ECE 3318

Applied Electricity and Magnetism

# Spring 2023

## Homework #2

**Date Assigned:** Thursday, Jan. 26, 2023

**Due Date:** Thursday, Feb. 2, 2023

1. A parallel-plate capacitor has a plate separation of *h*. The capacitor is connected to a battery of voltage *V*0 [V], with the anode connected to the top plate. The top plate at *x* = 0 is taken as a reference, where the voltage is zero. The bottom plate at *x* = *h* is grounded. Find a formula for the potential Φ(*x*) inside the capacitor. What is the potential of the bottom plate?
2. Derive the identity .
3. Derive the identity .
4. An electric field is described by  [V/m].

Find the voltage drop *VAB* by integrating along the circular arc, as shown below.

(This is similar to one of the examples in the Appendix of Notes 6 -- you might want to look at this example.)

A

*x*

C

3 [m]

*B*

*y*

*A*

*y*

*C*

1. Repeat the previous problem by integrating first from point *A* to the origin, and then from the origin to point *B*. Do you get the same result as above?
2. An electric field is described in cylindrical coordinates as

  [V/m].

Find the voltage drop *VAB* where *A* is the point (1, 0, 0) and *B* is the point (2, *π* / 2, 1). (In the description of these points, the cylindrical coordinate notation (*ρ*, *φ*, *z*) is used, where dimensions are in [m] and angles are in radians.) Assume that the voltage drop is path independent (i.e., this is a valid electrostatic field), so you can use any path that you wish. (Hint: Consider choosing part of the path to be an arc of a circle, part of the path to be a straight horizontal line, and part of the path to be a straight vertical line.)

Question for your consideration: In this problem, do you have to actually choose a path, or does the calculus let you evaluate the integral without doing so? Even if you do not have to choose a path, go ahead and use the one suggested in the hint.

1. A hemispherical surface defined by a radius *r* = 5 [m] and *z* > 0 has a surface charge density of  [C/m2]. Find the total charge on the surface.
2. A cylindrical volume is defined by  [m],  [m], and . Inside this region there is a volume charge density

 [C/m3].

Find the total charge inside the region.

1. A current density vector is given as

.

Find the total current crossing the surface of a hemisphere that is defined by *r* = 2 [m] and *z* > 0. The current is defined to be the current crossing the surface in the upward (outward) sense.

1. A surface current density *Js* is one that flows on a surface, corresponding to a surface charge density in motion on the surface. The units are [A/m]. The 2-D form of the charge-velocity equation states that the surface current density vector is given by

.

The current crossing a contour *C* on the surface is given by

.

Consider a uniform surface charge density *ρs*0 [C/m2] in the *z* = 0 plane moving in the *x* direction at a constant velocity (speed) of *v* [m/s].

a) Determine the surface current density vector.

b) Find the current in amps that crosses, from left to right, a straight line of length two meters, lying in the *z* = 0 plane and making a 45o angle with respect to the *x* axis (i.e., lying along a line *φ* = 45o).

**Extra Problems (not to be turned in – for extra practice only):**

Shen and Kong: None

Hayt and Buck, 7th Edition: 1.18 – 1.30, 5.2, 5.3, 5.5(a), 5.6(a,b)