ECE 3318
Applied Electricity and Magnetism

Spring 2023

Homework #8

Date assigned: Thursday, March 23, 2023
Date due: Thursday, March 30, 2022

Due Probs. 1-6, 8-10

1) The height of a mountain is for $z > 0$ is described by

$$z = 10^3 \left(1 - a^2 x^2 - b^2 y^2\right),$$

where $a = 0.001$ and $b = 0.002$, and all dimensions are in meters. Note that the peak of the mountain is at $x = y = 0$. Assume that a mountain climber is on the side of the mountain at $x = 500$, $y = 200$.

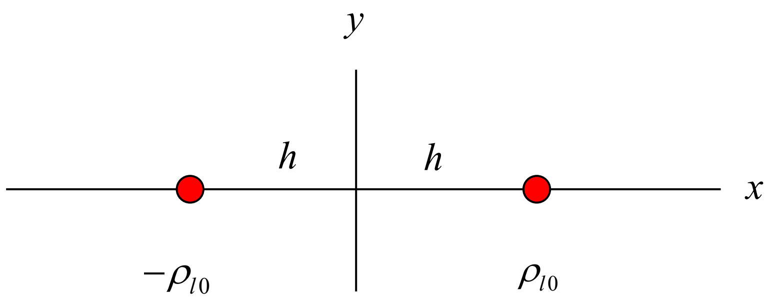
Find a unit vector parallel to the xy plane (i.e., a horizontal unit vector) that points in the direction that the climber located at (x, y, z) should head towards, in order to ascend the mountain as rapidly as possible from her present location. (This corresponds physically to the compass heading that she must head towards.) Also, find the horizontal unit vector that points from where the climber is towards the top of the mountain. Are the two unit vectors the same?

2) As continuation of the previous problem, let's assume that the mountain climber is heading from her position in the direction of the compass heading that makes her go uphill the fastest. How much altitude (in meters) does she gain with every meter that she goes horizontally?

3) As a continuation of the previous problem, what is the slope of the mountain at the point where the mountain climber is located? That is, what is the angle (in degrees), measured from horizontal, on the path that the mountain climber would be climbing on, assuming that she is heading from her position in the direction of the compass heading that makes her go uphill the fastest.

4) Consider a circular disk of uniform surface charge density ρ_{s0} of radius a, lying in the $z = 0$ plane with the center of the disk at the origin. Calculate the potential at any point on the z axis by using the potential-charge formula (i.e., find the potential by integrating over the charge without finding the electric field first). Then use the gradient to find the electric field on the z axis, for both $z < 0$ and $z > 0$.

5) Two infinite line charges (running in the z direction) are located at $x = \pm h$ as shown below. Calculate the potential $\Phi(x,y)$ at any point (x,y), assuming zero volts on the z axis. When calculating the potential, you may start with the potential of a single infinite line charge and use superposition. (The potential of a single infinite line charge was derived in class in Notes 14.)



6) Use the gradient to calculate the electric field vector at any point (x, y) in space, for the problem of the two infinite line charges in the previous problem.

7) Consider a finite-length uniform line charge density ρ_{l0} running along the z axis between z = - $h/2$ and $z = h/2$. Using the potential-charge formula (i.e., the formula that lets you calculate the potential directly by integrating over the charge without finding the electric field first), you can find the electric potential $\Phi(\rho)$ at a point that is located in the x-y plane at a distance ρ from the z axis, assuming that the potential is zero at infinity. This was done in Prob. 8 of HW 6. Starting with this answer for the potential, use the gradient to find the electric field at any point in the $z = 0$ plane at a distance ρ from the z axis. The answer for the potential from HW 6 is

$$\Phi(\rho) = \frac{\rho_{l0}}{4\pi\varepsilon_0}\left[\ln\left(\frac{\sqrt{h^2+4\rho^2}+h}{\sqrt{h^2+4\rho^2}-h}\right) - \ln\left(\frac{\sqrt{h^2+4b^2}+h}{\sqrt{h^2+4b^2}-h}\right)\right].$$

8) Consider a sphere of uniform charge density ρ_{v0} [C/m^3] having a radius a, centered at the origin. Perform the following steps:

a) Calculate the electric field both inside and outside the sphere using Gauss' law.

b) Calculate the potential Φ inside and outside the sphere by integrating the electric field to get the potential, assuming zero volts at infinity.

c) Find the electric field inside and outside the sphere by taking the gradient of the potential. You should be able to verify that your answer is the same as the electric field that you started with from Gauss's law.

9) Verify that the Laplacian of the following potential from a point charge at the origin is zero (that is, the potential satisfies Laplace's equation), as long as we are at any point in space except the origin:

$$\Phi = \frac{q}{4\pi\varepsilon_0 r} \ .$$

10) Consider a sphere of uniform volume charge density ρ_{v0} [C/m^3] having a radius a, centered at the origin. Use the potential Φ inside and outside the sphere found from Prob. (8). Take the Laplacian of this potential, and verify that the Poisson equation is satisfied. That is,

$$\nabla^2\Phi = -\frac{\rho_v}{\varepsilon_0} = \begin{cases} -\dfrac{\rho_{v0}}{\varepsilon_0}, & r < a \\ 0, & r > a. \end{cases}$$