DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: SOLN

Student Number: ____________________

Instructor: ____________________

ECE 2317

Applied Electricity and Magnetism

Exam 1

February 24, 2001

1. This exam is closed book and closed notes. A calculator and one crib sheet (one 8.5" X 11" piece of paper) are allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the paper provided.
4. Write neatly. You will not be given credit for work that is not easily legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. If you have any questions, ask the instructors. You will not be given credit for work that is based on a wrong assumption.
9. You will have a total of 90 minutes to work the entire exam.

_____/25 Prob. 1 _____/25 Prob. 3

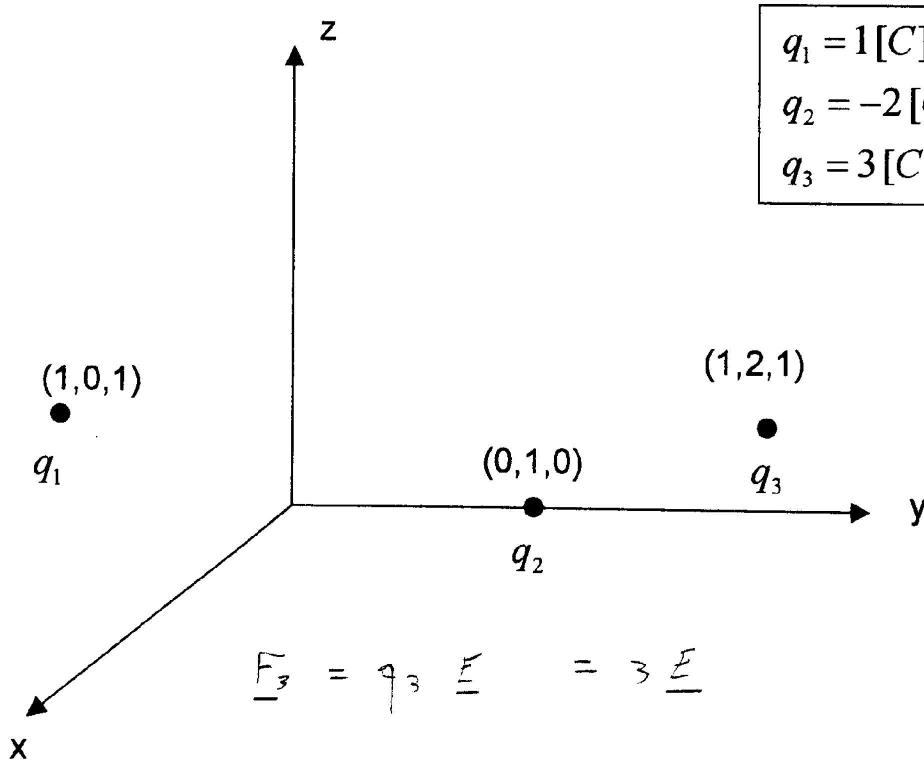
_____/25 Prob. 2 _____/25 Prob. 4

ROOM FOR EXTRA WORK

Problem 1 (25 pts)

A system of three charges is shown in the diagram below. Find the total force on the charge q_3.

$q_1 = 1\,[C]$
$q_2 = -2\,[C]$
$q_3 = 3\,[C]$



$$\underline{F_3} = q_3\,\underline{E} = 3\,\underline{E}$$

$$= 3\,(\underline{E_1} + \underline{E_2})$$

$$\underline{E_1} = \frac{q_1}{4\pi\epsilon_0 R_1^2}\,\hat{\underline{R_1}} \qquad \text{So} \quad \underline{E_1} = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{4}\right)\hat{\underline{y}}$$

$$\underline{R_1} = (0,2,0)\,, \quad R_1 = 2$$

$$\hat{\underline{R_1}} = \hat{\underline{y}}$$

$$\underline{E_2} = \frac{-2}{4\pi\epsilon_0}\left(\frac{1}{3}\right)\frac{1}{\sqrt{3}}\,(\hat{\underline{x}} + \hat{\underline{y}} + \hat{\underline{z}})$$

$$\underline{R_2} = (1,1,1)\,, \quad R_2 = \sqrt{3}$$

$$\hat{\underline{R_2}} = \frac{1}{\sqrt{3}}\,(\hat{\underline{x}} + \hat{\underline{y}} + \hat{\underline{z}})$$

ROOM FOR EXTRA WORK

Hence

$$\underline{F_3} = 3 \left(\frac{1}{4\pi \epsilon_0} \right) \left[\frac{1}{4} \hat{\underline{y}} - \frac{2}{3\sqrt{3}} \left(\hat{\underline{x}} + \hat{\underline{y}} + \hat{\underline{z}} \right) \right]$$

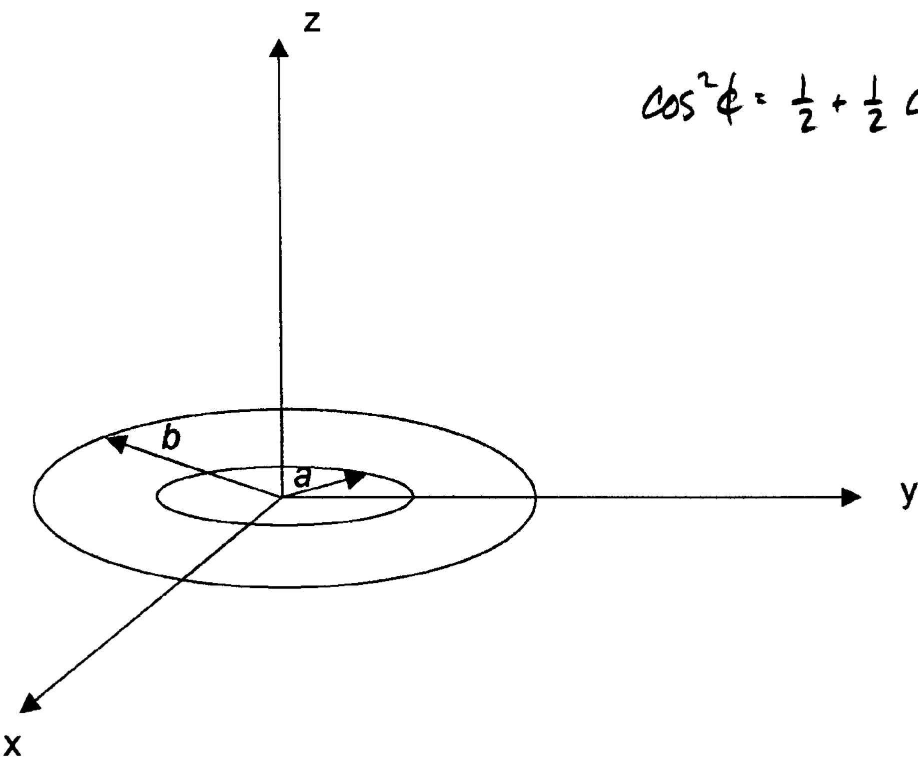
Problem 2 (25 pts)

Given the electric flux density

$$\mathbf{D} = 5\cos^2(\phi)\hat{\boldsymbol{\rho}} - 17\rho^2\hat{\boldsymbol{\varphi}} + 9\rho\cos^2(\phi)\hat{\mathbf{z}} \quad [C/m^2]$$

find the net electric flux in the upward direction through the annulus shown in the figure ($a \le \rho \le b$).

$\cos^2\phi = \frac{1}{2} + \frac{1}{2}\cos(2\phi)$



$\Psi_e = \iint_S \bar{D}\cdot\hat{n}\,ds = \iint_S \bar{D}\cdot\hat{z}\,\rho\,d\phi\,d\rho$

$= \int_a^b \int_0^{2\pi} 9\rho\cos^2\phi\,\rho\,d\phi\,d\rho$

$= \int_a^b \int_0^{2\pi} 9\rho^2\left(\frac{1}{2}+\frac{1}{2}\cos 2\phi\right)d\phi\,d\rho$

$= \int_a^b 9\rho^2\pi\,d\phi = 9\pi\frac{1}{3}(b^3-a^3) = \boxed{3\pi(b^3-a^3) = \Psi_e}$

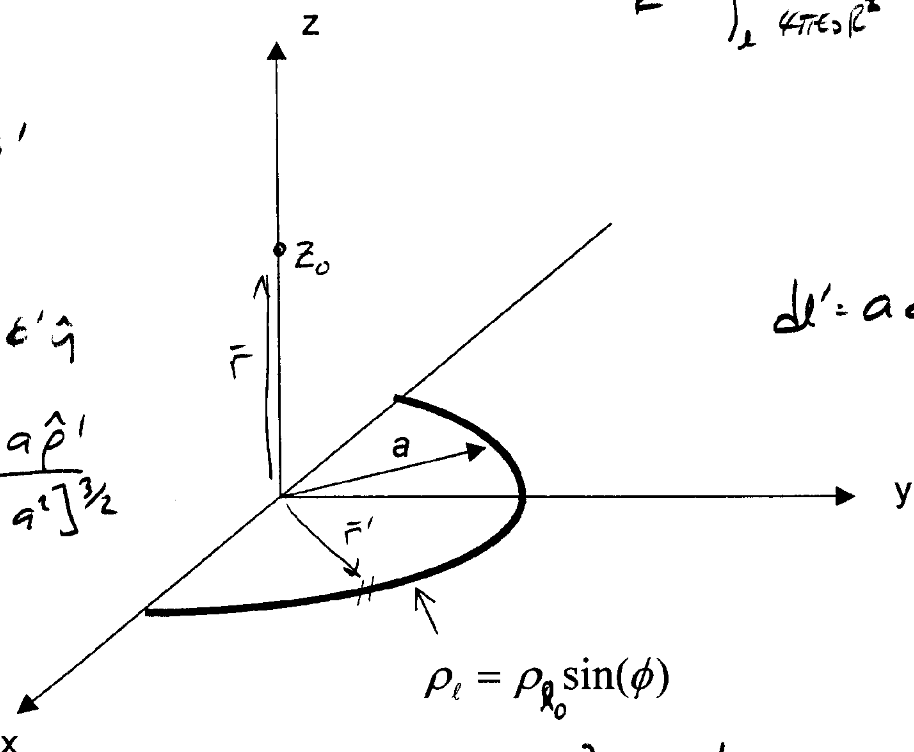
ROOM FOR EXTRA WORK

Problem 3 (25 pts)

A thin line charge of density $\rho_\ell = \rho_{\ell 0}\sin(\phi)$ is in the form of a semi-circle of radius a lying on the *x*-*y* plane with its center located at the origin. The semicircle starts at $\phi = 0$ and ends at $\phi = \pi$. Find the electric field vector ***E*** at a location z_0 along the *z*-axis.

$$\bar{E} = \int_\ell \frac{\rho_\ell}{4\pi\epsilon_0 R^2}\hat{R}\, d\ell'$$

$$\bar{R} = \bar{r} - \bar{r}' = z_0\hat{z} - a\hat{\rho}'$$



$$d\ell' = a\, d\phi'$$

$$\hat{\rho}' = \cos\phi'\,\hat{x} + \sin\phi'\,\hat{y}$$

$$\Rightarrow \frac{\hat{R}}{R^2} = \frac{\bar{R}}{R^3} = \frac{z_0\hat{z} - a\hat{\rho}'}{[z_0^2 + a^2]^{3/2}}$$

$$\bar{E} = \int_0^\pi \frac{\rho_{\ell 0}\sin\phi'}{4\pi\epsilon_0} \frac{[z_0\hat{z} - a\cos\phi'\,\hat{x} - a\sin\phi'\,\hat{y}]}{[z_0^2 + a^2]^{3/2}}\, a\, d\phi'$$

$$= \frac{\rho_{\ell 0}\, a}{4\pi\epsilon_0 [z_0^2 + a^2]^{3/2}} \int_0^\pi (z_0\sin\phi'\,\hat{z} - a\sin^2\phi'\,\hat{y})\, d\phi'$$

$$= \frac{\rho_{\ell 0}\, a}{4\pi\epsilon_0 [z_0^2 + a^2]^{3/2}} \left(2 z_0\,\hat{z} - \frac{\pi a}{2}\hat{y}\right)$$

$$\boxed{\bar{E} = \hat{z}\left(\frac{\rho_{\ell 0}\, a\, z_0}{2\pi\epsilon_0 [z_0^2 + a^2]^{3/2}}\right) - \hat{y}\left(\frac{\rho_{\ell 0}\, a^2}{8\epsilon_0 [z_0^2 + a^2]^{3/2}}\right)}$$

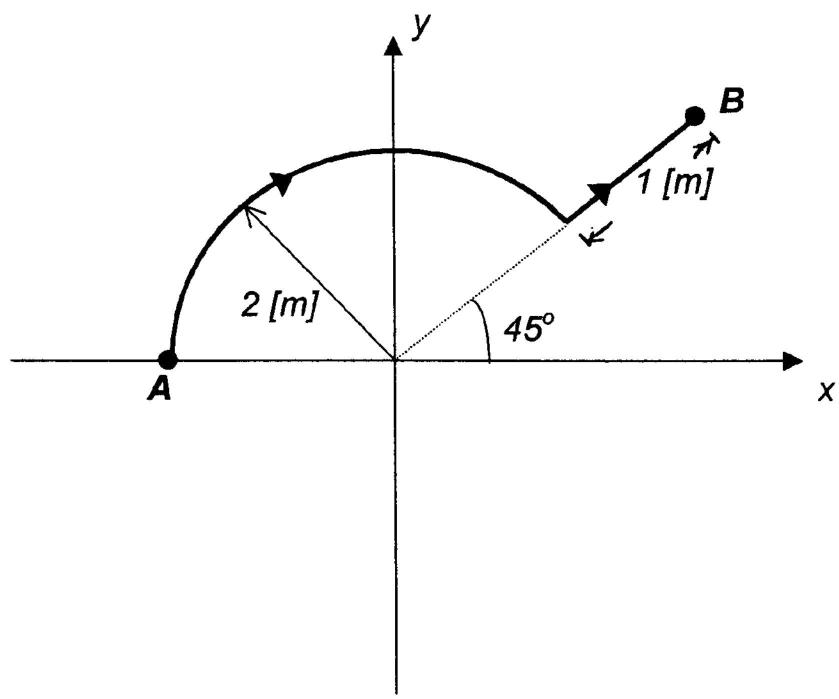
ROOM FOR EXTRA WORK

Problem 4 (25 pts)

Given the electric field

$$\mathbf{E} = (3 + 4\sin\phi)\hat{\boldsymbol{\rho}} + (\phi\rho^2)\hat{\boldsymbol{\varphi}} + (2z\rho^2 + 1)\hat{\mathbf{z}} \quad [V/m]$$

find the result of the line integration $V_{AB} = \int_A^B \mathbf{E} \cdot \mathbf{d\ell}$ by integrating along the path shown in the figure.



ROOM FOR EXTRA WORK

Part #1:

$$\underline{dr} = \underline{dl} = (a\, d\phi)\, \hat{\underline{\phi}}$$

$$\underline{E} \cdot \underline{dr} = (\phi \rho^2 \hat{\underline{\phi}}) \cdot (a\, d\phi\, \hat{\underline{\phi}}) = a^3 \phi$$

$$\int_{c_1} \underline{E} \cdot \underline{dr} = \int_{\pi}^{\pi/4} a^3 \phi \, d\phi = a^3 \left[\frac{\phi^2}{2} \right]_{\pi}^{\pi/4}$$

$$= \frac{a^3}{2} \pi^2 \left[\frac{1}{16} - 1 \right]$$

$$= -\frac{15}{32} \pi^2 a^3$$

$$= -\frac{15}{32} \pi^2 (2)^3 = -\frac{15}{4} \pi^2$$

Part #2:

$$\underline{dr} = \hat{\underline{\rho}}\, d\rho$$

$$\underline{E} \cdot \underline{dr} = \left[\hat{\underline{\rho}} (3 + 4 \sin\phi) \right] \cdot \hat{\underline{\rho}}\, d\rho$$

$$= (3 + 4 \sin\phi)\, d\rho$$

$$\int_{c_2} \underline{E} \cdot \underline{dr} = (3 + 4 \sin\phi) \int_2^3 d\rho$$

$$= 3 + 4 \left(\frac{1}{\sqrt{2}} \right) (1)$$

$$\boxed{V_{AB} = \left(-\frac{15}{4} \pi^2 \right) + \left(3 + \frac{4}{\sqrt{2}} \right) = -31.183}$$

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