# Name: \_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 2**

#### April 25, 2019

1. This exam is open-book and open-notes. A calculator is allowed (as long as it cannot be used to communicate), but no other device (laptop, phone, tablet, etc.) is allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

**TABLE OF INTEGRALS**

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Problem 1 (30 pts.)

Two horizontal circular disks of uniform surface charge density are situated as shown below. One disk has *ρs* = *ρs*0 and is located at *z* = *h*. The other disk has *ρs* = - *ρs*0 and is located at *z* = -*h*.

Find the potential Φ(*z*) on the *z* axis for -*h* < *z* < *h*, assuming that the potential is zero at the center of the bottom disk.



**Room for Work**

Using the potential formula, we have

.

Hence, we have



or



or



We then need to add a constant *C* to the answer to make the potential zero at the center of the bottom disk. We then have



where

.

Problem 2 (40 pts.)

A hollow (air-filled) perfectly conducting spherical shell has a radius *a* and is at a potential *Va*. It is surrounded by a larger perfectly conducting spherical shell of radius *b* that is at a potential of *Vb*. Between the two spherical conducting shells is a dielectric material with a relative permittivity of *εr*. Outside the larger spherical shell is air. The potential is zero at infinity.

a) Find the potential Φ in all three regions: *r* < *a*, *a* < *r* < *b*, and *r* > *b*.

b) Find the total charge that is on the inner and outer surfaces of the inner shell, and the total charge that is on the inner and outer surfaces of the outer shell. That is, find





**Room for Work**

**Part (a)**

For *r* < *a* there is no electric field, and the potential is constant. Hence, we have

.

For *a* < *r* < *b* we have

.

Enforcing the boundary conditions at *r* = *a* and *r* = *b* gives us



Hence, we have



For *r* > *b* we have

.

Enforcing the boundary conditions at *r* = *a* and *r* = *b* gives us



Hence, we have



Hence, we have



**Part (b)**

From boundary conditions on a PEC, we know that



Hence we have

,

where the plus sign is for an inner surface and the minus sign is for an outer surface.

This gives us:







.

Problem 3 (30 pts.)

A perfectly conducting sphere of radius *a* has its center at a height of *h* about a perfectly conducting infinite floor. The sphere has a charge *Q* on it, and this corresponding surface charge density may be assumed to be uniformly distributed on the surface of the sphere.

a) Find the electric field vector *E*(*z*) along a vertical line that runs between the floor and the bottom of the sphere (on the *z* axis with 0 < *z* < *h*-*a*).

b) Find the maximum charge *Q*max that can be put on the sphere before the surrounding air begins to break down. Assume that the air has a dielectric breakdown strength of *Ec*.

c) Find the capacitance between the sphere and the floor.



**Room for Work**

**Part (a)**

The image picture has a point charge *Q* at *z* = *h* and an image charge –*Q* at *z* = -*h*.

We then have

.

**Part (b)**

The air is breaking down at the bottom of the sphere. Hence, we set

.

This gives us



so

.

**Part (c)**

Top get the capacitance we need to find the voltage between the sphere (object *A*) and the floor (object *B*). We can integrate along a vertical line, between the bottom of the sphere and the floor, to get



This gives us

.

Hence, we have



or

.

This gives us



or

.

The capacitance is given by

.

Hence, we have

.