##### DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

# Name: \_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 2317

#### Applied Electricity and Magnetism

**Exam 2**

#### April 29, 2014

1. This exam is closed-book and closed-notes notes. A formula sheet is provided. A calculator is allowed (as long as it cannot be used to communicate), but no other device (laptop, phone, tablet, etc.) is allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

FORMULA SHEET































































































**Electrostatic Triangle**

**TABLE OF INTEGRALS**

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TABLE OF COORDINATE SYSTEM FORMULAS

 

 

 

 

 

 

 

























Problem 1 (20 pts.)

**Please provide a short answer to the following questions:**

a) In most naturally occurring materials, the relative permittivity is always larger than one. Why is this?

The molecules (modeled as electrostatic dipoles) always rotate so that the dipole moments align with the field, not against it.

b) If dielectric breakdown occurs in a coaxial cable, where will the breakdown first occur at?

At the surface of the inner wire.

c) There can be only one solution of the Poisson equation in a region of space that corresponds to a given charge distribution and a given potential on the boundary of the region. What is the name of the principle that guarantees this?

The uniqueness theorem (or principle).

d) What is the name of the group of electrons that starts descending from the bottom of a cloud to initiate a lightning strike?

The stepped leader.

e) What is the name of the law (one of Maxwell’s equations) that explains how transformers and generators work?

Faraday’s law

Problem 2 (25 pts.)

A spherical shell of uniform surface charge density *ρs*0 has a radius *a*. This charge density is inside of a perfectly conducting spherical shell of radius *b* (where *b* > *a*). The spherical conducting shell of radius *b* is grounded.

Find the stored electric energy of the system, using the electric field formula.

*b*

*a*

*ρs*0

PEC

ROOM FOR WORK

Solution

The electric field only exists for *a* < *r* < *b*. In this region the electric field vector is

.

The stored energy is thus

.

Hence we have

.

This gives us

.

The answer is thus

.

Problem 3 (25 pts.)

A power line has a radius of *a*. The center of the power line is at a height of *h* above the ground, at *x* = 0 and *y* = *h*. (The power line runs parallel to the *z* direction, and is infinite in this direction.) The surface of the earth at *y* = 0 may be considered to be a perfect electric conductor. If the voltage of the power line (with respect to the earth) is high enough, corona discharge will occur. The air will begin to breakdown first at the bottom of the wire (*x* = 0, *y* = *h* - *a*). Assume that the breakdown field strength for air is denoted as *Ec*.

Derive a formula for the maximum voltage *V*0 that can be placed on the power line (assuming that the earth is at zero volts) before corona discharge begins to occur (at the bottom of the wire). In your derivation, make sure that you account for the presence of the earth!

Helpful hint: There is no need to derive anything that is already on the formula sheet.

*h*

Earth

*x*

*y*

ROOM FOR WORK

Solution

The effective line charge density that models the power line is

.

Accounting for the image, the electric field at the bottom of the power line is

.

We then set the magnitude of the electric field to the critical breakdown field strength, so that

.

This gives us

.

From the first equation we then have

.

This gives us

.

Simplifying, we have

.

Although it is not necessary to simplify this, this can also be written as

.

Since *a* << *h*, we can also write

.

Problem 4 (30 pts.)

A spherical conducting sphere of radius *a* is surrounded by an infinite medium of air (*ε* = *ε*0). The potential on the surface of the sphere is *V*0 and the potential at infinity is zero.

a) Solve the Laplace equation to find the potential Φ at all points in space outside the sphere.

b) Find the charge density on the outer surface of the sphere, using your answer to part (a).

c) Find the capacitance between the sphere and infinity, using your answers to the previous parts.

*a*

PEC

*ε*0

*V*0

Note that in spherical coordinates we have the following formulas, which may be helpful:







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ROOM FOR WORK

Solution

From Laplace’s equation we have

.

This gives us



or



or



or

.

Since the potential is zero at infinity, we have

.

Hence, we have

.

Applying the boundary condition on the surface of the sphere, we have

.

Hence,

.

The potential outside the sphere is then

.

The electric field outside the sphere is



which gives us

.

From this we have

.

The charge density is then

,

with the field evaluated at *r* = *a*.

We thus have



or

.

The total charge on the sphere is

.

Using

,

we have

.