#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 2**

#### April 28, 2022

Name \_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**General Instructions**

1. This exam is open-book and open-notes.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Write neatly. You will not be given credit for work that is not **easily** legible.
4. Leave answers in terms of the parameters given in the problem.
5. Show units in all of your final answers.
6. Circle your final answers.
7. Double-check your answers. For simpler problems, partial credit may not be given.
8. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.

**TABLE OF INTEGRALS**

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Problem 1 (35 pts.)

An annulus of surface charge density lying in the *z* = 0 plane is described by

.

a) Calculate the potential function  at an observation point located at  in rectangular coordinates on the *z* axis, assuming that the potential is zero at infinity.

b) How would your answer change if the potential is now zero at the origin?

SOLUTION

Part (a)



We have

.

In cylindrical coordinates, we the have



or

.

The integrand is separable, and the limits are fixed, so we can write



Hence, evaluating the  integral, we have

.

This gives us



or



Part (b)



or



or



Problem 2 (30 pts.)

An infinite power line carrying a current in the *z* direction runs along the *z* axis. This produces a magnetic field that is described by

,

where

.

Find the output voltage of a rectangular loop sensor that is in the *xz* plane near the power line as shown below. Make sure that you get the sign right! (You can use Lenz’s law to help with this.)



SOLUTION

Invoking Lenz’s law, we have

.

We also note that

.

Hence,



or

.

Hence,

.

Hence, we have

.

Problem 3 (35 pts.)

A spherical conducting PEC shell of radius *a* is surrounding by another PEC shell of larger radius *b*. Between the two shells is a dielectric material having a relative permittivity . The inner shell is at a potential of  volts and the outer shell is at a potential of volts.

a) Find the potential function Φ in the region *a* < *r* < *b*.

b) Find the surface charge density  on the outer surface of the inner PEC shell, at *r* = *a*.

c) Find the capacitance between the two PEC shells, using your answers to the above parts.

SOLUTION

Part (a)

After solving Laplace’s equation in spherical coordinates, we find the general solution is

.

We apply boundary conditions:



Subtracting the two equations and solving for *A*, we have

.

Then solving for *B*, we have

.

The potential is then

.

(Note that the potential does not depend on )

Part (b)

We have

.

We also have

,

so that, since Φ has only an *r* variation,

,

and therefore

.

Therefore, we have

.

The surface charge density on the outer surface of the inner PEC sphere is then

,

so that

.

Part (c)

We have

The capacitance is given by

.

This gives us

.

BONUS PROBLEM (5 pts.)

A video was recently add to the Blackboard site, showing a car being struck by lightning. Give the two basic electrostatic principles that were illustrated in the video, which explain the following:

(1) why the antenna on the car was struck;

(2) why the driver inside the car was safe.

SOLUTION

(1) The antenna was acting as a lightning rod, which sent out a good streamer due to the sharp point property.

(2) The driver inside the car was safe due to the Faraday cage effect.