#### ECE 3318

#### Applied Electricity and Magnetism

**Final Exam**

#### May 11, 2023

Name: **SOLUTION**

**General Instructions**

1. This exam is open-book and open-notes.
2. Cell phones, laptops, ipads, and any other devices that have communication functionality are not allowed during the exam.
3. Show all of your work. No credit will be given if the work required to obtain the solutions is not shown.
4. Write neatly. You will not be given credit for work that is not easilylegible.
5. Leave answers in terms of the parameters given in the problem.
6. Show units in all of your final answers.
7. Circle your final answers.
8. Double-check your answers. For simpler problems, partial credit may not be given.
9. If you have any questions, ask the instructor. You will not be given credit for work that is based on a wrong assumption.
10. Remember the UH Academic Honesty Policy. You must not receive or give assistance to anyone else during the exam, or communicate with anyone other than the instructor during the exam.

**TABLE OF INTEGRALS**

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**Problem 1 (20 pts.)**

A spherical conductor of radius *a* is located at a height *h* above the ground (as measured from the center of the sphere) in air. Assume that the frequency is low enough so that the ground may be treated as a perfect conductor. Also assume that *h* >> *a*, so that that surface charge density on the sphere may be assumed to be uniform.

a) Derive a formula for the capacitance between the sphere and the ground.

b) Assume that a bird sits on a 60 [Hz] power line that is at a voltage of 10,000 [V] (the magnitude of the peak voltage phasor) that is 20 [m] above the ground. Assume that the bird is modeled as a sphere that has a radius of 5 [cm]. How much current (the magnitude of the peak current phasor) flows through the bird down to the ground, due to the capacitance between the bird and the ground?



**Solution**

**Part (a)**

From image theory, and the formula for the potential due to a point charge, we have

,

where *Q* is the charge on the sphere and  is the voltage of the sphere (with the earth being at zero volts).

The capacitance is

.

We have

.

Hence, we have

.

**Part (b)**

From the impedance of a capacitor, the current is

.

Hence, we have

.

Thus,

.

Plugging in the values, we have

*I* = 21.0 [μA].

This current is not enough to hurt the bird. In fact, the bird probably does not even feel it. Note that the smallest current that people can feel is around 1 [mA]. (The dangerous range for people starts at around 10 [mA].)

**Problem 2 (20 pts.)**

A conducting system consists of two conducting spherical shells, one of radius *a* and a larger one of radius *b*. Between the two spheres is a dielectric with a relative permittivity .

a) Assume that the inner sphere is at a potential of *Va* volts and the potential of the outer sphere is at *Vb* volts. Solve for the potential function Φ in the region between the two spheres, *a* < *r* < *b*.

b) Solve for the surface charge density  that is on the outer surface of the inner sphere.



**Solution**

**Part (a)**

From the solution of the Laplace equation in spherical coordinates, we have

.

Applying boundary conditions, we have



Subtracting the two equations gives us



or

.

From the first boundary condition equation we then have



so that

.

**Part (b)**

The surface charge density is

.

We also have

,

so that

.

We then have

.

Hence, we have

.

Therefore, we have

.

**Problem 3 (20 pts.)**

A transmission line consists of a center metal slab of width *w* and height *h* that is below a thin metal sheet that also has width *w*, as shown below. The structure is infinite in the *z* direction. The slab carries a DC current *I* Amps in the *z* direction. The metal sheet carries a DC current of *I* Amps in the negative *z* direction. Assume that  so that you may neglect fringing.

a) Find the magnetic field inside the slab, for .

b) Find the magnetic field in the air region .

c) Find the magnetic field in the air region .



**Solution**

The magnetic field is in the *x* direction, so that

.

We use ampere’s law, using a rectangular path with the bottom at a point below the structure (where the magnetic field is zero) and the top of the path though the observation point. The path *C* is chosen to go counterclockwise. Ampere’s law is

.

Using a loop path of width Δ in the *x* direction, we have



or

.

**Part (a)**

We have

.

We then have

.

**Part (b)**

We have

.

We then have

.

**Part (c)**

We have

.

We then have

.

**Problem 4 (20 pts.)**

Find the magnetic field at the observation point from the wire shown below. The wire carries a DC current of *I* Amps, and it extends to infinity in the positive *x* direction for the top and bottom sides. Do this by calculating the magnetic field from the three parts of the wire, as indicated below.

a) Find the magnetic field *H* from the bottom part of the wire.

b) Find the magnetic field *H* from the top part of the wire.

c) Find the magnetic field *H* from the vertical side part of the wire.



**Solution**

We use the Biot-Savart law, which is

.

**Part (a)**

We have









or

.

We use

****.

We then have

.

The final answer is then

.

Note that this is exactly 1/2 of what an infinite wire along the *x* axis would produce, if it were carrying current in the negative *x* direction.

**Part (b)**

We have









or

.

We use

****.

We then have

.

The final answer is then

.

Again, this is exactly 1/2 of what a corresponding infinite wire would produce.

**Part (c)**

We have









The cross product is zero, so we immediately see that

.

**Problem 5 (20 pts.)**

Two coils (coil 1 and coil 2) are wound on a transformer core as shown below (note how they are wound). Note that *L*1 is the side length of all sections. Also note that the middle vertical segment of the core has a cross-sectional area *A*2 that is twice that of the other segments (which have *A*1).

a) Draw the magnetic circuit for this structure. (Make sure that you show the polarity of the voltage sources in your circuit.)

b) Find a formula for the self inductance .

c) Find a formula for the mutual inductance .



**Solution**

**Part (a)**

The magnetic circuit is shown below.



In this circuit we have

 .

**Part (b)**

The self inductance of coil one is given by

,

where  is the flux going up in the left vertical core segment, due to the current , and  is the total reluctance seem by coil 1, meaning the total resistance seen by source 1 in the magnetic circuit.

Hence, we have that

.

We have

.

Hence,

.

**Part (c)**

The mutual inductance  is given by

.

Hence we have

,

where the last term in parenthesis is due to the current divider law. The negative sign is due to the fact that the unit normal for coil 2 points up, while the current in the load resistor at the right end of the circuit goes down.

Hence, we have

.

This gives us



or

.