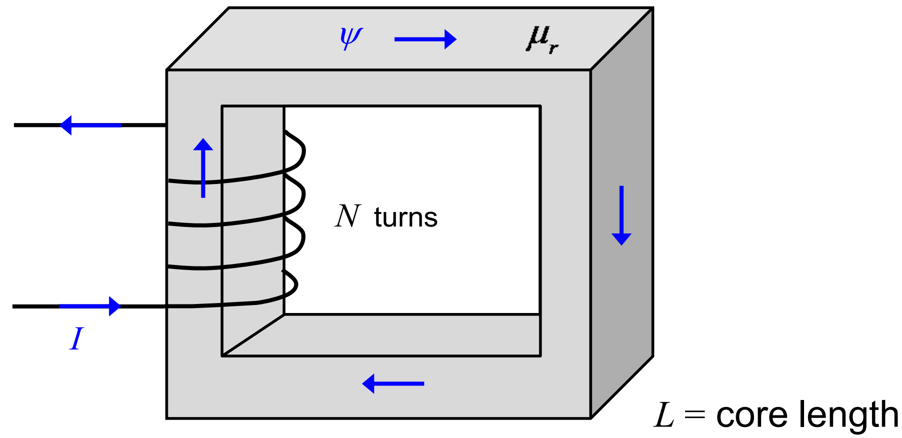
Magnetic Circuits (cont.)

Using a magnetic circuit, we can calculate inductance or mutual inductance (for a pair of coils).

Example (Find coil inductance L_{coil}):

$$L_{\text{coil}} = \frac{N\psi}{I}$$

$$\psi = I_m$$

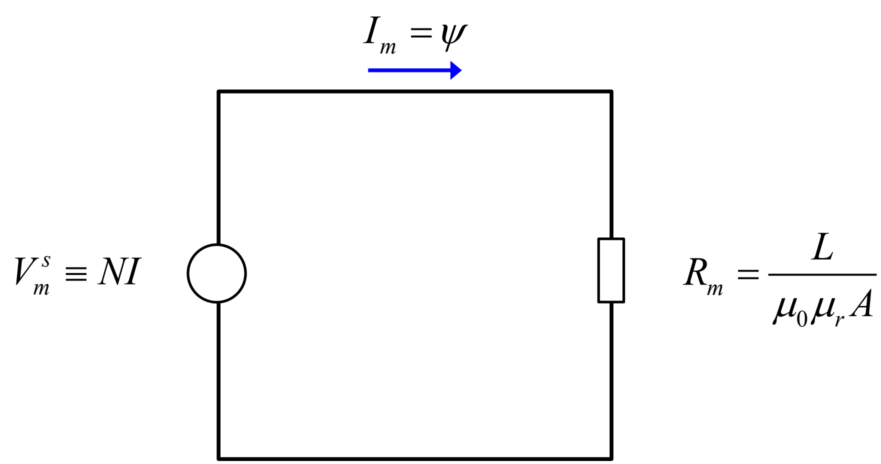


Hence,

$$L_{\text{coil}} = \frac{N}{I}\left(\frac{V_m}{R_m}\right) = \frac{N}{I}\left(\frac{NI}{R_m}\right)$$

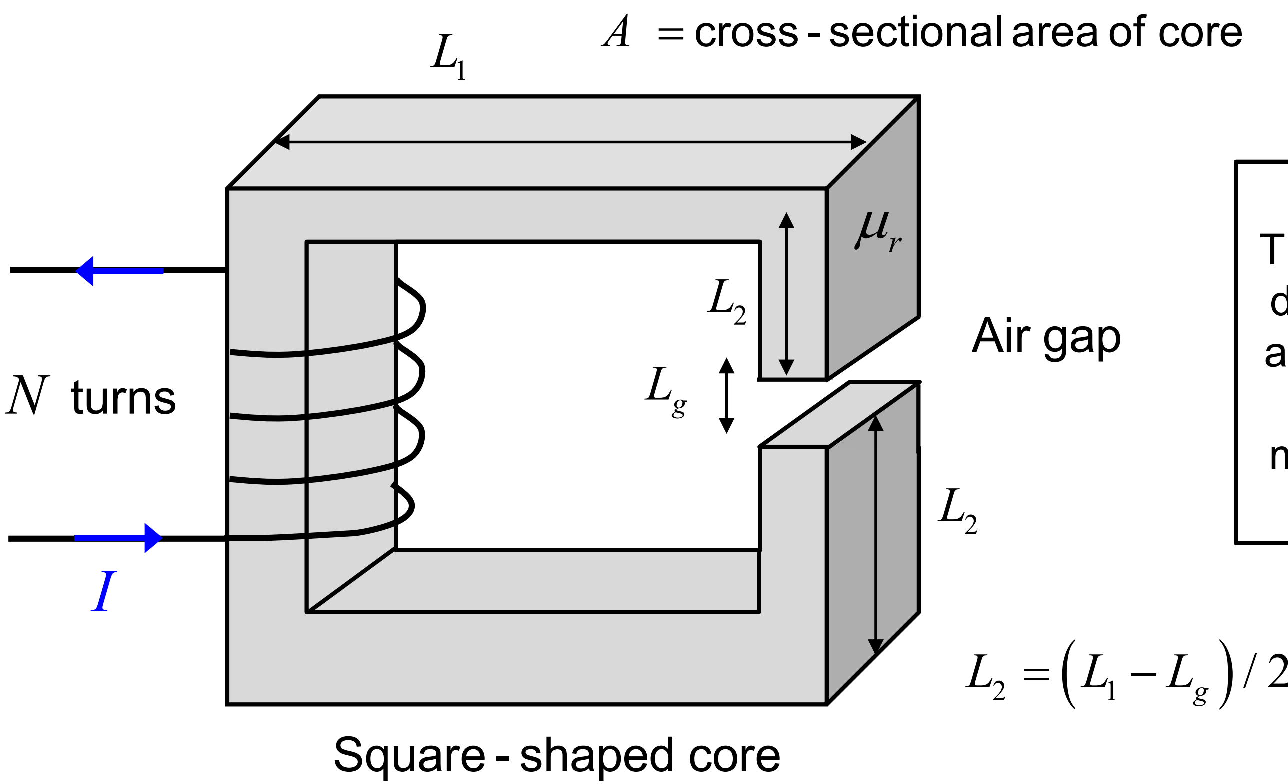
so

$$L_{\text{coil}} = \frac{N^2}{R_m}$$



Magnetic Circuits (cont.)

The concept of reluctance allows us to easily solve more complicated magnetic circuit problems.

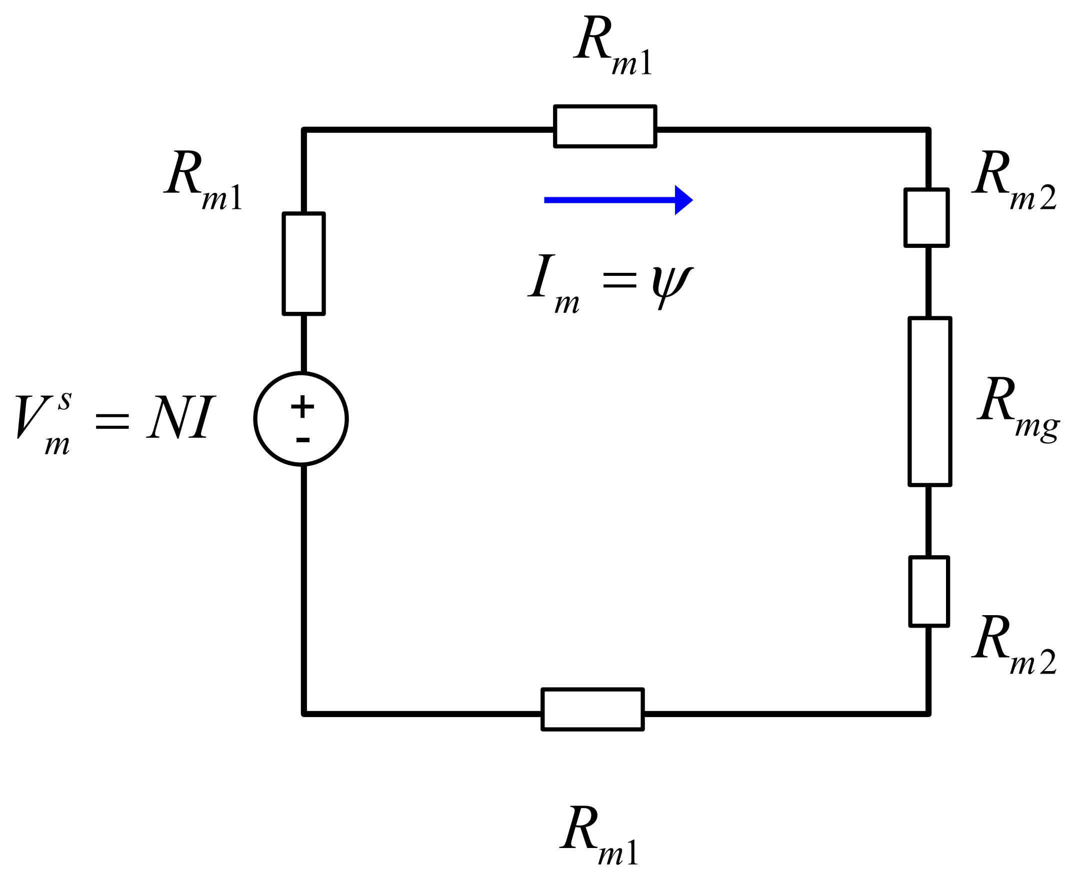


Note: The magnetic flux does not change as we go through the air gap (so magnetic current is continuous).

Each segment of the structure is a reluctance.

Magnetic Circuits (cont.)

This is the circuit model of the previous structure.



$$R_{m1} = \frac{L_1}{\mu_0 \mu_r A}$$

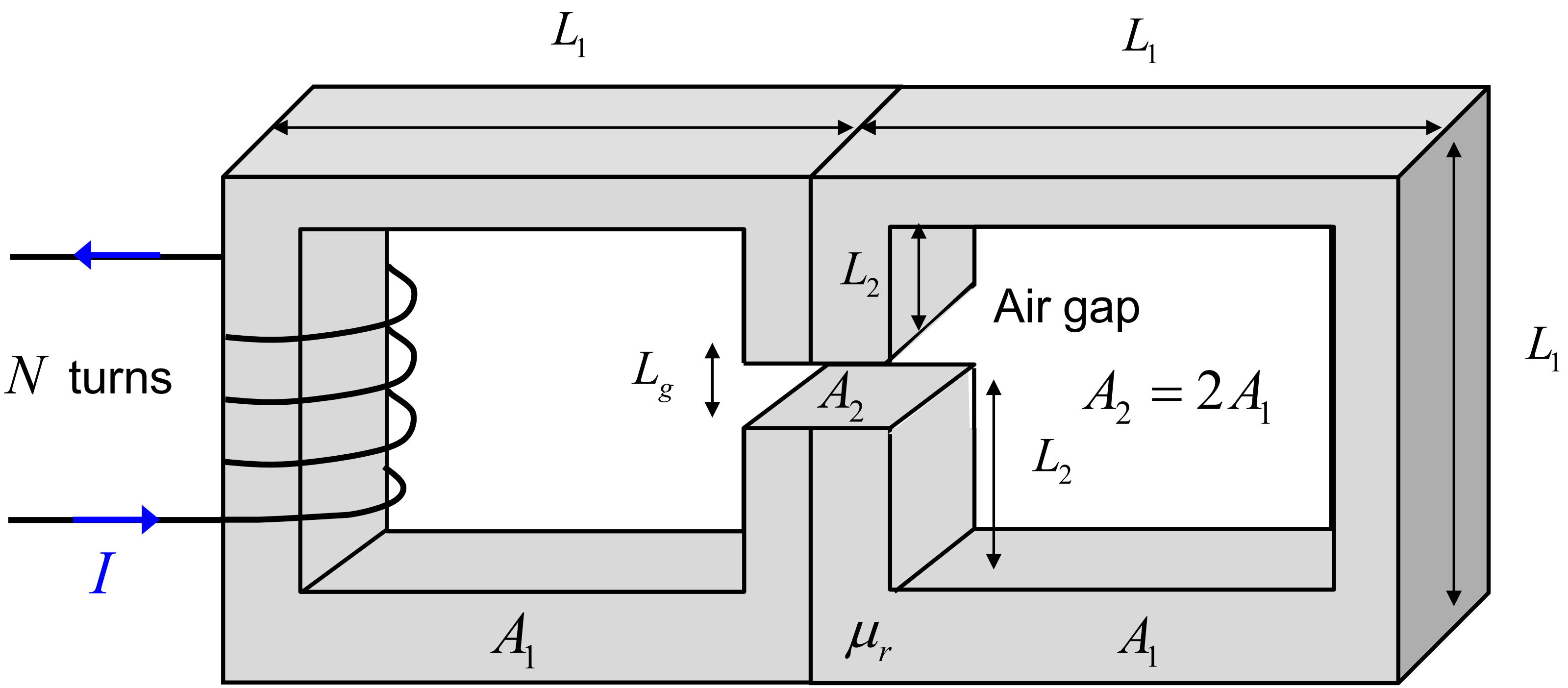
$$R_{m2} = \frac{L_2}{\mu_0 \mu_r A}$$

$$R_{mg} = \frac{L_g}{\mu_0 A}$$

Note : $R_{mg} >> \left(R_{m1}, R_{m2}\right)$ if $\mu_r >> 1$ (Air gaps in a core choke off the magnetic flux!)

Magnetic Circuits (cont.)

Another example is shown here.



$$L_2 = \left(L_1 - L_g\right)/2$$

Each segment of the structure is a reluctance.

Note: The cross sectional area A is different for the middle segments.

$$R_{m1} = \frac{L_1}{\mu_0 \mu_r A_1}$$

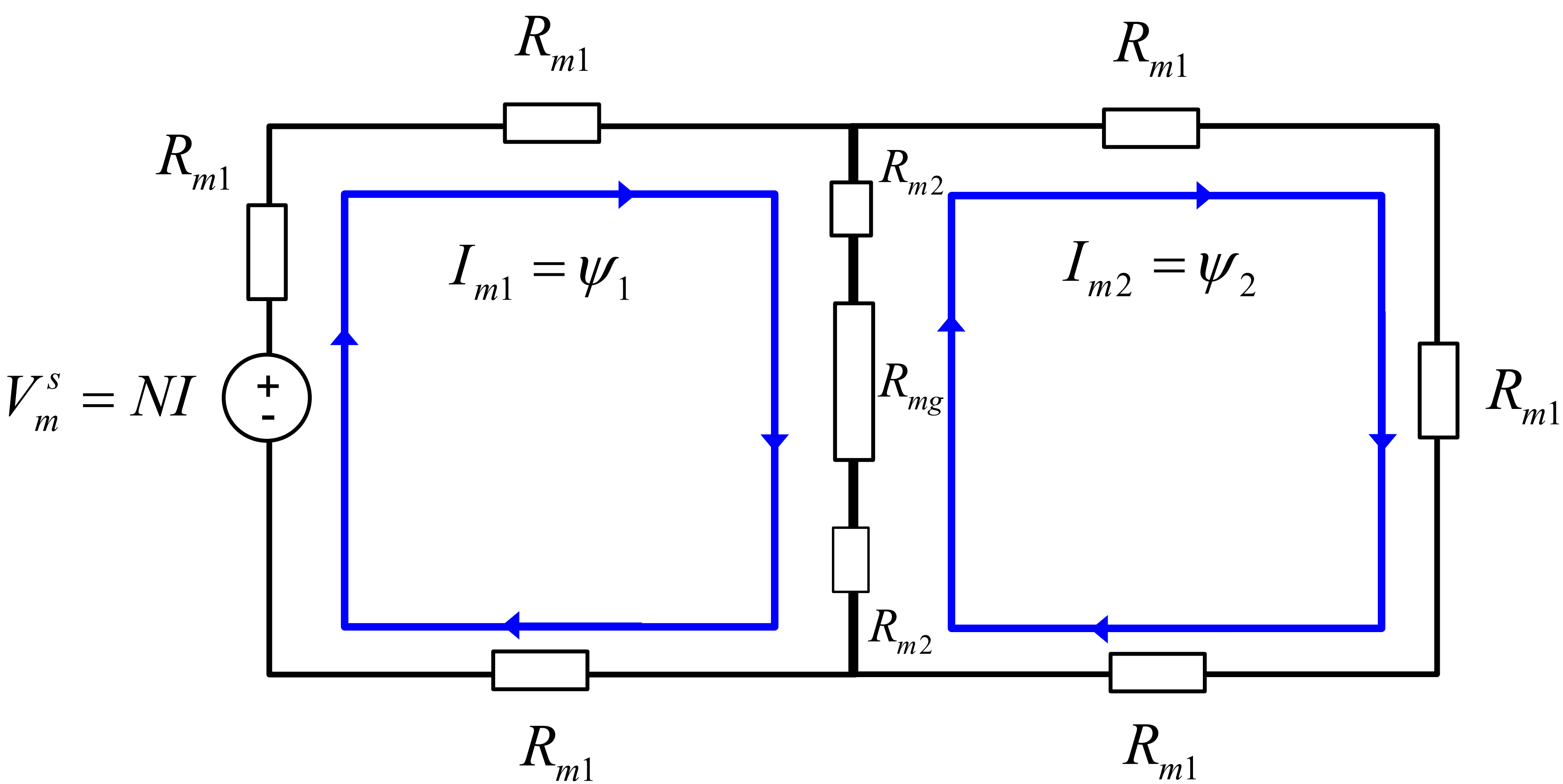
$$R_{m2} = \frac{L_2}{\mu_0 \mu_r A_2}$$

$$R_{mg} = \frac{L_g}{\mu_0 A_2}$$

Magnetic Circuits (cont.)

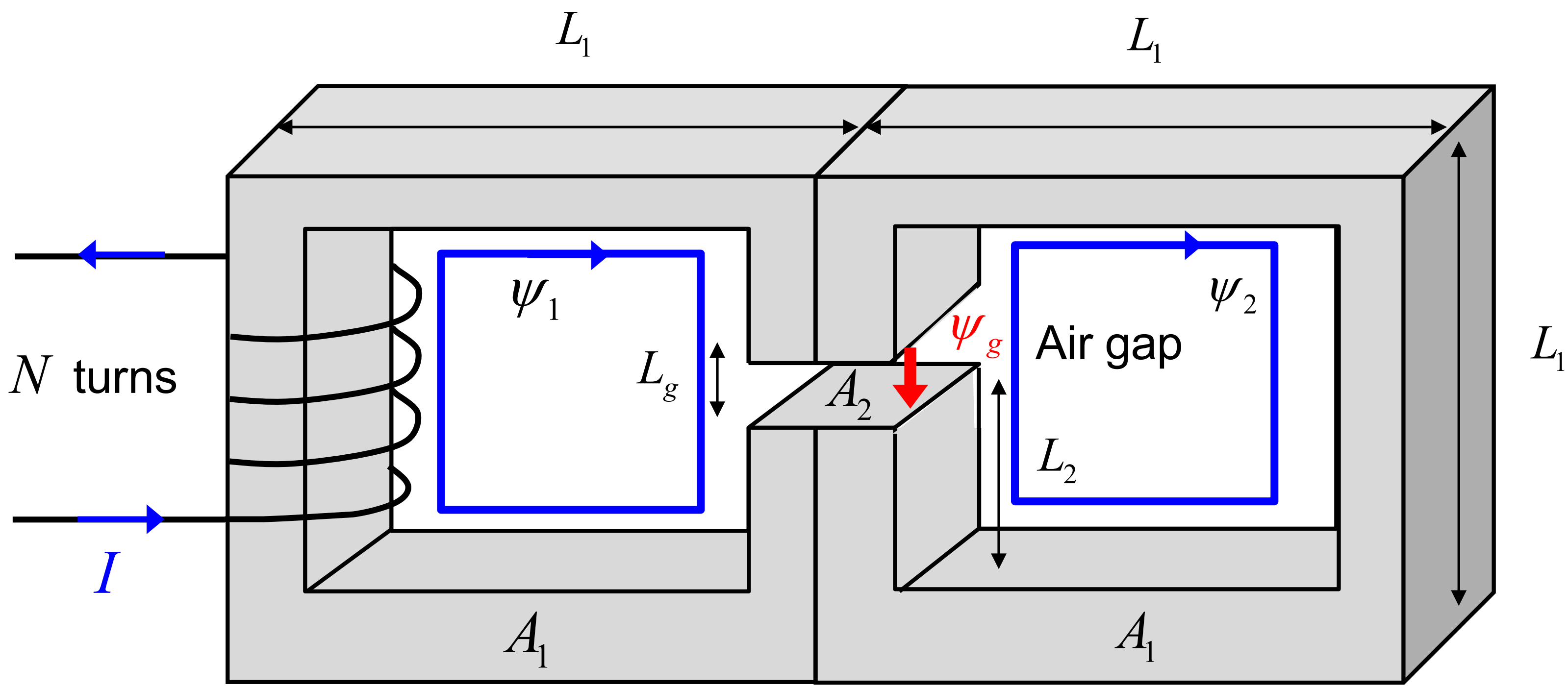
This is the circuit model of the previous structure.

Note that meshed currents have been defined.



We can solve this using a mesh-current approach (details omitted).

Magnetic Circuits (cont.)



Once the fluxes ψ_1 and ψ_2 are known (from the mesh-current analysis), we can find the value of B and H in any part of the structure.

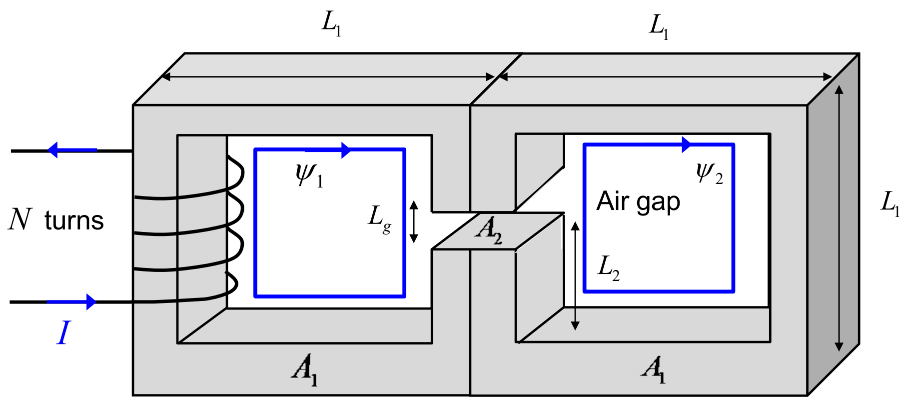
Example: $|\underline{H}|$ inside air gap: $\psi_g = \psi_1 - \psi_2, \quad B_g = \frac{\psi_g}{A_2}, \quad H_g = \frac{B_g}{\mu_0}$

Magnetic Circuits (cont.)

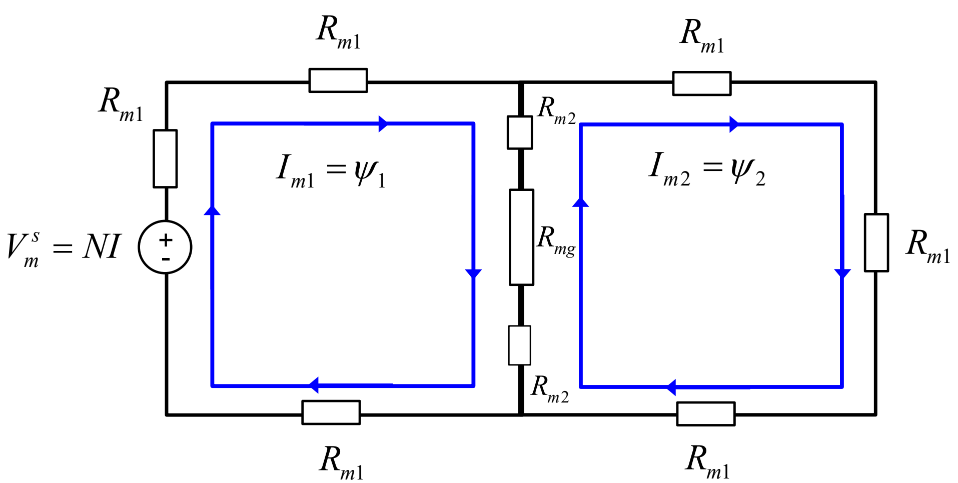
Note on Modeling

We have a perfect duality in the modeling equations.

(This is why the method works for any structure.)



- The magnetic flux going out of a junction must be zero (magnetic Gauss law).
- Around any loop, we have Ampere's law.



- The current going out of a junction must be zero (KCL).
- Around any loop, we have the KVL law.