##### DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

# Name: \_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 2317

#### Applied Electricity and Magnetism

**Exam 2**

#### Dec. 3, 2014

1. This exam is open-book and open-notes. A calculator is allowed (as long as it cannot be used to communicate), but no other device (laptop, phone, tablet, etc.) is allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

Problem 1 (35 pts.)

A spherical shell of uniform surface charge density *ρsa*has a radius *a*. Surrounding this shell is another spherical shell of uniform surface charge density *ρsb*thathas a radius *b*. Both shells are centered at the origin.

a) Find the potential in all three regions (*r* > *b*, *a* < *r* < *b*, *r* < *a*), assuming that the potential at infinity is zero. Do this by integrating the electric field.

b) Find the stored energy in the system by using the formula for stored energy that has the potential in it.

*z*

*ρsb*

*ρsa*

*b*

*a*

*y*

*x*

**Room for Work**

**Part (a)**

The potential is given by



which gives us



or

.

From Gauss’s law, the electric field is given by:

*r* > *b*: 

*a*  < *r* < *b*: 

*r* < *a*: 

where



.

Hence, we have

*r*  > *b*:



*a*  < *r* < *b*:



*r* < *a*:

.

The final result is:

*r*  > *b*:



*a*  < *r* < *b*:



*r* < *a*:

.

**Part (b)**

The electric stored energy is

.

Hence, we have



or

.

This gives us

.

Problem 2 (35 pts.)

A twin-lead transmission line consists of two parallel wires as shown below. Each wire has a radius of *a* and the separation between the centers of the wires is 2*h*. Assume that the wires are far enough apart so that the surface charge densities on each wire are uniform, and that each wire has the opposite charge from the other one. This means that each wire can be modeled as an effective line charge running down the middle of the wire. The top wire will have *ρ*l0 [C/m] and the bottom wire will have -*ρ*l0 [C/m].

a) Calculate the electric field vector *E*(*y*) along the vertical line *x* = 0 for -(*h*-*a*) < *y* < *h*-*a*. (This line which runs between the two wires, connecting points *A* and *B*.)

b) Calculate the voltage drop *VAB* between the two wires, where point *A* is at *y* = *h*-*a* and point *B* is at *y* = -(*h*-*a*).

c) Calculate the capacitance per unit length of the twin-lead transmission line.

*y*

*B*

*A*

2*h*

*x*

**Room for Work**

**Part (a)**

The electric field for the infinite line charge is given by

.

Applying this to the two line charges that model the twin lead transmission line, we have

.

**Part (b)**

The general formula for the voltage drop is

.

The voltage drop between the top wire and the bottom wire is given as

.

Hence, we have

,

or

.

Performing the integration, we have

.

Hence, we have

.

This gives us

.

The final result is then

.

**Part (c)**

The capacitance per unit length is given by

.

Hence, we have

.

Problem 3 (30 pts.)

A perfectly conducting hollow spherical shell has a radius of *a*. The shell is at a potential of *V*0 volts, and the potential at infinity is zero. Determine the following quantities below, leaving your answers in terms of *V*0 and the radius *a*.

a) Find potential Φ outside the shell (*r* > *a*), by solving Laplace’s equation.

b) Find potential Φ inside the shell (*r* < *a*).

c) Find the electric field vector outside the shell in terms of *V*0.

d) Find the surface charge density on the outer surface of the shell.

e) Find the maximum value of *V*0 that can be placed on the shell before dielectric breakdown occurs at the surface of the shell, assuming that the dielectric breakdown of the surrounding air is *Ec*.

*q*

*a*

PEC

*x*

*y*

*z*

**Room for Work**

**Part (a)**

Laplace’s equation is

,

so that

.

Since the potential is only a function of  *r*, we have

.

This gives us

.

Since the potential must be *V*0 at *r* = *a* and zero at *r* = ∞, we have

.

**Part (b)**

The potential is constant inside the spherical shell (the electric field is zero), and hence it is the same as it is in the above solution at *r* = *a*. Hence, we have

.

**Part (c)**

We have

.

Therefore, we have

.

This gives us

.

**Part (d)**

The charge density on the outer surface of the metal shell is given from boundary conditions as

,

Where the electric field is evaluated at the surface *r* = *a*.

Hence, we have

.

**Part (e)**

From part (c) we have

.

Hence, we have

.