ECE 3318
Applied Electricity and Magnetism

Spring 2023

Homework #9

Date assigned: Thursday, March 30, 2023
Date due: Thursday, April 6, 2023

1) An infinite uniform line charge density ρ_{l0} [C/m] is along the z axis. Surrounding this line charge is a cylindrical shell of dielectric, having an inner radius a and an outer radius b. The relative permittivity of the dielectric is ε_r. Find the electric field vector in all three regions.

2) Consider two infinitely long perfectly conducting wires in air, each of radius a, parallel to each other. The distance between the axes of the wires is h. Assume that $h >> a$, so that the charge density on each wire may be assumed to be uniform. Assume that ρ_{l0} [C/m] is the charge per unit length on one wire and $-\rho_{l0}$ [C/m] is on the other wire. (We don't need it, but the surface charge density on each metal wire would be the charge per unit length divided by the circumference, assuming uniform surface charge density.) Derive a formula for the voltage drop (or potential difference) V_{AB} between the two wires by integrating the electric field between the surfaces of the two wires. The conductor A is the one with the positive charge on it. (Hint: Choose a path of integration that is a straight line, corresponding to the shortest distance between the two cylindrical surfaces. Although the coordinate system should not matter, you can assume that the wires are parallel to the z axis, with the positively charged wire centered $x = h/2$ and the negatively-charged wire centered at $x = -h/2$.)

3) Assume that an electrostatic air ionizer (air purifier) ionizes the air by using two wires as in the previous problem, with $a = 0.1$ [mm] and $h = 5$ [mm]. What is the voltage that must be placed across the wires in order to ionize the air at the surface of the wires? Assume that E_c is 3.0 [MV/m]. (Note: After the particles in the air are ionized, they can be collected by an electrode. This is how the filter removes particles from the air. For more information, please see https://en.wikipedia.org/wiki/Air_ioniser.)

4) There are two metal towers. Tower 1 is 50 meters tall and the base of the tower is located at (0, 0, 0) meters in rectangular coordinates (the surface of the earth is at $z = 0$). Tower 2 is 100 meters tall and the base of the tower is located at (200, 0, 0) meters in rectangular coordinates. In each part below, a stepped leader from a thundercloud is descending vertically, traveling approximately vertically downward along a vertical line located at $x = x_0$, $y = 0$. Which tower will get struck by lightning? (Your answer could be Tower 1, Tower 2, both of them, or neither of them.)

a) $x_0 = 0$ [m]
b) $x_0 = 30$ [m]
c) $x_0 = 50$ [m]
d) $x_0 = 100$ [m]
e) $x_0 = 175$ [m]
f) $x_0 = 250$ [m]
g) $x_0 = 275$ [m]

Assume that streamers emanating from the towers can reach a maximum length of 50 meters.

5) Assume that a typical lighting strike delivers -25 [C] to the earth, and the average voltage drop between the cloud and ground (voltage of cloud minus voltage of ground) is -75 [MV] during the time the charge is delivered. Assume that a lightning strike hits the earth from the cloud every 10 [s], and that the thunderstorm lasts one hour. Assume that somehow all of the energy in all of the lightning strikes could be captured. How long would this stored energy be able to supply a city, assuming that the supply rate is the same as that coming from a large power plant, rated at 1,000 [MW]?

6) A metal sphere in free space has a radius of a. A total charge q is placed on the sphere. Assume that the resulting surface charge density is distributed uniformly on the surface of the sphere. Solve for the electric field vector at the surface of the sphere (just outside the sphere), by using only knowledge of boundary conditions. (Note that there is no electric field inside the sphere, due to the Faraday cage effect.)

7) Solve Laplace's equation in cylindrical coordinates to obtain the potential function inside of a coaxial cable, having inner radius a and outer radius b. The cable is filled with a dielectric having a relative permittivity ε_r. The potential on the inner PEC conductor is V_0 volts, while the outer PEC conductor (the shield) has a potential of zero volts. Note that the potential should only be a function of the radial distance ρ.

8) A spherical capacitor consists of a PEC conducting spherical shell of radius a, surrounded by a larger PEC conducting shell of radius b. Both shells have negligible thickness. Starting from Laplace's equation, obtain an expression for the potential in spherical coordinates between the two shells. Assume that the outer shell is at zero volts and the inner shell is at V_0 volts. The material between the two shells has a relative permittivity of ε_r (although this shoujd not appear in your final answer for the potential). (Question: Would the answer change if the inner spherical shell is replaced by a solid PEC sphere?)