ECE 3318
Applied Electricity and Magnetism

Spring 2023

Homework #11

Date Assigned: Thursday, April 13, 2022
Due Date: Thursday, April 20, 2022

Do Probs. 2-6 and 8-10. (The other problems may be done for extra practice, but only these should be turned in.)

1) The specific heat s of a material in [J / (kG deg C)] is the amount of energy in Joules required to raise the temperature of 1 [kg] of material by one degree C. Hence, the energy given to a piece of material of mass m in raising the temperature by ΔT (in centigrade) is

$$U = m\left(\Delta T\right)s\,.$$

If a current density exists inside a material for a time Δt, show that the rise in temperature ΔT in degrees C is given by the formula

$$\Delta T = \frac{\left|\underline{J}\right|^2}{\sigma s \rho}\Delta t\,,$$

where ρ is the mass density of the material in [kg/m^3].

2) A lightning rod is connected to ground through a copper cable having a diameter of 0.5 [cm]. During a lightning strike, a current of 50 [kA] flows for a time of 100 [μS]. Determine the rise in temperature of the cable in degrees C. Assume that the conductivity of the copper is 5.8×10^7 [S/m] and that the specific heat of the copper is 385.60 [J / (kg deg. C)]. The density of copper is 8.96×10^3 [kg / m^3]. Use the answer from Prob. 1 to solve this problem.

3) A coaxial cable has an inner radius of a and an outer radius of b. Between the two conductors (which are assumed to be perfectly conducting) is a lossy material with conductivity σ. Determine the conductance G between the two conductors for a one meter length of cable. What would the resistance R be? What would the conductance be for a 100 [m] length of cable? Explain why the cable is characterized by a conductance per unit length G_l (not a resistance per unit length R_l).

4) A metal sphere of radius a is connected to the positive lead (anode) of a battery through an insulated wire. The sphere acts as an electrode that is placed in the middle of a large metal tank of water. (The tank is perfectly conducting and is large enough that it may be assumed

to be at infinity as far as the metal sphere is concerned.) The water has a conductivity σ. The negative terminal of the battery (cathode) is connected to the metal tank. What is the resistance seen by the battery? Use the *RC* analogy method, or the *RC* formula method (which is appropriate for a homegeneous medium, as we have here). Your first step is to find the capacitance of the corresponding capacitor problem. That is, you need to find the capacitance of the metal sphere in an infinite dielectric region when the outer conductor is at infinity. (You might want to revisit Prob. 3 on HW 10 for this.)

5) A block of lossy Teflon is inserted between the plates of a parallel-plate capacitor, to make a *lossy capacitor*. The area of the plates is 100 [cm^2]. The thickness of the slab is 1 [cm]. Assume that the relative permittivity of the Teflon is 2.1, and that the conductivity of the Teflon is 10^{-15} [S/m]. Determine the DC resistance of the structure. Then show the equivalent circuit of this lossy capacitor. Determine the time constant $\tau = RC$ for this circuit. Use this to answer the following question: If the capacitor is charged to a voltage of 1,000 [V] at $t = 0$, is the capacitor still dangerous to touch after letting it sit for one hour? Justify your answer by calculating the voltage across the capacitor after one hour.

6) A 60 [Hz] high-voltage power line carries a current of 1000 [A]. The power line is at a height of 50 [m] above the earth. What is the magnitude of the magnetic flux density $\underline{B}$ [T] at a point on the surface of the earth directly below the power line? Since the surface of the earth is made up largely of nonmagnetic materials ($\mu = \mu_0$), assume that the presence of the earth can be neglected in calculating the magnetic field at this frequency.

7) Two infinitely long wires run parallel to the *z* axis. One wire passes through the point $(x,y) = (h/2, 0)$ and carries a current of *I* [A] in the *z* direction. The other wire passes through the point $(x,y) = (-h/2\ ,0)$ and carries a current $-I$ [A] in the *z* direction. Calculate the magnetic field vector $\underline{H}(x,y)$ [A/m] at any point in space. Express the answer in rectangular coordinates.

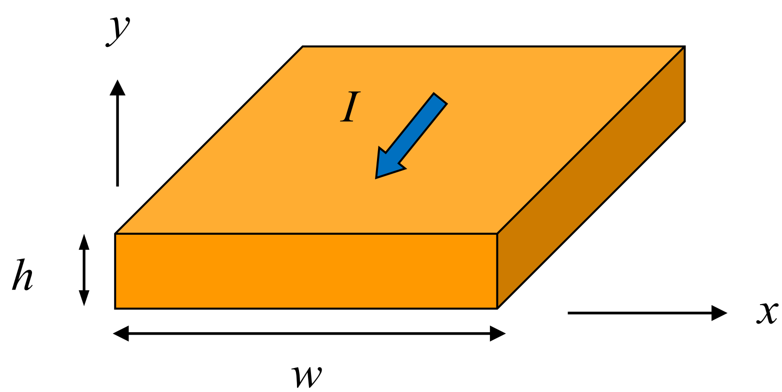
   Hint: Use superposition together with Ampere's law. Also, you might want to review how to convert a unit vector in cylindrical coordinates into rectangular coordinates. Try finding the magnetic field for a line current that is along the *z* axis first, and express your answer in rectangular coordinates. Then try to find out what the answer would be if the line current were moved to $(x,y) = (h/2, 0)$ or $(x,y) = (-h/2, 0)$.

   (Note: This is a model for a common type of transmission line known as the "twin-lead." Unlike the coaxial line, the twin-lead is not perfectly shielded, as you can tell from your result. That is, a magnetic field is produced by the transmission line away from the structure.)

8) A copper slab is carrying a total current of *I* [A] in the *z* direction, as shown below (the slab is infinite in the *z* direction). If the vertical distance *y* to the observation point is not too large in magnitude (compared to the width *w*), the magnetic field may be calculated assuming that the same current density extends to infinity in the $\pm\, x$ directions. (That is, the slab of metal can be assumed to be infinite in the *x* direction, with the *same current density* J_z inside as the original finite slab that has a total current *I*.) With this approximation, use Ampere's law to calculate the magnetic flux density vector $\underline{B}$ [T] for all values of *y*, including points below the

slab, inside the slab, and above the slab. Note that copper is nonmagnetic ($\mu = \mu_0$). The final answer should be in terms of the current I, the dimensions w and h, and the variable y.

Hint: The magnetic field is zero at $y = h/2$, by symmetry. Can you explain why? This may help you choose an Amperian path.



9) A rectangular loop of wire has a length $2a$ in the x direction and a width $2b$ in the y direction. The loop lies in the $z = 0$ plane and is centered at the origin. Assuming that the loop carries a current of I Amps counterclockwise, as viewed from the positive z axis, find the magnetic field vector $\underline{H}$ at the center of the loop.

Hint: Consider how the contributions from opposite sides of the rectangle compare, to save yourself some time. Also, consider what direction the total $\underline{H}$ vector be in at the center of the loop? This can also save you time, since you only need to calculate the component of the $\underline{H}$ vector from each edge of the loop that the need for the total $\underline{H}$ vector at the center of the loop.

10) Find the magnetic flux density vector $\underline{B}$ at the origin for the problem shown below, consisting of an infinite wire with a semi-circular bend in free space.