

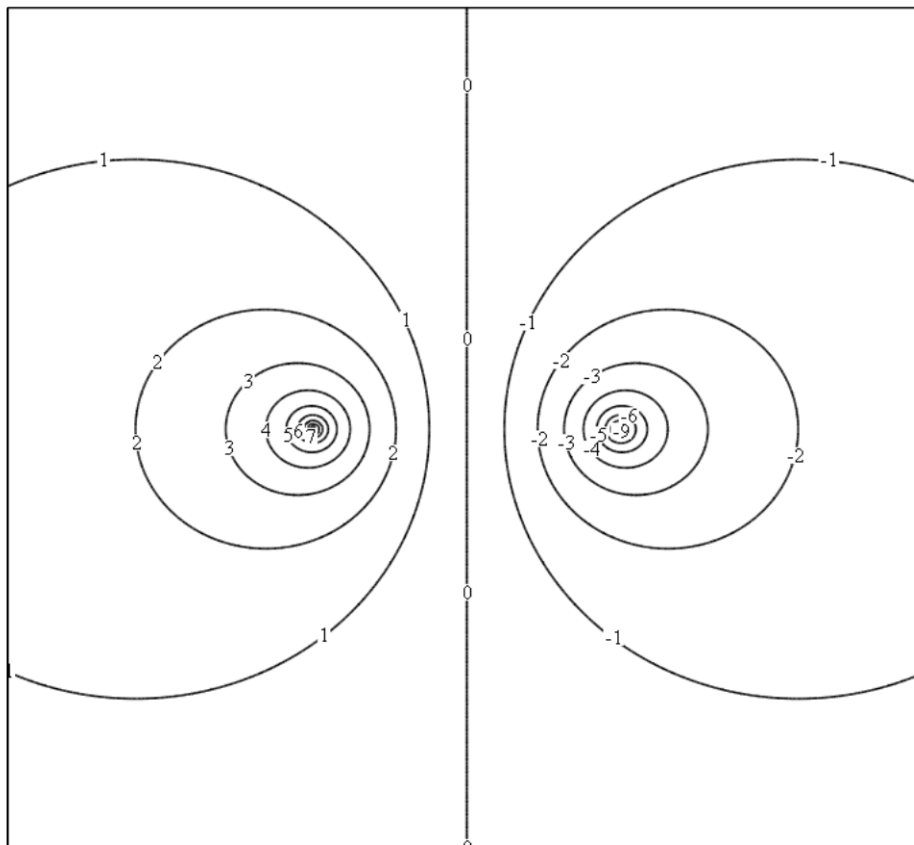
**ECE 3318**  
**Applied Electricity and Magnetism**  
**Spring 2023**  
**Homework #4**

**Date Assigned:** Thursday, Feb. 16, 2022

**Due Date:** Thursday, Feb. 23, 2022

Do Problems 1–4, 6–11.

- 1) Make a flux plot for the case shown below, which has two infinite uniform line charges parallel to the  $z$  axis ( $\rho_{l0}$  [C/m] on the left and  $-\rho_{l0}$  [C/m] on the right). The equipotential lines are already there (labeled with voltage values (in Volts) on the curves), so you only need to add the flux lines (the lines with the arrows). As you add the flux lines, keep the aspect ratio of the curvilinear squares to be  $L / W = 1$ . You may cut and paste the figure below onto your own paper in order to make your plot if you wish, or you may do your work on this page and attach this page with the rest of your homework.



- 2) On a piece of graph paper, make a flux plot for an infinite uniform line charge along the  $z$  axis. (The  $z$  axis should correspond to the origin on your graph, which shows the  $xy$  plane.) The flux plot should show both the flux lines and the equipotential contours. Assume that there are 28 flux lines that emanate from the line charge. Draw the outermost equipotential contour at a radius of 5 [cm] from the origin. Your plot should be drawn to full-scale, so that 1 [cm] on your plot corresponds to an actual distance of 1 [cm]. Draw the plot so that the aspect ratio  $L / W$  of the curvilinear squares is unity. Draw at least seven equipotential contours.
- 3) Assume that in Prob. 2 the reference point is taken as 5 [cm] from the origin, and that the potential is zero at the reference point. Also assume that the electric field has a magnitude of 1.0 [V/m] at a distance of 5.0 cm from the line charge.

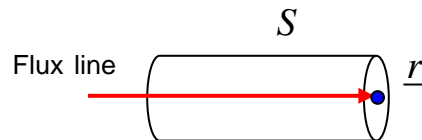
- a) Starting with the formula for the electric field of a line charge, show that the electric field in this example must be

$$\underline{E} = \hat{\underline{\rho}} \left( \frac{0.05}{\rho} \right) [\text{V/m}],$$

where  $\rho$  is in meters.

- b) Using the formula in part (a), find a formula for the voltage drop  $V_{AB}$  between two points  $\underline{A}$  and  $\underline{B}$  that are located at  $\rho = \rho_A$  and  $\rho = \rho_B$ .
- c) Using the result from part (b), determine what the value of  $\Delta V$  is for your flux plot (the voltage drop between adjacent equipotential contours). To do this, calculate  $\Delta V$  between two adjacent equipotential lines in your plot using the values of  $\rho$  that you read off from your plot, together with your formula from part (b). Pick at least a couple of different adjacent equipotential contours and verify that you get the same result for  $\Delta V$  from each of them (or something pretty close to the same result). Report the average that you have obtained as your final answer for  $\Delta V$ .
- d) Using your value of  $\Delta V$  from part (c), label the voltage for each of the equipotential lines on your plot (with the outer one being at zero volts, since this is where the reference point is). Label at least four of the equipotential contours, starting from the outermost one.
- 4) An engineer makes an accurate flux plot for a pair of infinite uniform line charges (infinite in the  $z$  direction). One line charge has a fixed line charge density of  $\rho_{l0} = 1$  [C/m] and is located on the  $x$  axis at  $x = -h$ . The other line charge has  $\rho_{l0} = -0.5$  [C/m] and is located on the  $x$  axis at  $x = h$ . The axes of both line charges are parallel to the  $z$  axis. Assume that the engineer chooses to have 100 flux lines emanating from the left line charge.
- a) How many flux lines enter the second line charge?

- b) How many flux lines end up going to infinity?
- c) Very close to the left line charge, what is the angle (in degrees) between adjacent flux lines?
- d) Very far away from the  $z$  axis, what is the angle (in degrees) between adjacent flux lines?
- e) How many flux lines from the left line charge end up crossing the  $y$  axis, going from left to right, if this line charge were present by itself?
- f) How many flux lines cross (going from left to right) the  $y$  axis in the flux plot when both line charges are present? (Hint: Use superposition, treating each line source separately. The total flux crossing the  $y$  axis is the sum of the fluxes that cross the line from each line source, treating each line source as if it were alone.)
- 5) Consider a flux line in free space that suddenly ends at a point  $\underline{r}$  in space. Why does this violate Gauss's law? Give a convincing explanation by considering Gauss's law applied to a small cylindrically-shaped Gaussian surface  $S$  as shown below. (Hint: If a flux line suddenly ends, the electric field must be zero there or else we would be able to continue the flux line.)



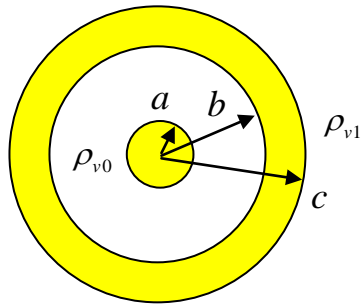
(Note: A flux line cannot suddenly begin at a point in free space either, by similar reasoning.)

- 6) The following spherically-symmetric inhomogeneous (non-uniform) charge distribution is set up in air (the charge density exists everywhere in space,  $r \geq 0$ ):

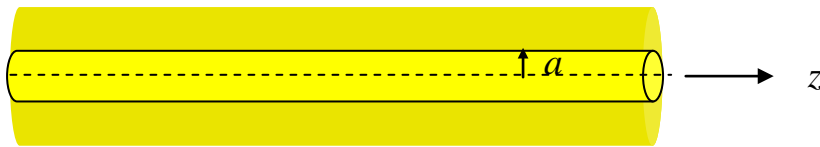
$$\rho_v = e^{-r} \text{ [C/m}^3\text{]}.$$

Use Gauss's law to find the electric field vector at any point in space.

- 7) A sphere of uniform volume charge density  $\rho_{v0}$  with radius  $a$  is surrounded by a spherical shell of uniform volume charge density  $\rho_{v1}$  that has an inner radius  $b$  and an outer radius  $c$ . Determine the electric field vector in all regions:  $r < a$ ,  $a < r < b$ ,  $b < r < c$ , and  $r > c$ .



- 8) A infinitely long electron beam has a uniform volume charge density of  $\rho_{v0}$  for  $\rho < a$ . For  $a < \rho < \infty$  the charge density is non-uniform and is given by  $\rho_v = \rho_{v0} e^{-(\rho-a)}$ . Find the electric field vector in the region  $\rho < a$  and the region  $\rho > a$ .

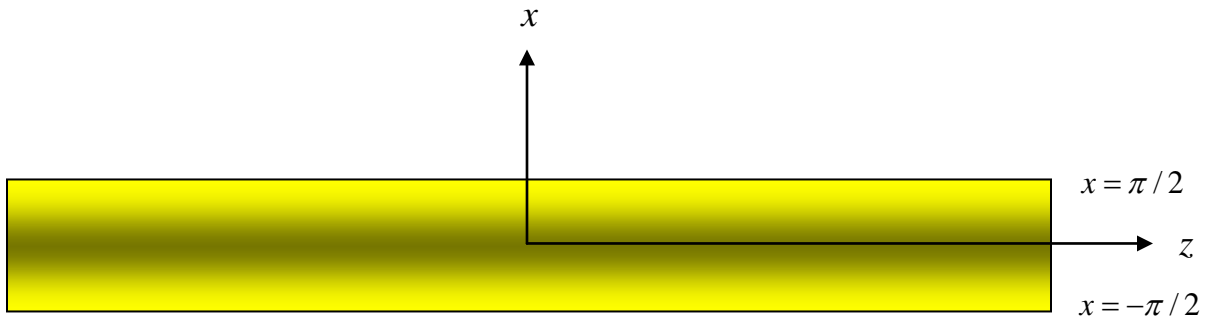


- 9) A planar slab of non-uniform charge shown below has a volume charge density

$$\rho_v = \cos x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Find the electric field vector in all three regions (above the slab, inside the slab, below the slab), using Gauss's law.

(Note that there are two good choices for what the Gaussian surface could look like. Can you think of what they both are? Use whichever one that you wish. Try both for extra practice!)



- 10) Find the equivalent surface charge density  $\rho_{s0}^{eq}$  that models the slab of nonuniform volume charge density in Prob. 9. Do this by equating the total charge in a 1 [m<sup>2</sup>] area in the  $yz$  plane in both problems (the actual slab of volume charge density and the equivalent sheet of surface charge density). Then assume that you are outside the slab in Prob. 9. Find the electric field from the equivalent sheet of surface charge density  $\rho_{s0}^{eq}$  when placed at  $x = 0$ , and show that you get the same results that you obtained using Gauss's law in Prob. 9.

(Note that the field of an infinite sheet of uniform surface charge density is given in the class notes (Notes 11), so you do not have to re-derive this.)

- 11) Repeat Prob. 9, assuming that the non-uniform charge density varies as

$$\rho_v = \sin x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Note that this charge density is an odd function of  $x$  instead of an even one (as in Prob. 9), so this requires a different choice of Gaussian surface.

(Hint: Choose one side of the Gaussian surface (either the top or bottom) to be at a point where the electric field is zero. This will be at any point *outside* the slab – can you justify this? Think about what the equivalent surface charge density for this problem would be.)