DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: ____

Student Number: __________________________________

Instructor: ____

ECE 2317

Applied Electricity and Magnetism
Exam 1
October 14, 2000

1. This exam is closed book and closed notes. A calculator and one crib sheet (one 8.5" X 11" piece of paper) are allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the paper provided.
4. Write neatly. You will not be given credit for work that is not easily legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. If you have any questions, ask the instructors. You will not be given credit for work that is based on a wrong assumption.
9. You will have a total of 90 minutes to work the entire exam.

_____/25 Prob. 1 _____/25 Prob. 3

_____/25 Prob. 2 _____/25 Prob. 4

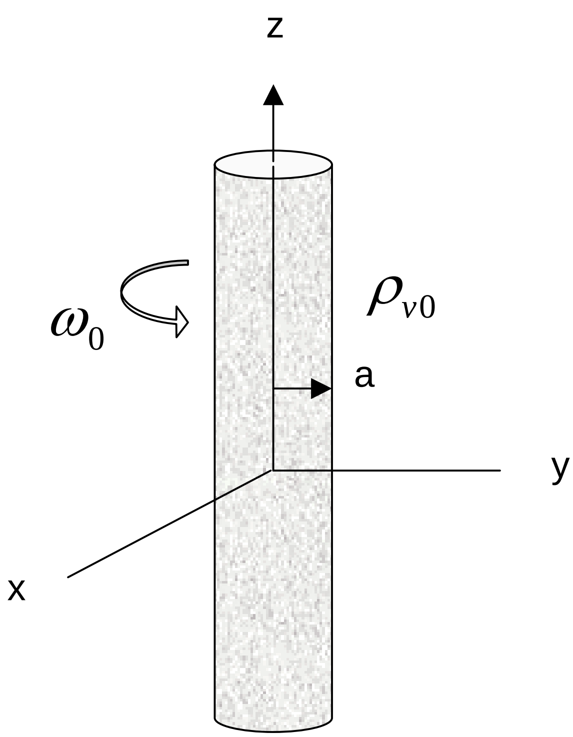
ROOM FOR EXTRA WORK

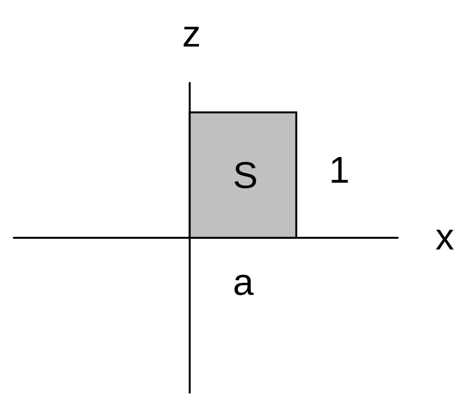
Problem 1 (25 pts)

A cylindrical column of uniform charge density ρ_{v0} [C/m^3] is shown below. The column is rotating at a constant angular speed of ω_0 [radians/s] in the counterclockwise direction. Recall that the speed at any distance ρ from the z axis is given by $\omega_0 \rho$ [m/s].

a) Determine the current density vector $\boldsymbol{J}$ at any point inside the column. Express your answer in cylindrical coordinates.

b) Find the total current that crosses through the surface S shown below, in the direction *out of the paper*. This surface is one meter high in the z direction, and has width a, running from the z axis to the edge of the column.

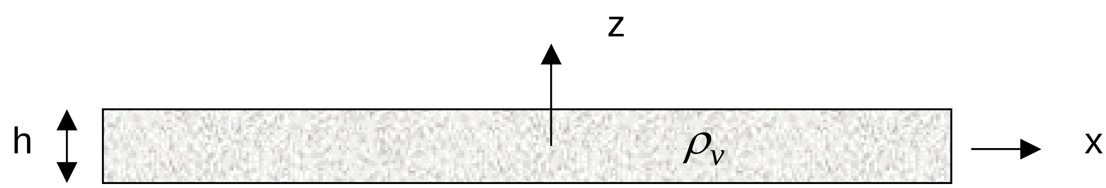




ROOM FOR EXTRA WORK

Problem 2 (25 pts)

a) Consider a slab of charge of thickness h, located between $z = -h/2$ and $z = +h/2$. Within the slab there is a volume charge density $\rho_v = z$ [C/m^3]. The slab is infinite in the *x* and *y* directions. Find the electric field vector in *all* regions: $z < -h/2$, $-h/2 < z < h/2$, and $z > h/2$.



b) Verify your answers from part (a) above by using the differential (point) form of Gauss's Law. That is, show that your solutions satisfy the differential form of Gauss's law.

ROOM FOR EXTRA WORK

Problem 3 (25 pts)

A hemispherical shell of uniform charge density ρ_{s0} is shown below. The hemispherical shell lies above the z = 0 plane, with its base (a circle) lying on the z = 0 plane. The radius of the shell is a.

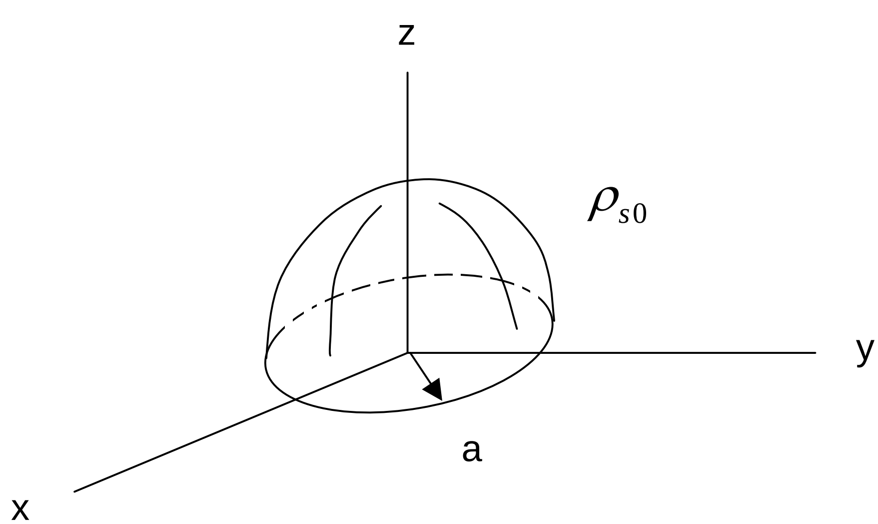
Determine the electric field vector at the origin.

The following equations may be helpful to you:

$$\hat{\boldsymbol{r}} = \hat{\boldsymbol{x}}\sin\theta\cos\phi + \hat{\boldsymbol{y}}\sin\theta\sin\phi + \hat{\boldsymbol{z}}\cos\theta$$

$$\hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{x}}\sin\phi + \hat{\boldsymbol{y}}\cos\phi$$

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{x}}\cos\theta\sin\phi + \hat{\boldsymbol{y}}\cos\theta\cos\phi - \hat{\boldsymbol{z}}\sin\theta\ .$$



ROOM FOR EXTRA WORK

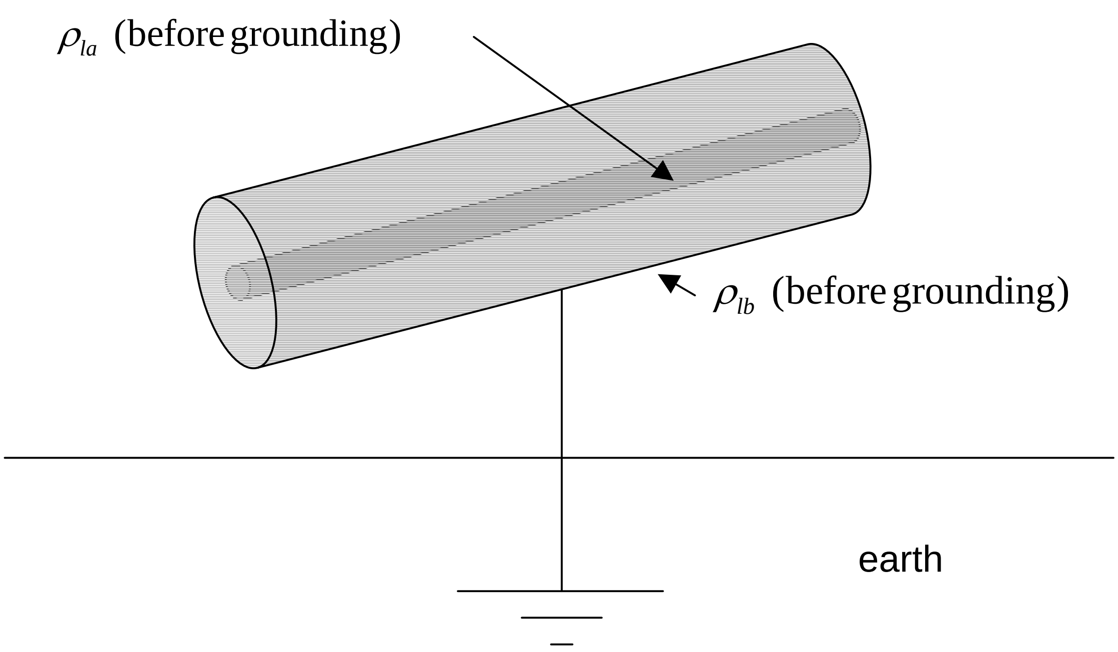
Problem 4 (25 pts)

A long coaxial cable has a uniform surface charge density on the inner conductor of radius *a*, which corresponds to an effective line charge density of ρ_{la} [C/m]. The outer conductor of radius *b* has a uniform surface charge density that corresponds to an effective line charge density of ρ_{lb} [C/m]. The values ρ_{la} and ρ_{lb} are arbitrary. Assume free space between the conductors.

The coaxial cable is then grounded by attaching a conducting ground wire to the outside of the coax (no connection is made to the inner conductor of the coax).

a) Determine the electric field vector inside the coax ($a < \rho < b$) and outside the coax ($\rho > b$) before the coax is grounded, assuming the earth is not present.

b) Determine the electric field vector inside the coax ($a < \rho < b$) and outside the coax ($\rho > b$) after the coax is grounded.



ROOM FOR EXTRA WORK

Prob. 1

a)

$$\underline{J} = \rho_v \underline{v}$$

$$= \rho_{v0} (\omega_0 \rho) \hat{\underline{\phi}} \quad [A/m^2]$$

b)

$$I = \int_S \underline{J} \cdot \hat{\underline{n}} \, ds = \int_S \underline{J} \cdot (-\hat{\underline{y}}) \, ds$$

$$@ \ \phi = 0°, \quad \hat{\underline{\phi}} = \hat{\underline{y}}$$

Hence $\underline{J} = \rho_{v0} \omega_0 \rho \hat{\underline{y}}$

$$I = \int_S -\rho_{v0} \omega_0 \rho \, ds$$

$$= -\rho_{v0} \omega_0 \int_0^a \int_0^1 \rho \, d\rho \, dz$$

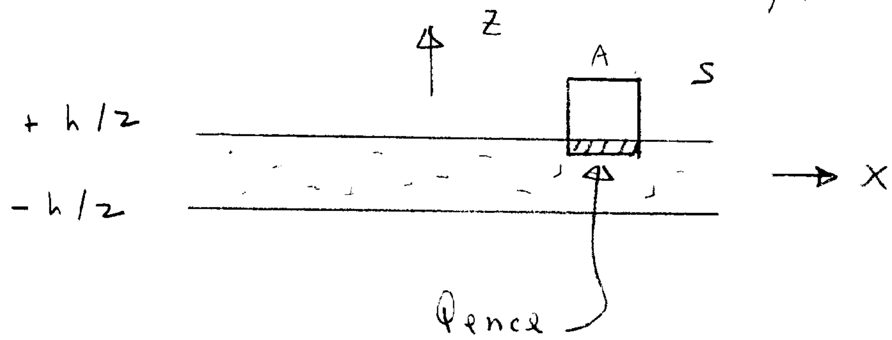
$$= -\rho_{v0} \omega_0 \int_0^a \rho \, d\rho = -\rho_{v0} \omega_0 \left[\frac{\rho^2}{2} \right]_0^a$$

$$I = -\frac{1}{2} \rho_{v0} \omega_0 a^2 \quad [A]$$

Prob. 2

$\rho_v = z$

(a)



$|z| > h/2$: $\underline{E} = \underline{0}$ since $\rho_s^{eff} = \int_{-h/2}^{+h/2} \rho_v \, dz$

$= 0,$

$|z| < h/2$:

$$\oint_S \underline{D} \cdot \hat{\underline{n}} \, ds = Q_{encl}$$

$$\int_{Top} \cancel{\underline{D}}^{0} \cdot \hat{\underline{z}} \, ds + \int_{Bottom} \underline{D} \cdot (-\hat{\underline{z}}) \, ds = A \int_z^{h/2} \rho_v \, dz$$

$$-D_z \cancel{A} = \cancel{A} \int_z^{h/2} z \, dz$$

$$D_z = -\int_z^{h/2} z \, dz$$

$$D_z = -\frac{z^2}{2}\Big|_z^{h/2}$$

$$D_z = -\frac{1}{2}\left(\frac{h^2}{4} - z^2\right)$$

so

$$\underline{E} = \hat{\underline{z}}\left(\frac{1}{2}\right)\left(z^2 - \frac{h^2}{4}\right) \;[V/m], \quad |z| < h/2$$

$$\underline{E} = \underline{0} \;[V/m], \quad |z| > h/2$$

(b)

$$\nabla \cdot \underline{D} = \rho_v$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\partial D_z}{\partial z}$$

$$= \frac{d}{dz}\left(\frac{1}{2}\left(z^2 - \frac{h^2}{4}\right)\right) = z = \rho_v \;\checkmark$$

$(|z| < h/2)$

For $|z| > h/2$ $\quad \nabla \cdot \underline{D} = 0 = \rho_v$ ✓

Prob. 3

$$\underline{E} = \int_S \frac{\rho_s(\underline{r}')\ \hat{\underline{R}}}{4\pi\epsilon_0 R^2}\, ds'$$

$$= \frac{\rho_{s0}}{4\pi\epsilon_0 a^2} \int_S \hat{\underline{R}}\, ds'$$

$$= \frac{\rho_{s0}}{4\pi\epsilon_0 a^2} \int_S -\hat{\underline{r}}\, ds'$$

$$= \frac{\rho_{s0}}{4\pi\epsilon_0 a^2} \int_0^{2\pi}\int_0^{\pi} -\left[\hat{\underline{x}} \sin\theta\cos\phi + \hat{\underline{y}} \sin\theta\sin\phi + \hat{\underline{z}}\cos\theta\right] a^2 \sin\theta\, d\theta\, d\phi$$

$$\int_0^{2\pi} \cos\phi\, d\phi = 0\ , \quad \int_0^{2\pi} \sin\phi\, d\phi = 0$$

So

$$\underline{E} = -\hat{\underline{z}}\left(\frac{\rho_{s0}}{4\pi\epsilon_0}\right)(2\pi)\int_0^{\pi/2} \cos\theta\sin\theta\, d\theta$$

$$\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \int_0^1 u \, du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}.$$

Hence

$$\underline{E} = -\hat{\underline{z}} \left(\frac{\rho_{s0}}{4 \epsilon_0} \right) \quad [V/m]$$

Prob. 4

(a) Gauss's law:

$$a < \rho < b \qquad \underline{E} = \underline{\hat{\rho}} \left(\frac{\rho_{\ell a}}{2\pi \epsilon_0 \rho} \right) \quad [V/m]$$

$$\rho > b \qquad \underline{E} = \underline{\hat{\rho}} \left(\frac{\rho_{\ell a} + \rho_{\ell b}}{2\pi \epsilon_0 \rho} \right) \quad [V/m]$$

(b) $\rho_{\ell b}' = -\rho_{\ell a}$

(inner surface : $\rho_\ell = -\rho_{\ell a}$
outer surface : $\rho_\ell = 0$)

$$a < \rho < b \qquad \underline{E} = \underline{\hat{\rho}} \left(\frac{\rho_{\ell a}}{2\pi \epsilon_0 \rho} \right) \quad [V/m]$$

$$\rho > b \qquad \underline{E} = \underline{0} \quad [V/m]$$