#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 1**

#### March 24, 2022

**Name: \_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Instructions**

1. This exam is open-book and open-notes.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Write neatly. You will not be given credit for work that is not **easily** legible.
4. Leave answers in terms of the parameters given in the problem.
5. Show units in all of your final answers.
6. Circle your final answers.
7. Double-check your answers. For simpler problems, partial credit may not be given.
8. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.

**TABLE OF INTEGRALS**

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**Problem 1 (35 pts.)**

An infinite slab of volume charge density is shown below. The charge density inside the slab is given by

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(a) Find the electric field vector in all three regions: , , .

(b) Find the voltage drop *VAB* between the points *A* and *B*.

(c) Find the equivalent surface charge density  which, when placed at *x* = 0, will produce the same electric field outside the slab as the original slab does.

**Solution**

*x*



*z*



**Part (a)**

Because the charge density is an odd function, the electric field is zero outside the slab. Inside the slab we apply Gauss’s law with the top of the Gaussian surface above the slab (where the electric field is zero) and the bottom part of the Gaussian surface at the observation point, located at *x*. The base area of the Gaussian surface is denoted as *A*.

For an observation point inside the slab, Gauss’s law then gives us



Hence, we have

.

We then have



**Part (b)**



or

.

**Part (c)**



so



**Problem 2 (30 pts.)**

A uniform surface charge density *ρs*0 has a cylindrical shape as shown below. The radius of the cylinder is *a* and the height of the cylinder in the *z* direction is *h*. The cylinder is centered at the origin. The cylinder is hollow (there are no top and bottom parts, only the side part at *ρ* = *a*).

Find the electric field vector at the point  as shown below (*x*0 > *a*).

You do not have to evaluate any integrals that appear in your answer. However, your answer should make it clear what the direction of the electric field vector is.



**Solution**

We have from Coulomb’s law

.

We have



Therefore,

.

By symmetry, the electric field at the observation point will be in the *x* direction. Therefore, we can write



**Problem 3 (35 pts.)**

A spherical uniform surface charge density *ρs*0 with radius *a* is surrounded by a perfectly conducting shell of inner radius *b* and outer radius *c*. This shell has a total net charge of  (it is not neutral). At the origin, there is a point charge *q* [C].

a) Find the electric field vector in all four regions (*r* < *a*, *a* < *r* < *b*,  *b* < *r* < *c*, *r* > *c*).

b) Find the surface charge density  on the inner surface of the PEC shell, at *r* = *b*.

c) Find the surface charge density  on the outer surface of the PEC shell, at *r* = *c*.

d) If the PEC shell is now grounded, how do the answers to parts (a)−(c) change? (If an answer does not change you can simply say this.)



**Solution**

**Part (a)**

From Gauss’s law we have









**Part (b)**



**Part (c)**



**Part (d)**

All previous answers remain the same except that:



