# Name: \_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 3318

#### Applied Electricity and Magnetism

**Final Exam**

#### May 8, 2018

1. This exam is open-book and open-notes. A calculator is allowed (as long as it cannot be used to communicate), but no other device (laptop, phone, tablet, etc.) is allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Make sure you sign the academic honesty statement on the next page.

Academic Honesty Statement

I agree to abide by the UH Academic Honesty Policy during this exam. I understand that the punishment for violating this policy will be most severe, including the possibility of getting an F in the class and/or getting expelled from the University.

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Signature

**TABLE OF INTEGRALS**

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Problem 1 (15 pts.)

Sometimes people propose to use lighting as a way to extract and store energy. Of course, there would be concerns about the very high levels of voltage involved, but let’s ignore this and do a calculation to see how much power can extracted from a thunderstorm, to see how practical this scheme is.

Assume that during a typical thunderstorm, there is a lightning strike to earth every 10 seconds. the voltage between the base of the cloud and the earth is 150 [MV]. Assume that the current during the strike is at a level of 10 [kA] and that this current (the return stroke current) lasts for 100 [μs]. The total charge that gets transferred during each strike is thus -1 [C]. Assume that the entire thunderstorm lasts for 30 minutes. Assume that the energy contained within all of the lightning strikes to earth during the entire thunderstorm can be captured and stored.

Assume that this stored energy is then delivered to a city at a power rate of 1000 [MW] (roughly equivalent to the power output of a large power plant).

For how long could this total stored energy be used to power the city?



**Room for Work**

**Solution**

The total energy delivered during each strike is

.

The total energy during the entire thunderstorm is

.

For a time *T* that we supply power *P* to the city, we then set



or

.

This gives us a time of

.

Problem 2 (20 pts.)

A uniform volume charge density *ρv*0 is inside of a parallel-plate capacitor. (This charge density is floating in free space between the plates.) The bottom plate of the capacitor (at *x* = 0) is at zero volts. The top plate of the capacitor (at *x* = *h*) is also at zero volts. The plates are large compared to the separation *h*, so fringing may be neglected.

a) Find the potential Φ(*x*) at any point inside the capacitor.

b) Find the surface charge densities  and on the top and bottom plates.



**Room for Work**

**Solution**

**Part (a)**

We have

.

Integrating twice, we have



Enforcing the boundary conditions, we have



.

Hence we have

.

**Part (b)**

We then have

.

We then have

.

Problem 3 (25 pts.)

A power line of radius *a* is located at a height *h* above the earth, as shown below. The earth may be modeled as a perfect conductor at the power line frequency.

a) Derive a formula for the capacitance per unit length between the power line and the earth. (You may use any formula in the class notes to help you.)

b) Now assume that the air has a conductivity of *σ*. Find the resistance between a length *L* of the power line and the earth.

c) Assume the following dimensions:



Find the resistance (in Ohms) between the power line and the earth.

Practical note: This would be the resistance between the power line and the earth assuming that no corona discharge is taking place. If the air was breaking down and corona discharge was taking place, the resistance would be much lower than this.



**Room for Work**

**Solution**

**Part (a)**

We have

.

Hence, we have

.

**Part (b)**

From the RC analogy we then have

.

The conductance for a length *L* of line is then

.

The corresponding resistance is

.

**Part (c)**

Inserting the values, we have



Problem 4 (25 pts.)

A bus line in a power substation consists of a hollow aluminum tube of inner radius *a* and outer radius *b*. (The aluminum is nonmagnetic, so *μ* = *μ*0.) The bus (tube) runs in the *z* direction. Assume that this bus is carrying a current of *I* amps, and that this current is uniformly distributed within the cross section of the conductor. (The power line frequency is low enough that we can consider this to be a DC current problem.)

a) Find the magnetic field *H* in all three regions (*ρ* < *a*, *a* < *ρ* < *b, ρ* > *b*)

b) Find the magnetic stored energy per unit length inside the tube (i.e., in the region *a* < *ρ* < *b*).

c) Find the “internal inductance per unit length” of the tube, which is based on the energy per unit length stored inside the tube (i.e., in the region *a* < *ρ* < *b*).



**Room for Work**

**Solution**

**Part (a)**

We have



For *ρ* < *a* there is no current enclosed, and hence we have

.

For *a* < *ρ* < *b* we have

.

We then have



where

.

Hence we have

.

Hence

.

For *ρ* > *b* the current enclosed is *I*, and hence we have

.

**Part (b)**

The stored energy in a length *h* of tube is

.

Hence



so

.

**Part (c)**

We have

.

From this we have



Problem 5 (30 pts.)

An infinite wire is shown below. The wire consists of two semi-infinite vertical segments and a horizontal segment of length *h*.

Find the magnetic field at a point on the *x* axis at *x* = *x*0 (where *x*0 > *h*).



**Room for Work**

**Solution**

We have

.

The horizontal segment does not contribute since the differential length vector is parallel to the *R* vector.

For the vertical segments we have

.

Hence we have



or



or



or

.

Bonus Problem (20 pts.)

Two coils (primary and secondary) are wound on a square transformer core as shown below.

a) Draw the equivalent “magnetic circuit” for the system, with all elements and the polarity of all sources in the circuit labeled clearly. Explain how you chose the polarity of the sources.

b) Use your magnetic circuit to show that

,

where *Bc* is the magnetic flux density that circulates clockwise in the core. (The currents and the magnetic flux density are all phasor-domain quantities here.)

Please note: In order to get credit, you must show all of your steps clearly so that your deviation is easy to understand!

Practical note: For an ideal transformer, , so .



**Room for Work**

**Solution**

**Part (a)**

The equivalent magnetic circuit is shown below. The polarity of the voltage sources is found by the right-hand rule for determining the direction of the magnetic fields due to each of the two coils.



**Part (b)**

We have



so

.

Hence, we have

