Part A: Problem # 1

[redacted]

Region 1: $r < a$

$\vec{E} = \emptyset \rightarrow$ no charge enclosed

Region 2: $a < r < b$

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0} \qquad Q_{encl} = \int_S \rho_s \, ds$$

$$E_r 4\pi r^2 = \frac{4\pi a^2}{\varepsilon_0}(5k)$$

$$\vec{E} = \frac{5ka^2}{r^2 \varepsilon_0} \hat{r}$$

Region 3: $b < r < c$

$\vec{E} = 0 \rightarrow$ by definition of conductor

Region 4:

$r > c$

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

See Region 2 solution

$$\vec{E} = \frac{5ka^2}{r^2 \varepsilon_0} \hat{r}$$

Region 4: $r > c$

Find V

$$V = -\int \vec{E} \cdot dl$$

$$= -\int_{\infty}^{r} \frac{5ka^2}{r^2 \varepsilon_0} \, dr$$

$$= \frac{5ka^2}{r\varepsilon_0} \Big|_{\infty}^{r} = \frac{5ka^2}{r\varepsilon_0}$$

Region 3 $b < r < c$

using boundary conditions (ie V continuous across boundary)

$$V = \frac{5ka^2}{c\varepsilon_0}$$

Region 2 $a < r < b$

$$\vec{E} = \frac{5ka^2}{r^2 \varepsilon_0} \hat{r}$$

$$V = -\int \vec{E} \cdot \vec{dl} = -\int \frac{5ka^2}{r^2 \varepsilon_0}$$

$$V = \frac{+5ka^2}{r\varepsilon_0} + C_1$$

at $r = b$

$$V = \frac{5ka^2}{c\varepsilon_0} = \frac{+5ka^2}{b\varepsilon_0} + C_1$$

Region 2 $a < r < b$

V (continued)

at $r = b$

$$V = \frac{5ka^2}{c\varepsilon_o} = \frac{+5ka^2}{b\varepsilon_o} + C_1$$

$$C_1 = \frac{5ka^2}{\varepsilon_o}\left[\frac{1}{c} + \frac{-1}{b}\right]$$

$$\therefore V(a < r < b) = \frac{+5ka^2}{r\varepsilon_o} + \frac{5ka^2}{\varepsilon_o}\left[\frac{1}{c} + \frac{-1}{b}\right]$$

Region 1

$r < a$

$$\vec{E} = \emptyset$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$= C_2$$

solve for C_2 at $r = a$

$$V(r=a) = \frac{+5ka}{\varepsilon_o} + \frac{5ka^2}{\varepsilon_o}\left[\frac{1}{c} + \frac{-1}{b}\right] = C_2$$

$$C_2 = \frac{5ka^2}{\varepsilon_o}\left[+\frac{1}{a} + \frac{1}{c} + \frac{-1}{b}\right]$$