ECE 3318
Applied Electricity and Magnetism

Spring 2023

Homework #6

Date Assigned: Thursday, March 2, 2023
Due Date: Thursday, March 9, 2023

Do Problems 1–4, 6–8.

1) A spherical shell of uniform surface charge density ρ_{s0} has a radius a. Find the potential at the center of the sphere, assuming that the potential at infinity is zero. Do this by integrating the electric field.

2) Repeat the previous problem by using the potential-charge formula (i.e., find the potential by integrating over the charge without finding the electric field first).

3) A line of uniform charge density ρ_{l0} extends from $z = -L/2$ to $z = L/2$. Obtain an expression for the potential at the point $(0,0,z)$ (where $z > L/2$) using the potential-charge formula (i.e., find the potential by integrating over the charge without finding the electric field first). Assume that the reference point is at infinity, and that the potential at the reference point is zero.

4) An electrostatic "dipole" is formed by placing a positive charge of q [C] on the z axis at $z = d/2$ and a negative charge of $-q$ [C] on the z axis at $z = -d/2$. Determine the potential $\Phi(x,y,z)$ (with respect to infinity) at a point (x,y,z). Note: Determining the potential at a point "with respect to infinity" is equivalent to determining the potential at the point when the reference point is at infinity and the potential there is zero.

5) As a continuation of the previous problem, assume that $d << r$. This gives us an approximation to an "ideal point dipole". By using mathematical approximations, show that the potential can be written in spherical coordinates as

$$\Phi \approx \frac{qd}{4\pi\varepsilon_0 r^2}\cos\theta \,.$$

Note that the potential from the dipole falls off with distance faster than that from a single point charge.

Hint: Use the following approximations for $d << r$ (which you should derive), that

$$R_1 = \sqrt{x^2 + y^2 + (z - d/2)^2} \approx r\left[1 - \frac{1}{2}\frac{d}{r}\cos\theta\right],$$

and

$$R_2 = \sqrt{x^2 + y^2 + (z + d/2)^2} \approx r\left[1 + \frac{1}{2}\frac{d}{r}\cos\theta\right].$$

These approximations come from using the Taylor series expansion

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots \ .$$

Also, use the first two terms of a Taylor series expansion to approximate $1/(1-x)$ when x is small. This Taylor series is

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots .$$

(This is also called the geometric series.)

6) Find the electric potential function $\Phi(x)$ at a point that is located at $x > 0$ from an infinite uniform planar sheet of charge density ρ_{s0} located at $x = 0$, when the reference point is located at a distance $x_0 > 0$ from the sheet and the potential at the reference point is zero. Do this by integrating the electric field. Describe the surfaces on which the potential is constant.

   Then answer the following questions:

   - Is it possible to locate the reference point at $x_0 = \infty$? If not, why not?
   - Is it possible to locate the reference point at $x_0 = 0$? If not, why not?
   - Can the potential-integral formula be used for this problem? If not, why not?

7) A circular disk of uniform surface charge ρ_{s0} and radius a lies in the x-y plane with center at $(0,0,0)$. Find the potential $\Phi(z)$ along the z axis, when the potential is V_0 is at the center of the disk. Use the potential-charge formula (i.e., find the potential by integrating over the charge without finding the electric field first). (Hint: Remember, you can always add a constant to a valid potential solution to change the reference point.)

8) Use the potential-charge formula (i.e., the formula that lets you calculate the potential directly by integrating over the charge without finding the electric field first) to find the electric potential $\Phi(\rho)$ at a point that is located in the x-y plane at a distance ρ from the z axis, due to a line charge of uniform charge density ρ_{l0} and length h. The line charge is coincident with the z axis and extends from $z = -h/2$ to $z = h/2$. Assume that the potential is zero in the x-y plane at a distance of b [m] from the origin. (Hint: Remember, you can always add a constant to a valid potential solution to change the reference point.)

The following integration formula might be helpful:

$$\int \frac{dx}{\left(x^2+a^2\right)^{1/2}} = \ln\left(x+\sqrt{x^2+a^2}\right).$$

9) Explain why the potential-charge formula cannot be used to directly determine the potential from an infinite uniform line charge ρ_{l0} that is on the z axis. Then use your solution to the previous problem to show how in the limit $h \to \infty$ the potential becomes

$$\Phi(\rho) = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0}\ln\left(\frac{b}{\rho}\right),$$

which agrees with the result derived in class using an integration of the electric field.

Hint: In your expression for the potential of the finite line charge, factor out a term h from the numerator and denominator of each expression that is inside of the logarithm. Then use the following approximation (accurate for $x << 1$) to approximate all of the square root terms:

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x\,.$$