##### DO NOT BEGIN THIS EXAM UNTIL TOLD TO START

# Name: \_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 2317

#### Applied Electricity and Magnetism

**Final Exam**

#### May 7, 2015

#### 11:00 a.m. – 2:00 p.m.

1. This exam is open-book and open-notes. A calculator is allowed (as long as it cannot be used to communicate), but no other device (laptop, phone, tablet, etc.) is allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

Prob. 1 (30 pts.)

A parallel-plate capacitor is filled with a material having a relative permittivity of *εr*. The bottom plate is at a potential of *V*1 volts and the top plate is at a potential of *V*2 volts, with *V*2 > *V*1. Fringing at the edges of the capacitor can be neglected.

a) Solve for the potential Φ inside the parallel-plate capacitor.

b) Find the electric field vector inside the parallel-plate capacitor, using your answer to part (a).

c) Find the surface charge density on the top plate, using your answer to part (b) .

d) Find the capacitance, using your answer to part (c)



ROOM FOR WORK

After applying boundary conditions, the potential is

 

or

  .

Using the gradient, the electric field is

 .

From boundary conditions, the surface charge density is

 .

The capacitance is

 .

Prob. 2 (40 pts.)

A small metal sphere of radius *a* is suspended by a conducting wire over a large ground plane as shown below. The metal wire connects the sphere to a top plate at a potential of *V*0 volts. The center of the sphere is at a height *d* above the ground plane. The height *h* of the top plate is large enough so that the top plate can be neglected. The metal wire can also be neglected. The metal sphere is close to the ground plane, however, so that the effects of the ground plane cannot be neglected. Because the sphere is small, it can be assumed that the surface charge density on the sphere is approximately uniform.

a) Find the charge *q* on the sphere.

b) Assuming that the air surrounding the metal sphere has a breakdown field strength of *Ec*, find the maximum voltage *V*0*max* that can be placed on the metal sphere before breakdown of the air occurs. Assume that breakdown occurs at the bottom of the sphere.

c) Imagine now that the air region is replaced by a region of conductive liquid having a conductivity *σ*. What would the resistance be seen between the metal sphere and the lower plate?

ROOM FOR WORK

Image theory is used to find the electric field, and this is integrated to find the voltage drop, which is set equal to *V*0.

Using the potential formula and superposition, we have (since the ground plane is at zero volts)

.

Hence, we have

 .

We could also get this from the electric field along a vertical line between the sphere and the ground plane, which is

 .

The voltage drop is given by

 .

This yields

 

or

 .

Hence, we have

 .

Dielectric breakdown occurs when

 

so that

 .

Therefore, we have

 

The capacitance is given by

 .

Using the RC analogy, we then have

 

so that

 

Prob. 3 (30 pts.)

A wire is shown below. The wire is infinite in the positive and negative *y* directions, and is located at *x* = 0. At *y* = 0 the wire changes height from *z* = 0 to *z* = *h*.

Find the magnetic field vector *H* at a point on the *z* axis that is located at a height of *z* above the origin, where *z* > *h*.



ROOM FOR WORK

The vertical part of the wire does not contribute. For the right part of the wire we have



so that

.

This gives us

.

Performing the integration, we have

.

This gives us

.

The result is then

.

Similarly, the contribution from the left part of the wire is

,

which gives us

.

The total magnetic field is then

.

Prob. 4 (30 pts.)

An infinite power line carries a low-frequency current in the *z* direction that is

 .

The power line is at a height of *h* above the surface of the earth. A small loop of width *w* is placed in proximity of the power line, as shown below. The center of the loop is at (*x*0, *y*0). (A separate top view of the loop is also shown for clarity, looking down from the positive y direction.) Because the earth is nonmagnetic and the frequency is low, the effects of the earth may be neglected in the calculation of the magnetic field.

a) Find the magnetic field components *Hx*, *Hy*, *Hz* at the center of the loop.

b) Find the magnetic flux *ψ* going through the loop in the upward direction, assuming that the magnetic field is approximately constant over the small dimensions of the loop.

****c) Find the voltage *v*(*t*) across the output terminals of the loop.

ROOM FOR WORK

The magnetic field at the center of the loop is

,

which in rectangular coordinates becomes



or



or

.

Hence, we have





.

The magnetic flux is approximately



and thus

.

The open circuit voltage on the loop is



so that



Prob. 5 (30 pts.)

An infinite wire carrying a current *I*1 in the *z* direction produces a force on a loop that is situated as shown below. The left end of the loop is at *x* = *d*, while the right end of the loop is very far away (at *x* = ∞). The loop carries a current *I*2.

Find the force vector *F* on the loop due to the infinite wire carrying current *I*1.



ROOM FOR WORK

The force is given by

.

the force on the top and bottom parts of the loop cancel so, we only need to consider the left edge of the loop.

We then have

.

The magnetic field from the wire at the left edge of the loop is

.

Therefore, we have



which give us

.

Hence, the result is

.