##### DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

# Name: \_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 2317

#### Applied Electricity and Magnetism

**Final Exam**

#### May 1, 2014

#### 11:00 a.m. – 2:00 p.m.

1. This exam is closed-book and closed-notes notes. A formula sheet is provided.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

FORMULA SHEET













































































































































**Electrostatic TriangleTABLE OF INTEGRALS**



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TABLE OF COORDINATE SYSTEM FORMULAS

 

 

 

 

 



























Prob. 1 (25 pts.)

A perfectly conducting metal sphere with a radius *a* is inside of a large metal container that is filled with a liquid having a conductivity *σ*. The dimensions of the container are large enough that the container may be approximated as being infinitely large (and therefore the shape of the container is not important).

This system can be used to determine the conductivity of an unknown liquid, by measuring the resistance seen between the two terminals at *A* and *B*.

Determine the resistance seen between the points *A* and *B*, which are connected to the sphere and the container, respectively.

Liquid

*A*

*B*

Sphere

Container

ROOM FOR WORK

Solution

First, we will find the capacitance assuming a medium that has a permittivity *ε*, and then use the *RC* analogy.

To find the capacitance, we assume a charge *Q* on the inner sphere. From Gauss’s law the electric field is

.

The voltage drop between the sphere and infinity is

.

Hence, the capacitance is

.

Hence,

.

Using



then gives us

.

Prob. 2 (25 pts.)

A coaxial cable is filled with two different dielectric materials as shown below. The inner material (region 1) exists in the region *a* < *ρ* < *b*. The outer material (region 2) exists in the region *b* < *ρ* < *c*. The inner region has a relative permittivity of *εr*1 and a dielectric breakdown field strength of *Ec*1. The outer material (region 2) has a relative permittivity of *εr*2 and a dielectric breakdown field strength of *Ec*2. Assume that *Ec*1 > *Ec*2.

Assuming that all of the parameters except the radius *b* are fixed, derive a formula for the radius *b* (in terms of all of the other given parameters) so that dielectric breakdown occurs at the same time at *ρ* = *a* in the inner material and *ρ* = *b* in the outer material. (That is, as the effective line charge density *ρl* on the inner conductor is increasing, it will reach a value where breakdown occurs at these two points simultaneously.)

*b*

*c*

*a*

*z*

ROOM FOR WORK

Solution

At the breakdown point we have



and

.

Dividing the two equations gives us

.

Hence we have

.

Prob. 3 (25 pts.)

A spherical shell of uniform surface charge density *ρs*0 has a radius *a*. This charge density is inside of a perfectly conducting spherical shell of radius *b* (where *b* > *a*). The spherical conducting shell of radius *b* is grounded.

Find the stored electric energy of the system, using the potential formula (not the formula that has the electric field in it).

*b*

*a*

*ρs*0

PEC

ROOM FOR WORK

Solution

The stored energy is given by

.

This formula assumes that the potential is zero at infinity. There is no electric field outside the PEC shell, since it is grounded. The shell is therefore also at zero volts.

We then have

.

The potential on the surface of the spherical charge shell is

.

Hence, we have

.

Therefore, we have

.

The stored energy is then

.

Hence, we have

.

Prob. 4 (30 pts.)

A parallel-plate transmission line consists of a slab connector that is suspended above a flat strip conductor as shown below. The rectangular-shaped slab conductor has a width *w* and a thickness *t*, and a relative permeability of *μr*. The slab conductor carries a total current of *I* in the *z* direction. The flat strip conductor is also of width *w*, and it carries a current of *I* in the negative *z* direction. The flat strip conductor is located at *y* = 0. The lower surface of the slab conductor is at *y* = *h*. Assume that *w* is large compared to *h* and *t*, so that you may neglect fringing. The current in the slab conductor is uniformly distributed throughout the cross section of the slab. The surface current in the flat strip is uniformly distributed across the width of the strip.

Determine the magnetic field *H* in the following regions:

*y* < 0

0 < *y* < *h*

*h* < *y* < *h*+*t*

*y > h*+*t.*

# *I*

*t*

*x*

*y*

*w*

*I*

*z*

*I*

*h*

*μr*

ROOM FOR WORK

Solution

The magnetic field is found from Ampere’s law:

.

A rectangular Amperian path *C* is used that has the top part of the path in the region above the entire structure, where the magnetic field is zero. The bottom part of the path is at the observation point. We integrate counterclockwise around the path, so that the current enclosed is the current cutting through the path in the positive *z* direction. The width of the path is called Δ*x*.

The magnetic field is in the *x* direction. We then have

.

In the four different regions we have:







.

Hence we have:



Prob. 5 (30 pts.)

A square loop of current *I* lies in the *z* = 0 plane, centered at the origin, as shown below. Find the magnetic field *H* on the *z* axis.

You may use symmetry as much as possible to help you. For example, what direction will the magnetic field be in at the observation point? How will the contributions from the four different sides compare? You may use your answers to these questions to save you time.

*z*

*r* (0, 0, *z*)

*y*

2*a*

2*a*

*x*

ROOM FOR WORK

Solution

The magnetic field is found from the Biot-Savart law:

.

From symmetry, the total magnetic field will be in the *z* direction, and the *z* component will be four times that from a single side of the loop.

Consider a single side, the one at *x* = *a*. For this side we have





so

.

Hence,



and thus

.

We then have

.

Since we are only interested in the *z* component, we have

.

Performing the integration, we use (from the Table of Integrals)

**.**

This gives us

.

Evaluating, we have

.

Simplifying,

.

Therefore, the magnetic field from all four sides is



or

.

Prob. 6 (15 pts.)

Find the mutual inductance *M*21 between two circular loops in free space. One current loop has a radius *a* and lies in the *z* = 0 plane, and is centered at the origin as shown below. The other loop has a radius *b* and lies parallel to the *xy* plane at a height *z* = *h*, with its center on the *z* axis. Assume that the radii *a* and *b* are both very small compared to the height *h*. This means that the magnetic field from one loop can be assumed to be approximately constant over the surface of the other loop. Note the reference directions of the defined currents on the loops. (Make sure that you get the sign correct for the mutual inductance!)

The magnetic field on the *z* axis produced by the lower loop of current is

.

*z*

*a*

*b*

*I*1

*I* 2

*y*

*x*

ROOM FOR WORK

Solution

The mutual inductance is given by

.

The mutual flux is

.

From the right-hand rule, we see that

.

Therefore,

.

Since the magnetic field form loop 1 is approximately constant over the area for loop 2, we have

.

We then have

.

The mutual inductance is then

.