DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

Name: KEY

Student Number: ______________________________

Instructor: ______________________________

ECE 2317

Applied Electricity and Magnetism

Exam 2

November 18, 2000

1. This exam is closed book and closed notes. A calculator and one crib sheet (one 8.5" X 11" piece of paper) are allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the paper provided.
4. Write neatly. You will not be given credit for work that is not easily legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. If you have any questions, ask the instructors. You will not be given credit for work that is based on a wrong assumption.
9. You will have a total of 90 minutes to work the entire exam.

_____/25 Prob. 1	_____/25 Prob. 3
_____/25 Prob. 2	_____/25 Prob. 4

ROOM FOR EXTRA WORK

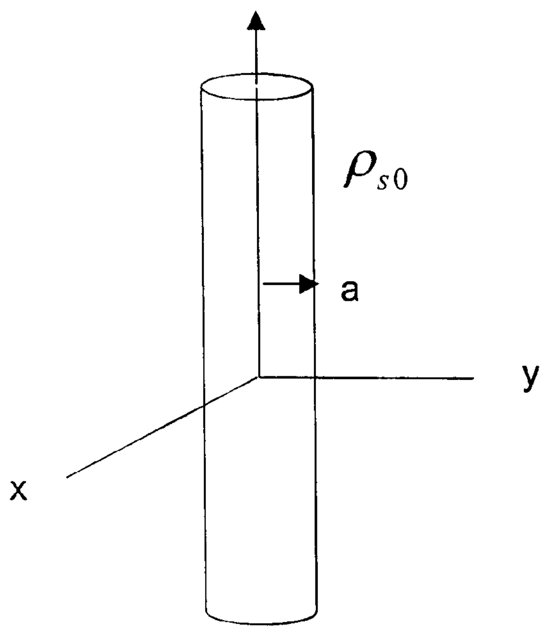
Problem 1 (25 pts)

A infinitely long cylindrical shell of uniform charge density ρ_{s0} [C/m^2] is shown below.

a) Calculate the potential inside the shell, by integrating the electric field. Assume that the potential is zero on the z axis.

b) Calculate the potential outside the shell, by integrating the electric field. Assume again that the potential is zero on the z axis.

c) Modify your answers to the above two parts to obtain the solutions if the potential on the z axis is 10 [V].



By GAUSS's LAW,

$$E_\rho = \begin{cases} 0, & \rho < a \\ \dfrac{\rho_{s0}}{\epsilon}\dfrac{a}{\rho}, & \rho > a \end{cases} \quad [V/m]$$

Note $\Phi(\rho = 0) = 0$

a) $\Phi(\rho) - \Phi(0) = -\int_0^\rho 0 \cdot d\rho = 0 \text{ [V]}, \quad \rho < a$

b) $\Phi(\rho) - \Phi(0) = -\int_0^a 0 \cdot d\rho - \int_a^\rho \frac{\rho_{s0}}{\epsilon}\frac{a}{\rho}\, d\rho = \frac{\rho_{s0} a}{\epsilon} \ln a/\rho \text{ [V]}, \quad \rho > a$

c) $\Phi'(\rho) = \begin{cases} 10 \text{ [V]} & \rho < a \\ \frac{\rho_{s0} a}{\epsilon} \ln a/\rho + 10 \text{ [V]}, & \rho > a \end{cases}$

ROOM FOR EXTRA WORK

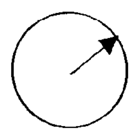
Problem 2 (25 pts)

A metal sphere of radius a is in air. The dielectric breakdown of the air is E_c [V/m].

a) What is the maximum total charge Q_{max} that can be placed on the sphere before the air will break down?

b) Assuming that this charge is placed on the sphere, what is the stored energy of the system? Use the potential formula for your calculation, and express your answer in terms of E_c.

c) Repeat the calculation to find the stored energy in terms of E_c, using the electric-field formula



a) BY GAUSS'S LAW, $E_r = \frac{Q_{MAX}}{4\pi\epsilon_0 r^2}$, $r > a$

BREAKDOWN OCCURS WHERE FIELD IS A MAX, I.E. AT SURFACE $r = a$:

$$E_c = \frac{Q_{MAX}}{4\pi\epsilon_0 a^2} \; [V/m]$$

$$\text{OR} \quad Q_{MAX} = 4\pi\epsilon_0 a^2 E_c \; [C]$$

NOTE $E_r = E_c \frac{a^2}{r^2}$ [V/m], $r > a$

b) $$U_E = \frac{1}{2}\int \rho_s \Phi \, dS = \frac{1}{2}\Phi(r=a)\int \rho_s \, dS = \frac{1}{2}\frac{Q_{MAX}^2}{4\pi\epsilon_0 a}$$

(Φ const @ $r=a$; $\frac{1}{2}\Phi(r=a) = \frac{1}{2}\frac{Q_{MAX}}{4\pi\epsilon a}$; $\int \rho_s \, dS = Q_{MAX}$)

$$= \frac{1}{2}\frac{E_c^2(4\pi\epsilon a^2)^2}{4\pi\epsilon_0 a} = E_c^2 \, 2\pi\epsilon_0 a^3 \; [J]$$

ROOM FOR EXTRA WORK

c) $$U_E = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_a^{\infty} \epsilon_0 \underbrace{\left(\frac{E_c^2 a^4}{r^4}\right)}_{E_r^2} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 2\pi \epsilon_0 E_c^2 a^3 \ [J]$$

Problem 3 (25 pts)

A parallel-plate capacitor is shown below. The top plate is at a voltage of V_2 [V], while the bottom plate is assumed to be at a voltage of V_1 [V], with $V_2 > V_1$.

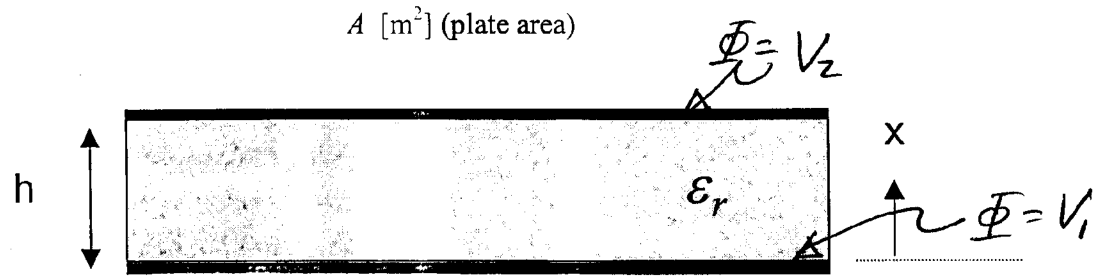
a) Solve the Laplace equation to obtain the solution for the potential function inside the capacitor.

b) Using your answer to part (a), find the electric field inside the capacitor.

c) Using your answer to part (b), along with boundary conditions, determine the charge density and the total charge Q [C] on the top plate.

d) Using your answer from part (c), determine the capacitance of the parallel-plate capacitor.

Φ IS A FUNCTION OF X-ONLY ($\Phi(x)$)



a) $\nabla^2 \Phi = \frac{d^2\Phi}{dx^2} = 0 \Rightarrow \frac{d\Phi}{dx} = A, \quad \Phi = Ax + B$

$@\, x=0, \quad \Phi = V_1 = A \cdot 0 + B \Rightarrow B = V_1$

$@\, x=h, \quad \Phi = V_2 = A \cdot h + V_1 \Rightarrow A = \frac{V_2 - V_1}{h}$

OR $\quad \Phi = \frac{V_2 - V_1}{h} x + V_1 \quad [V]$

b) $\underline{E} = -\nabla\Phi = -\frac{\partial \Phi}{\partial x}\hat{x} = -\frac{V_2 - V_1}{h}\hat{x} \quad [V/m], \; 0 < x < h$

c) $\rho_s = -\hat{x} \cdot \underline{D}\big|_{x=h} = -\epsilon E_x\big|_{x=h} = +\epsilon_r \epsilon_0 \frac{V_2 - V_1}{h} \quad [C/m^2]$

$Q = \rho_s A = \epsilon_r \epsilon_0 \frac{V_2 - V_1}{h} A$

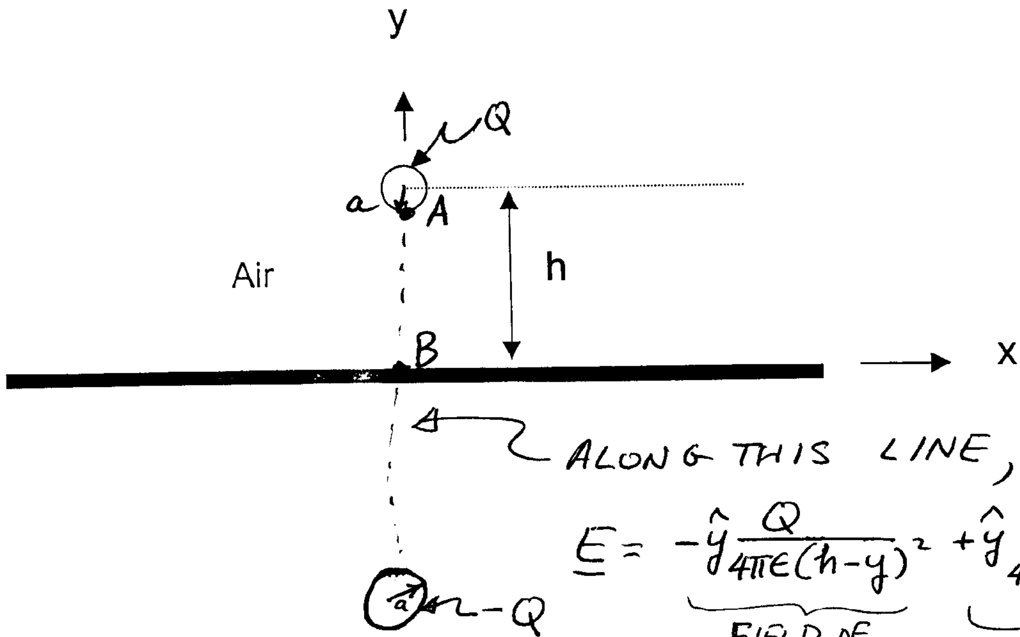
d) $C = \frac{Q}{V} = \frac{Q}{V_2 - V_1} = \frac{\epsilon_r \epsilon_0 A}{h} \quad [F]$

ROOM FOR EXTRA WORK

Problem 4 (25 pts)

A conducting sphere of radius a is above the ground, which may be considered to be a perfect conductor. Assume that the sphere is far enough above the ground so that the charge density on the sphere may be assumed to be uniform. The total charge on the sphere is Q [C].

Find the voltage drop between the sphere and the ground, V_{AB} (where the A conductor is the sphere and the B conductor is the ground). Do this by integrating the electric field between the two conductors.



$$\underline{E} = -\hat{y}\frac{Q}{4\pi\epsilon(h-y)^2} + \hat{y}\frac{-Q}{4\pi(h+y)^2} \; [V/m]$$

FIELD OF ORIGINAL CHG, FIELD OF IMAGE CHG,

ONLY FOR $0 < y < h-a$; $\underline{E} = 0$ FOR $-h+a < y < 0$.

$$\Phi(A) - \Phi(B) = -\int_B^A \underline{E} \cdot d\underline{\ell}$$

$$= -\int_0^{h-a} \left[\frac{-\hat{y}\,Q}{4\pi\epsilon(h-y)^2} - \hat{y}\frac{Q}{4\pi\epsilon(h+y)^2} \right] \cdot \hat{y}\,dy$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{-1}{y-h} - \frac{1}{y+h}\right]_{y=0}^{h-a} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{a} - \frac{1}{h} - \frac{1}{2h-a} + \frac{1}{h}\right]$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{a} - \frac{1}{2h-a}\right] \; [V]$$

ROOM FOR EXTRA WORK