ECE 3318
Applied Electricity and Magnetism
Spring 2023

Homework #2

Date Assigned: Thursday, Jan. 26, 2023
Due Date: Thursday, Feb. 2, 2023

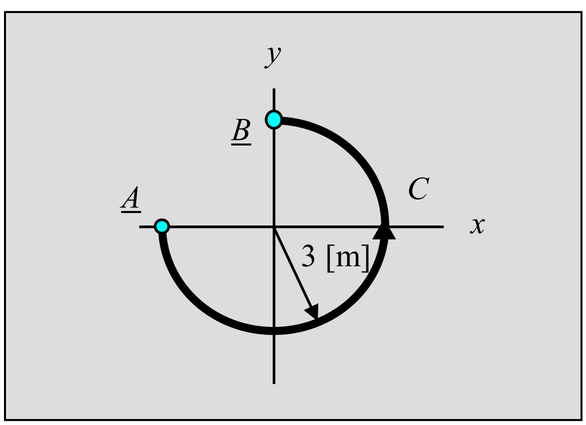
1) A parallel-plate capacitor has a plate separation of h. The capacitor is connected to a battery of voltage V_0 [V], with the anode connected to the top plate. The top plate at $x = 0$ is taken as a reference, where the voltage is zero. The bottom plate at $x = h$ is grounded. Find a formula for the potential $\Phi(x)$ inside the capacitor. What is the potential of the bottom plate?

2) Derive the identity $\underline{\hat{x}} = \underline{\hat{\rho}}\cos\phi + \underline{\hat{\phi}}(-\sin\phi)$.

3) Derive the identity $\underline{\hat{\theta}} = \underline{\hat{x}}\cos\theta\cos\phi + \underline{\hat{y}}\cos\theta\sin\phi + \underline{\hat{z}}(-\sin\theta)$.

4) An electric field is described by $\underline{E} = \underline{\hat{y}}(y) + \underline{\hat{x}}(x)$ [V/m].
   Find the voltage drop V_{AB} by integrating along the circular arc, as shown below.

   (This is similar to one of the examples in the Appendix of Notes 6 -- you might want to look at this example.)



5) Repeat the previous problem by integrating first from point $\underline{A}$ to the origin, and then from the origin to point $\underline{B}$. Do you get the same result as above?

6) An electric field is described in cylindrical coordinates as

$$\underline{E} = \hat{\underline{\rho}}\left(\frac{1}{\rho}\right) \text{ [V/m]}.$$

Find the voltage drop V_{AB} where $\underline{A}$ is the point (1, 0, 0) and $\underline{B}$ is the point (2, $\pi/2$, 1). (In the description of these points, the cylindrical coordinate notation (ρ, ϕ, z) is used, where dimensions are in [m] and angles are in radians.) Assume that the voltage drop is path independent (i.e., this is a valid electrostatic field), so you can use any path that you wish. (Hint: Consider choosing part of the path to be an arc of a circle, part of the path to be a straight horizontal line, and part of the path to be a straight vertical line.)

Question for your consideration: In this problem, do you have to actually choose a path, or does the calculus let you evaluate the integral without doing so? Even if you do not have to choose a path, go ahead and use the one suggested in the hint.

7) A hemispherical surface defined by a radius $r = 5$ [m] and $z > 0$ has a surface charge density of $\rho_s = 2\sin\theta\cos^2\phi$ [C/m^2]. Find the total charge on the surface.

8) A cylindrical volume is defined by $\rho < \rho_0$ [m], $0 < z < h$ [m], and $0 < \phi < 2\pi$. Inside this region there is a volume charge density

$$\rho_v = e^{-\rho^2}\cos^2\phi\,\sin\left(\frac{\pi z}{h}\right) \text{ [C/m}^3\text{]}.$$

Find the total charge inside the region.

9) A current density vector is given as

$$\underline{J} = \hat{\underline{z}}(z) \text{ [A/m}^2\text{]}.$$

Find the total current crossing the surface of a hemisphere that is defined by $r = 2$ [m] and $z > 0$. The current is defined to be the current crossing the surface in the upward (outward) sense.

10) A surface current density $\underline{J}_s$ is one that flows on a <u>surface</u>, corresponding to a surface charge density in motion on the surface. The units are [A/m]. The 2-D form of the charge-velocity equation states that the surface current density vector is given by

$$\underline{J}_s = \rho_s \underline{v}\ .$$

The current crossing a contour C on the surface is given by

$$I = \int_C \underline{J}_s \cdot \hat{\underline{n}}\, dl\ .$$

Consider a uniform surface charge density ρ_{s0} [C/m^2] in the $z = 0$ plane moving in the x direction at a constant velocity (speed) of v [m/s].

a) Determine the surface current density vector.

b) Find the current in amps that crosses, from left to right, a straight line of length two meters, lying in the z = 0 plane and making a 45° angle with respect to the x axis (i.e., lying along a line $\phi = 45°$).

Extra Problems (not to be turned in – for extra practice only):

Shen and Kong: None
Hayt and Buck, 7th Edition: 1.18 – 1.30, 5.2, 5.3, 5.5(a), 5.6(a,b)