#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 1**

#### March 23, 2023

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Instructions**

1. This exam is open-book and open-notes.
2. Cell phones, laptops, ipads, and any other devices that have communication functionality are not allowed during the exam.
3. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
4. Write neatly. You will not be given credit for work that is not easilylegible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Remember the UH Academic Honesty Policy. You must not receive or give assistance to anyone else during the exam, or communicate with anyone other than the instructor during the exam.

**TABLE OF INTEGRALS**

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**Problem 1 (30 pts.)**

Inside of a transformer core there is an electric field that is given (in cylindrical coordinates) as

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Find the voltage drop *VAB* between points *A* and *B* by using paths as indicated below. Point *A* is on the *x* axis at a distance *a* from the origin. Point *B* is on the *y* axis at a distance *a* from the origin.

(a) Use path *C*1. This path goes counter clockwise on a circle of radius *a* from *A* to *B*.

(b) Use path *C*2. This path goes clockwise on a circle of radius *a* from *A* to *B*.

(c) Use path *C*3. This path goes from *A* to the origin along the *x* axis, and then from the origin to point *B* along the *y* axis.

**Solution**

*z*

We have

.

When we stay on the circle (parts (a) and (b)), we have

.

Hence, we have

.

**Part (a)**



so

.

**Part (b)**



so

.

**Part (c)**

Along the axes, the *dr* vector is perpendicular to the electric field vector. Hence, we have

.

**Problem 2 (35 pts.)**

A uniform volume charge density *ρv*0 [C/m3] exists inside of a spherical region *a* < *r* < *b*. Surrounding this charge region is a PEC spherical shell of radius *c*. This PEC shell has a total net charge of *Q* [C] on it. (The total net charge on the shell is the sum of the charges on the inner and outer surfaces of the shell.)

(a) Find the electric field vector in all four regions: *r* < *a*, *a* < *r* < *b*, *b* < *r* < *c*, *r* > *c*.

(b) Find the charges *Q*1 and *Q*2 (units of [C]) on the inner and outer surfaces of the PEC shell.

(c) Find the surface charge densities *ρs*1 and *ρs*2 on the inner and outer surfaces of the PEC shell.

(d) The PEC shell is now grounded. Give the new answers for the electric field vector in all four regions. (If the answer in a certain region has not changed from part (a), you can simply say so.)

(e) Give the new answers for the charges *Q*1 and *Q*2 on the inner and outer surfaces of the PEC shell, after the PEC shell is grounded.



**Solution**

**Part (a)**

We have from Gauss’s law:

.







**Part (b)**

All of the flux lines end on the inner surface of the PEC shell, so we have that the charge on the inner surface is the negative of that inside the charge region with . Hence, we have

.

Since the total charge on the PEC shell is *Q*, we then have

,

and hence

.

**Part (c)**

We divide the charges on the two surfaces by their areas to get the two surface charge densities. Hence we have



.

**Part (d)**

After grounding, the electric field is the same in all regions except for the outmost region, *r* > *c*. In this region the electric field is now zero.

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**Part (e)**

After grounding, the surface charge density is the same on the inner surface of the PEC shell, but the surface charge density on the outer surface is now zero.

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**Problem 3 (35 pts.)**

A line charge density *ρl* lies along the *x* axis from *x* = 0 to *x* = *L*. The line charge density is nonuniform and is described by

.

 Find the electric field vector at the observation point (0, 0, *z*) on the *z* axis.

**Solution**

We have from Coulomb’s law:

,

which gives us

.

Hence, we have

.

We have

.

Hence, we have



We then have



or

.

Performing the integrals, we have



This gives us

