ECE 3318
Applied Electricity and Magnetism
Spring 2023

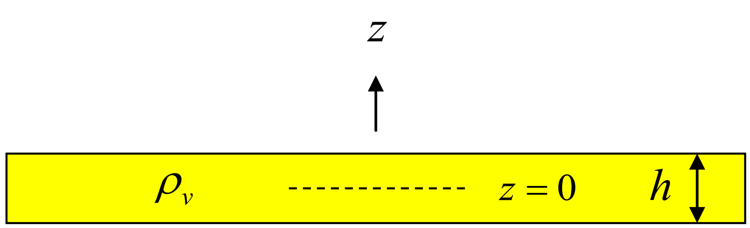
Homework #1

Date Assigned: Thursday, Jan. 19, 2023
Due Date: Thursday, Jan. 26, 2023

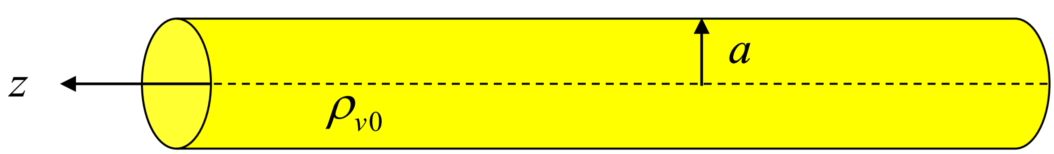
1) A surface charge density $\rho_s = 5x^2(y-1)$ [C/m^2] exists on the *xy* plane. Find the total charge that lies within the rectangle defined by the vertices (0,0,0), (2,0,0), (2,1,0), (0,1,0).

2) A surface charge density $\rho_s = 5x^2(y-1)$ [C/m^2] exists on the *xy* plane. Find the total charge that lies within the triangle defined by the vertices (0,0,0), (1,0,0), (0,2,0).

3) A slab of volume charge density $\rho_v = z^2$ (where $z = 0$ is at the center of the slab) has a thickness h as shown below (side view), and is infinite in the x and y directions. Determine an equivalent surface charge density ρ_s^{eq} (lying in the *xy* plane) that has the same amount of charge per unit area in the *xy* plane as the slab of charge does.



4) A cylindrical region of uniform charge density $\rho_v = \rho_{v0}$ having a radius a is shown below. Determine an effective line charge density ρ_l^{eq} lying along the z axis that has the same amount of charge per unit length (in the z direction) as the cylinder of charge does.



5) A wire is oriented along the z axis. Electrons inside the wire are moving along the z axis (moving in the positive z direction). The magnitude of the charge that passes any given point in one second is 15 [C]. The wire has a radius of 3.0 [mm].

   a) Calculate the current that is flowing in the positive z direction. (This means that the reference direction for the current is in the positive z direction.)

   b) Calculate the current density vector $\underline{J}$ inside the wire.

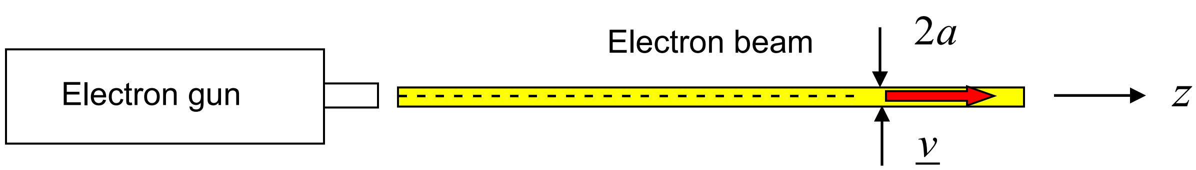
6) A copper wire has a radius of 1 [mm]. The wire is carrying a current of 1 [A] in the positive z direction. Determine the velocity vector of the electrons inside the copper wire (Hint: See the similar example in Notes 3, as use the same parameters for the copper as in this example.)

7) An electron beam coming from an electron gun (shown below) consists of electrons that are uniformly distributed within a cylindrical region of space having a radius a = 1 [mm]. The axis of the cylinder is along the z axis, and the electrons are moving with a velocity vector $\underline{v}$ in the positive z direction. The velocity of the electrons is $0.1c$ where c is the speed of light (2.99792458×10^8 [m/s]). The magnitude of the current in the beam is 1 Amp.

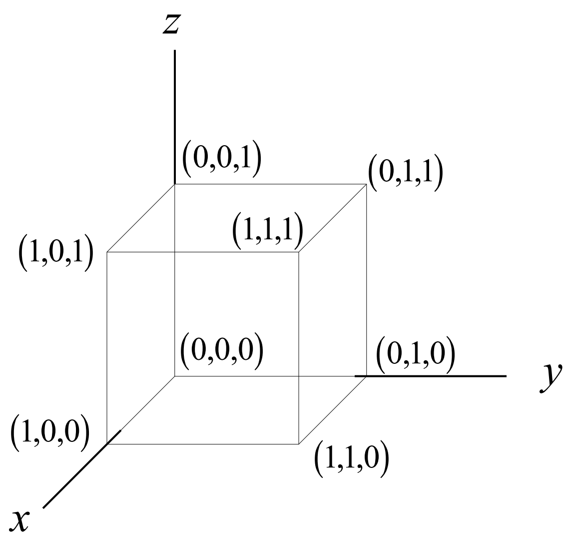
   a) Calculate the current density vector.

   b) Calculate the volume charge density ρ_{v0} inside the electron beam.

   c) Calculate the spacing between the electrons, assuming that they are arranged on a cubical lattice, with spacing Δ between the centers of adjacent electrons in the x, y, and z directions. (Hint: Think of one electron being at the center of each cube of length Δ in the lattice. The average charge density can be found from considering how much charge is inside each cube.)



8) You are given a current density vector field $\underline{J} = \left(x^4 z\right)\underline{\hat{x}} + \left(3yx\right)\underline{\hat{y}} + \left(x^2 y\right)\underline{\hat{z}}$ [A/m^2] and the unit cube shown below.



Evaluate the integral

$$I_{out} = \int_S \underline{J} \cdot \underline{\hat{n}} \; dS \,,$$

where $\underline{\hat{n}}$ is the outward-pointing unit normal vector. Do this by evaluating the contributions from each of the six faces, and then add the results together. Physically, I_{out} represents the total current (in amps) flowing out of the cube.

Extra Problems (not to be turned in – for extra practice only):

Shen and Kong: None
Hayt and Buck, 7^{th} Edition: 5.1, 5.4