# Name: \_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 1**

#### March 31, 2020

**Remote Exam Instructions**

You are not allowed to use your computer, or any other device, to communicate with anyone other than the instructor during the exam.

If you wish to ask a question during the exam, please use the “chat” feature of Zoom to chat with the instructor (please chat only with the instructor, not with “everyone”).

By taking this exam, you agree to the UH Academic Honesty Policy.

1. This exam is open-book and open-notes.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Write neatly. You will not be given credit for work that is not **easily** legible.
4. Leave answers in terms of the parameters given in the problem.
5. Show units in all of your final answers.
6. Circle your final answers.
7. Double-check your answers. For simpler problems, partial credit may not be given.
8. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.

**TABLE OF INTEGRALS**

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Problem 1 (30 pts.)

An electrode is at the origin in a conducting medium, and it spews out current equally in all directions in spherical coordinates. The total current coming out of the electrode is *I* amps. The current density vector is thus given in spherical coordinates as

.

Find the total current (in amps) that is going upward through a horizontal circular disk that has a radius *a* and is at a height *h* above the *z* = 0 plane, as shown below.

**Solution**



We have

.

The total current crossing the surface is

.

Hence, we have



or

.

We want to put the integrand in cylindrical coordinates. We therefore use:





.

We then have

.

Performing the *φ* integration, we have



Hence, we have

.

Performing the integration, we have

.

The final answer is then



or

.

Problem 2 (35 pts.)

A uniform surface charge density  lies in the *xy* plane inside of a circular region defined by  and , as shown below.

a) Find the electric field component  at the point  as shown below.

b) Find the electric field component  at the point  as shown below.

c) Without doing any calculation, how is  related to  at the point ?



**Solution**

For this problem we use Coulomb’s law. We thus have

.

Since the surface charge density is homogenous (a constant), we have

.

We have



and therefore



.

We then have

.

**Part (a)**

For the *z* component, we have

.

Performing the *φ* ′ integration gives us

.

Performing the *ρ* ′ integration gives us

.

We then have

.

**Part (b)**

For the *x* component we use

.

If we substitute this into

,

we then have

.

Since we have a separable integrand and fixed limits of integration, we can write

.

Performing the *φ* ′ integration gives us

.

Using the table of integrals, we have the formula

****.

Hence, we have

.

This gives us the final result

.

**Part (c)**

From the symmetry we have

.

Problem 3 (35 pts.)

An infinite tube of uniform volume charge density  having a radius *a* is surrounded by an infinite metal pipe as shown below.

The metal pipe has an inner radius *b* and an outer radius *c*. The metal pipe is neutral.

a) Find the voltage drop *VAB* between the *z* axis (point *A*) and the pipe (point *B*).

b) Find the surface charge densities  and  on the pipe surfaces at  and .

c) If the pipe is now grounded, how will the answers to part (b) change?



**Solution**

For this problem we first use Gauss’s law. We thus have

.

Using a cylindrical Gaussian surface of height *h*, we have



Hence, we have



.

**Part (a)**

The voltage drop is



Hence, we have



or

.

This gives us the result

.

**Part (b)**

Using Gauss’s law with a cylindrical Gaussian surface inside the metal pipe, we have

.

Hence, we have

.

Because the pipe is neutral, we have

.

Hence,

.

Hence, we have

.

**Part (c)**

After the pipe is grounded, the charge density on surface *b* remains the same. The charge density on surface *c*, the outer surface of the pipe, is now zero.