#### ECE 3318

#### Applied Electricity and Magnetism

**Exam 2**

#### April 27, 2023

**Name: SOLUTION**

**Instructions**

1. This exam is open-book and open-notes.
2. Cell phones, laptops, ipads, and any other devices that have communication functionality are not allowed during the exam.
3. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
4. Write neatly. You will not be given credit for work that is not easilylegible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Remember the UH Academic Honesty Policy. You must not receive or give assistance to anyone else during the exam, or communicate with anyone other than the instructor during the exam.

**TABLE OF INTEGRALS**

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**Problem 1 (35 pts.)**

An engineer attempts to draw power wirelessly from an overhead power line operating at 60 [Hz]. The power line carries a current in the *z* direction that is given by

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This current creates a magnetic field that is given by

.

A rectangular loop with length *L* and width *W* having *N* turns is placed near the power line as shown below. Assume the following parameters (needed for part (c)):



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(a) Find a formula for the output voltage of the loop  (make sure to get the sign right!).

(b) Find a formula for the Thévenin voltage  at the output of the loop in the phasor domain.

(c) Assume that the coil has a resistance of 50 [Ω]. What is the average power that that coil can deliver to a 50 [Ω] load? Use the above given parameters to find a numerical value.

**Solution**

*z*

 **Part (a)**

With the help of Lenz’s law, we have that

.

We have



Hence,

.

We then have

.

**Part (b)**

Converting from the time domain to the phasor domain, we have

.

Hence, we have

.

**Part (c)**

.

We then have

.

Plugging in values, we have

.

**Problem 2 (35 pts.)**

A conducting wire of radius *a* is surrounded by a dielectric coating that has a radius of *b*, as shown in the figure below (the structure is infinite in the *z* direction). Surrounding the coated wire is an outer cylindrical conductor (shield) of radius *c*. The purpose of the coating is to prevent dielectric breakdown when there is a high voltage applied between the wire and the shield. The dielectric has a relative permittivity of  and the air has a dielectric breakdown field strength of  [V/m].

a) Find the capacitance per unit length  between the wire and the shield.

b) Assuming that dielectric breakdown will occur in the air before it does in the dielectric, find a formula for the maximum voltage  that can be placed between the wire and the shield before the air begins to break down.



**Solution**

 **Part (a)**

Let’s assume that we have  [C/m] on the inner wire. The voltage drop between the wire and the shield is

.

From Gauss’s law we then have

.

This gives us

.

The capacitance per unit length is then

.

Hence, we have

.

**Part (b)**

At breakdown we have

.

Thus, at breakdown

.

We have

.

Hence, we have

.

**Problem 3 (30 pts.)**

A circular metal disk of radius *a* lies in the *z* = 0 plane and is centered at the origin. The metal disk has a surface charge density that is

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 where *A* is a constant. (Practical notes: The charge density has an edge singularly at the edge of the disk. The charge density on each side of the disk is one-half of the total charge density, which is given by the above equation.)

a) Find the potential on the *z* axis at (0, 0, *z*), assuming that the potential is zero at infinity. You may leave your answer in the form of a single integral for this part.

b) Use your answer to part (a) to find the electric field on the *z* axis at (0, 0, *z*). You may leave your answer in the form of a single integral for this part.

c) Find the total charge *Q* on the disk. (Your answer should be in closed form.)

d) Find the capacitance between the disk and the surrounding space (the edge of the universe). You may take the potential at the center of the disk (*z* = 0) to be the potential of the entire disk for convenience. (Your answer should be in closed form.)

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**Solution**

 **Part (a)**

From the potential formula we have

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Hence, we have



or

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Performing the integration in , we have

  .

Hence, we have

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**Part (b)**

We have

.

We also have

.

Hence, we have

.

We then have

.

This gives us


or

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**Part (c)**

The total charge on the disk is

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Therefore,



so that

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This gives us

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We use

**.**

This gives us

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Hence, we have

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**Part (d)**

We have

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where

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We therefore have

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We then use

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This gives us

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Hence,

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We then have

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We thus have

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