ECE 3318
Applied Electricity and Magnetism

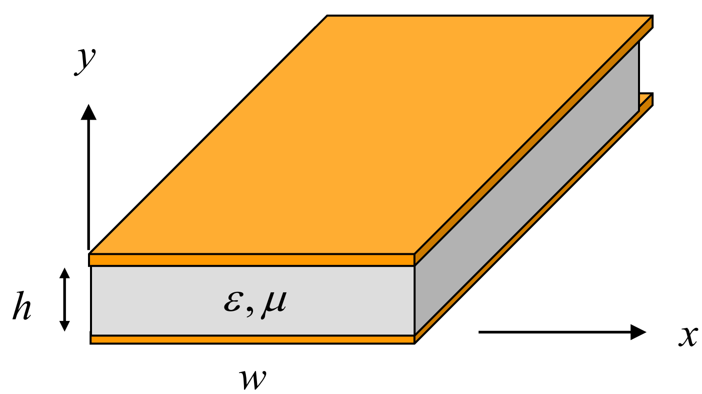
Spring 2023

Homework #12

Date Assigned: Thursday, April 20, 2023
Due Date: Thursday, April 27, 2023

1) Use the flux method to calculate the inductance per unit length L_l (i.e., inductance per meter in the z direction) of a transmission line consisting of a pair of parallel, planar conductors of width w and spacing h as shown below. Assume that the thickness of the plates is negligible. Assume $w >> h$, and therefore neglect fringing. Assume that the material between the plates has material properties ε and μ.

   Hint: Take the surface through which you calculate the flux to be a rectangle that lies in a plane that is parallel to the yz plane, with height h in the y direction and length l in the z direction (where l is arbitrary). Note that the magnetic field for this problem was calculated in the class notes, and you can use this.



2) For the parallel-plate transmission line of the previous problem, show that

$$L_l \, C_l = \mu\varepsilon \,,$$

   where L_l and C_l are the inductance and capacitance per unit lengths, respectively. (The value of C_l can be found directly from the parallel-plate capacitor formula.)

This property that you are proving for the parallel-plate transmission line actually holds for any transmission line, including a coaxial cable, twin lead, etc., as long as there is a single material present.

3) Repeat problem (1) using the stored-energy method to find the inductance per unit length.

4) Use the flux method to calculate the inductance per unit length of a transmission line consisting of a pair of parallel wires of radius a, with a center-to-center spacing h (see the figure below). Assume $h >> a$ so that the current on each wire is uniformly distributed on the surfaces of the wires.

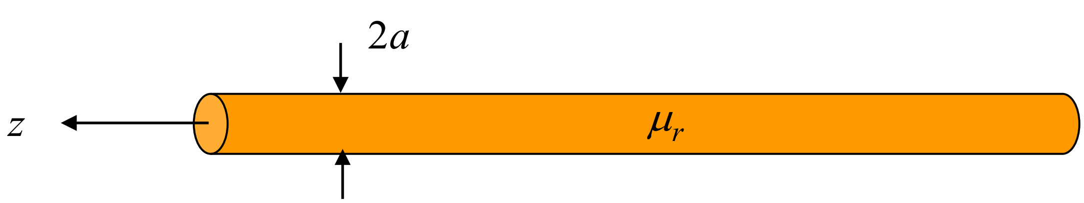
   Note: This problem would be very difficult using the stored energy method! Explain why.

5) Consider an infinitely long wire of radius a, as shown below. Assume that the wire carries a current I in the z direction, and this current in uniformly distributed throughout the cross section of the wire. The wire material has a relative permeability of μ_r.

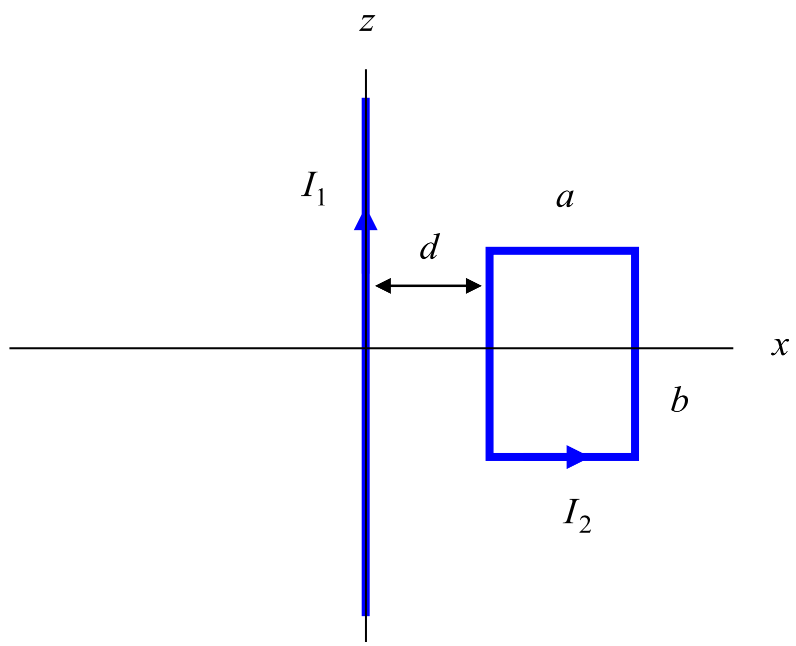
   Calculate the magnetic field vector $\underline{H}$ field inside the wire from Ampere's law. Then calculate the magnetic stored energy per unit length (per unit length in the z direction) inside the wire. Finally, calculate the "*internal inductance*" per unit length for the wire, which is based on the magnetic energy stored *inside* the wire. Show that the answer is given by

$$L_l^{\text{int}} = \frac{\mu}{8\pi} \; [\text{H/m}],$$

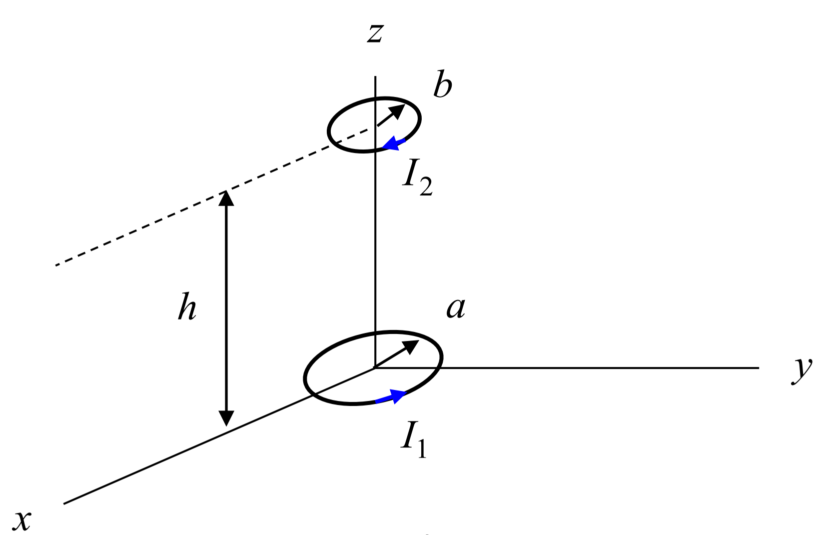
   where $\mu = \mu_0 \mu_r$.



6) An infinite wire oriented along the z axis carries a current I_1 [A] in the z direction. Next to the wire is a rectangular loop of current as shown below, carrying a current I_2 flowing counterclockwise. Derive a formula for the total force vector on the loop.



7) Find the mutual inductance M_{21} between two circular loops in free space. One current loop has a radius a and lies in the $z = 0$ plane, and is centered at the origin as shown below. The other loop has a radius b and lies parallel to the xy plane at a height $z = h$, with its center on the z axis. Assume that the radii a and b are both very small compared to the height h. This means that the magnetic field from one loop can be assumed to be approximately constant over the surface of the other loop. Note the reference directions of the defined currents on the loops. (Make sure that you get the sign correct for the mutual inductance!) You may take advantage of the formula derived in the class notes (using the Biot-Savart law) for the magnetic field on the z axis produced by a loop of current centered at the origin.



8) An electromagnet consists of an iron core with an air gap as shown below. The core is square shaped with a side length of L_1 and has a relative permeability of μ_r and a cross-sectional area of A. The air gap has a length of L_g, and $L_2 = (L_1 - L_g)/2$. The coil has N turns.

a) What direction will the magnetic field be in (up or down) inside the air gap?

b) Derive a formula for the magnitude of the magnetic flux density B and the magnitude of the magnetic field H inside the air gap.

c) Derive a formula for the inductance of the coil that is wound on this core.