ECE 3318
Applied Electricity and Magnetism

Spring 2023

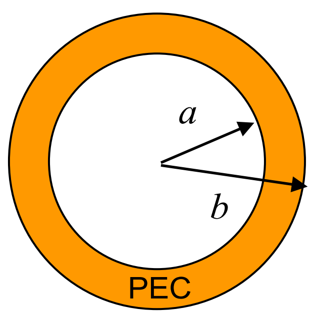
Homework #5

Date Assigned: Tuesday, Feb. 23, 2032
Due Date: Friday, March 2, 2023

Do Probs. 1–5, 7, 9–10

1) A perfectly conducting spherical shell has an inner radius a and an outer radius b as shown below. The region $r < a$ is hollow (filled with air). The entire shell has a net charge of Q [C] on it because it has been stuck by lightning.

   a) Determine the electric field vector in all three regions: $r < a$, $a < r < b$, and $r > b$.
   b) Determine the surface charge densities $\rho_{s0}{}^{a}$ and $\rho_{s0}{}^{b}$ on the two metal surfaces.
   c) Explain how this problem illustrates the Faraday cage effect.
   d) What are the two surface charge densities $\rho_{s0}{}^{a}$ and $\rho_{s0}{}^{b}$ after the conductor is grounded?
   e) What is the electric field vector in all three regions after the shell is grounded?



Next, use this as a model problem to explain what happens what a car gets struck by lightning. Answer the following questions (and give a brief explanation):

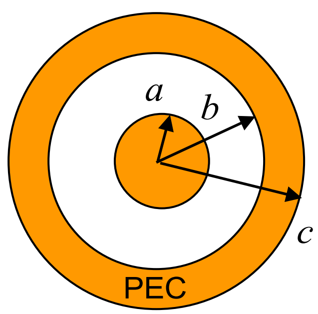
- Are you safe inside a car if it gets struck by lightning?
- Are you safe stepping out of the car immediately after it gets struck by lightning?
- Are you safe stepping out of your car if someone grounds your car after the lightning strike?
- Are you safe stepping out of your car if wait long enough for the charge on the outer surface of the car to eventually bleed off?

2) A coaxial cable has a conducting wire of radius of a surrounded by a cylindrical conducting shell (the outer "shield" of the coax) having an inner radius b and outer radius c. (Both are modeled as perfect electric conductors, or PEC). Between the wire and the shield there is air. A cross-sectional view is shown below (the problem is infinite in the z direction, coming out of the page). Assume that the inner conductor has a net line charge density of ρ_{l0} [C/m] and the outer shield has a net charge density of $-\rho_{l0}$ [C/m].

a) Determine the electric field vector in all four regions of the coax: $\rho < a$, $a < \rho < b$, $b < \rho < c$, and $\rho > c$.

b) Determine the net line charge densities ρ_l on all three surfaces ($\rho = a$, $\rho = b$, $\rho = c$). That is, determine ρ_l^a, ρ_l^b, ρ_l^c.

c) Determine the surface charge densities ρ_s on all three surfaces ($\rho = a$, $\rho = b$, $\rho = c$). (Hint: For each surface, consider how the surface charge density is related to the net line charge density on that surface.)



3) As a continuation of the previous problem, assume that the cable has been struck by lightning so that the shield takes on an extra charge density of ρ_{l0}^E [C/m]. Determine the electric field vector in all four regions: $\rho < a$, $a < \rho < b$, $b < \rho < c$, and $\rho > c$. Determine the surface charge densities on all three surfaces ($\rho = a$, $\rho = b$, $\rho = c$).

4) As a continuation of the previous problem, assume that the outer shield of the coax is now grounded. Determine the electric field vector in all four regions: $\rho < a$, $a < \rho < b$, $b < \rho < c$, and $\rho > c$. Determine the surface charge densities on all three surfaces ($\rho = a$, $\rho = b$, $\rho = c$). How much charge density (charge per meter length in the z direction) ρ_{l0}^G has flowed down to ground after the shield is grounded?

5) Explain in words the answers to the following questions:

(a) Why is a grounded coaxial cable safer than an ungrounded one in an explosive environment?

(b) Why is a grounded coaxial cable less likely to cause static interference with other pieces of equipment than an ungrounded one?

(c) Why is the high-frequency signal inside of a coaxial cable not disturbed when the cable runs near another object (such as a water pipe), as opposed to a transmission line such as twin lead or twisted-pair line? (Twin lead is a transmission line that consists of two wires running parallel to each other. Twisted line is a variation of that, in which the two wires are twisted around each other along the length of the line, in an effort to minimize coupling with other objects, and radiation loss.) Is this property true whether the cable is grounded or not? (Hint: High-frequency waves do not penetrate very far inside of good conductors, because of the "skin-depth" property.)

6) Consider an isolated block of practical metal (practical means a conductive body with a finite conductivity). The fact that it is isolated means that there is no electrical connection to it from anything on the outside. In electrostatics, it is still true that all charge must be on the outside of the body only, even though the conductivity is not infinity. Give a proof of this by assuming that there is a region inside the body where the charge density is not zero, and reach a contraction by applying Gauss's law (in a similar fashion as was done in the class notes for a perfect conductor). You may assume that in the static limit there is no current flowing at any point inside the isolated body. Also, assume that Ohm's law applies.

7) Calculate the divergence of the following vector functions:

$$\underline{A} = \hat{\underline{x}}\left(3x^2 yz\right) + \hat{\underline{y}}\left(4z^3 x\right) + \hat{\underline{z}}\left(2xyz\right)$$
$$\underline{A} = \hat{\underline{\rho}}\left(\rho\cos\phi\right) + \hat{\underline{\phi}}\left(z+\rho^2\right) + \underline{z}\left(ze^{-\rho}\right)$$
$$\underline{A} = \hat{\underline{r}}\left(r\sin\theta\cos\phi\right) + \hat{\underline{\theta}}\left(r^2+\sin\phi+\sin\theta\right) + \hat{\underline{\phi}}\left(\cos\theta\right)$$

8) A vector field

$$\underline{A} = \hat{\underline{\rho}}\,\rho^3$$

exists in the region between two concentric cylindrical surfaces defined by $\rho = 1$ and $\rho = 3$, with both cylinders extending from $z = 0$ to $z = 5$. Verify that the divergence theorem is satisfied for this region by evaluating the following:

a) $\oint_S \underline{A}\cdot\hat{\underline{n}}\,dS$

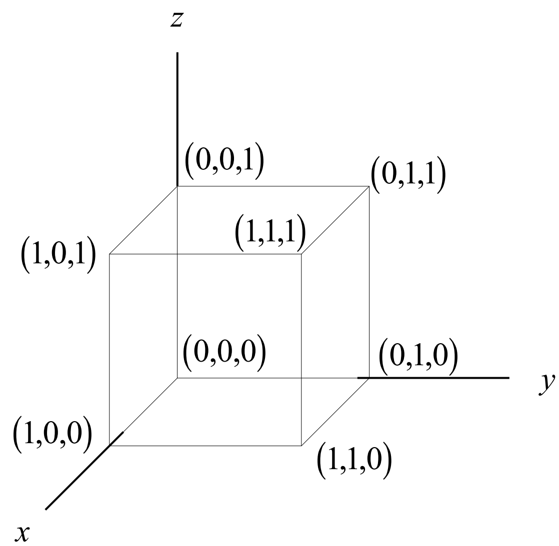
b) $\int_V \nabla\cdot\underline{A}\,dV$.

9) Assuming an electric field $\underline{E}$ that is the same as the vector function $\underline{A}$ in the above problem, use the point form (differential form) of Gauss's law to find the charge density at $\rho = 2$ [m].

10) You are given a current density vector field

$$\underline{J} = \left(x^4 z\right)\underline{\hat{x}} + \left(3\,yx\right)\underline{\hat{y}} + \left(x^2 y\right)\underline{\hat{z}} \text{ [A/m}^2\text{]}$$

and the unit cube shown below.



Evaluate the integral

$$I_{out} = \int_S \underline{J} \cdot \underline{\hat{n}}\, dS \ ,$$

where $\underline{\hat{n}}$ is the outward-pointing unit normal vector. I_{out} is the total current flowing out of the cube. Use the divergence theorem to perform the calculation (do not use a surface integration).

Note that this is the same problem that was encountered in HW 1, where you found the current I_{out} by using a surface integration. You may wish to compare your answer with what you found there.

Extra Problems (not to be turned in – for extra practice only):

Shen and Kong: 9.13, 9.14, 9.15, 9.16,
Hayt and Buck, 7th Edition: 3.1, 3.6, 3.7, 3.9, 3.10, 3.11, 3.13, 3.15, 3.16