#### ECE 3318

#### Applied Electricity and Magnetism

**Final Exam**

#### May 10, 2022

Name \_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**General Instructions**

1. This exam is open-book and open-notes.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
3. Write neatly. You will not be given credit for work that is not **easily** legible.
4. Leave answers in terms of the parameters given in the problem.
5. Show units in all of your final answers.
6. Circle your final answers.
7. Double-check your answers. For simpler problems, partial credit may not be given.
8. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.

**TABLE OF INTEGRALS**

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Problem 1 (20 pts.)

A cylindrical power bus (tube) in a power substation runs next to a tall grounded metal wall as shown below. The radius of the power bus is *a*. The spacing between the center of the bus and the wall is denoted as *h*. The air has a dielectric breakdown field strength of *Ec* [V/m].

Derive a formula for the maximum voltage *V*max that can be put on the bus (with respect to ground) before corona discharge takes place on the bus.

You may use any formula from the class notes that you wish. Make sure your answer is as accurate as possible (do not neglect anything in your derivaiton).

**Solution**

From the class notes we have:

.

Hence,

.

Also, using image theory, the breakdown condition is

.

Solving for  in terms of  and then substituting into the equation with , we have

.

Problem 2 (20 pts.)

A stepped leader is descending vertically down from the clouds. The stepped leader is descending halfway between two tall buildings, as shown below. Each building is 40 meters tall. The stepped leader will strike an object once the tip of the stepped leader is within 50 meters of any point on that object (due to the streamer coming from the object). A flagpole is located halfway between the two buildings, of height *h*. The two buildings are separated by a distance of 30 [m].

What is the maximum allowed height *h* of the flagpole, if we wish to be sure that the flagpole will not get struck by lightning?



**Solution**

Imagine the tip of the stepped leader getting ready to connect with the corner of the two buildings. the line between the tip of the stepped leader and the corner of one of the buildings forms a right triangle with a hypotenuse of 50 [m] and a horizontal base of 15 [m]. The vertical side of the right triangle will be

.

The height of the stepped leader (from the ground) when it makes contact with the buildings will be

.

We want to make sure that the top of the flagpole is at least 50 [m] below .

Hence, we have

 .

or

.

This gives us



In the “rolling sphere” method, we can also visual a giant sphere of radius 50 [m] sitting on top of the two buildings. The maximum height of the flagpole corresponds to the point at the bottom of the sphere.

Problem 3 (20 pts.)

A transmission line consists of a center metal slab surrounded by a top and bottom metal slab as shown below. All three slabs have the same thickness *h* and width *w*, and they are infinite in the *z* direction. The central slab carries a DC current *I* in the *z* direction. The top and bottom slabs each carry a DC current of *I*/2 in the negative *z* direction.

Find the magnetic field inside the central slab, for .

Assume that  so that you may neglect fringing. You may also assume that the current density inside each slab is uniform.

**Solution**

We use Ampere’s law. The magnetic field is zero above and below the entire structure, since the net total equivalent surface current (from all three slabs) is zero. So a good choice of Amperian path is a rectangular path that has a top part that is above the entire structure, and a bottom part that goes through the observation point at altitude *y* above the center.

Let’s choose a path *C* that goes counterclockwise along the rectangular loop path. The current enclosed is then the current that cuts through the path in the positive *z* direction.

Since we neglect fringing, the magnetic field is in the *x* direction.

Assuming that the width of the path is called  in the *x* direction, we have

.

Hence,

.

We then have

.

This simplifies to

.

Note that this is the same answer that we would have gotten by ignoring the top and bottom slabs, and taken only the central slab. This is because in the region between the top and bottom slabs, , the field from the top and nottom slabs cancels.

Problem 4 (20 pts.)

Find the magnetic field at the observation point on the *y* axis due to the wire shown below. The wire is infinite along the *x* and *z* axes, and carries a DC current of *I* Amps.



**Solution**

 We use the Biot-Savart law. We have two parts of the wire: part (1) on the *x* axis and part (2) on the *z* axis.

We have



.

This gives us



.

Hence, we have

.

Note that each part of the wire gives 1/2 of the magnetic field that it would produce if it were infinite.

Problem 5 (20 pts.)

Two coils (coil 1 and coil 2) are wound on a square transformer core as shown below.

a) Find a formula for the self inductances  and .

b) Find a formula for the mutual inductance .

Use a magnetic circuit model to help you with this calculation.





**Solution**

**Part (a)**

We excite coil 1 and leave coil 2 open-circuited. From the magnetic circuit, we have

.

Here *ψ* is the magnetic flux (in webers) going clockwise around the core. We also have

.

We thus have

.

**Part (b)**

In a similar way, we now excite coil 2 and leave coil 1 open-circuited. We then have

.

Here *ψ* is the magnetic flux (in webers) going counterclockwise around the core. We also have

.

We thus have

.

**Part (c)**

For  we excite coil 1 and leave coil 2 open-circuited. As for part (a), we have

.

Here *ψ* is the magnetic flux (in webers) going counterclockwise around the core. We also have

.

The minus sign is because the unit normal for coil 2 (according to the right-hand rule) is pointing up on the right leg of the core, while the flux *ψ* is going down in the right leg of the core.

We thus have

.

Note that

.

Sometimes people define a coupling coefficient *k* by using

.

Here we can see that in this case

.

It can be shown that the coupling coefficient measures the amount of flux from one coil that goes through the other coil, relative to the total flux coming out of the excited coil. We would expect that *k* = 1 here, since we are assuming that the core is ideal with no flux leakage.

BONUS PROBLEM (10 pts)

(1) What is the “rolling sphere” method, and how can it be used to predict if something is subject to being hit by lightning?

(2) Why it is important for a lightning rod to be well grounded?

**Solution**

(1) In the “rolling sphere” method, we imagine a large sphere of radius 50 [m] (the maximum length of a streamer), rolling along the ground, rolling over buildings, trees, etc., as it goes. Whatever it touches as it rolls along the terrain is subject to being struck by lightning.

(2) If a lightning rod is not well grounded, then the lightning rod will attract the lightning to it (due to its height and the sharp-point effect), but the current will not have a safe path to travel down to the earth once the lightning rod it is struck, so it will go through the building!