ECE 3318
Applied Electricity and Magnetism

Spring 2023

Homework #10

Date assigned: Thursday, April 6, 2023
Date due: Thursday, April 13, 2023

1) A high-voltage power line with a radius of $a = 1$ [cm] is at a height of $h = 50$ [m] above the ground (measured from the center of the line), which may be approximately modeled as an infinite ground plane. Assume that the voltage of the power line is

$$v(t) = 100,000 \, \cos(\omega t) \, [\text{V}].$$

Find the electric field vector at a point on the earth directly below the power line. Assume that the frequency $\omega = 2\pi f$ is low enough so that at any instance of time, the electric field vector $\underline{E}(x,y,t)$ is that same as in a <u>static</u> problem, if you replace V_0 in the static solution with $v(t)$. This assumption should be accurate when all dimenisons are small compared with a wavelength.

Note: You may use any equation that was already drived in the class notes, but make sure that you give proper reference in your solution to where you took it from!

2) A point charge q sits at a height h above an infinite ground plane (at $z = 0$). Find the force vector that acts on the charge. Note that the electric field in the region $z > 0$ from the charge density on the ground plane is equal to the electric field due to the image charge.

3) Calculate the capacitance between a spherical PEC conducting shell of radius a surrounded by a larger PEC spherical conducting shell of radius b. Both shells have negligible thickness. The material between the two metal shells has a relative permittivity of ε_r. Simplify your answer for the capacitance when $b \rightarrow \infty$ and $\varepsilon_r = 1$. (This represents the capacitance between a single conducting spherical shell of radius a in free space and a "sphere at infinity" that is the boundary of the universe.)

4) Use the results from the previous problem to caculate the following.

   (a) Calculate the capacitance between the sphere of a Van de Graaff generator and the surrounding enviornment (assuming that everything in the surrounding enviornment is very far away). Assume that $a = 10$ [cm] for the radius of the Van de Graaff sphere, and that air surrounds the Van de Graaff.

(b) Calculate the capacitance of the entire earth, assuming it is modeled as a big spherical conductor, with the flux lines from it terminating at infinity (on the stars and galaxies very far away). Assume that the mean radius of the earth is approximately 6378.1 [km].

5) The "Maxwell Durablue" model supercapacitor From Maxwell Technologies has a capacitance of 1,000 [F]. It is charged to 16 [V]. How does the energy it stores compare with the energy stored in a regular car battery that has an energy capacity of 720 [Wh] (watt hours)? To see this, calculate the ratio of energy stored by the supercapacitor to the energy stored by the car battery.

6) A "twin lead" transmission line consists of two infinite wires running parallel to each other in air (assume that the wires are running in the z direction). Each wire has a radius of a, and the separation between the centers of the wires is labeled as h. Assume that one wire has a net line charge of ρ_l [C/m] and the other wire has a net line charge density of $-\rho_l$ [C/m]. Find the voltage drop V_{AB} between the two wires (where A is the anode wire). The anode wire is centered at $x = 0$ and the cathode wire is centered at $x = h$. Do this by integrating the electric field along a straight line that runs between the centers of the wires, going from the inner edge of one wire to the inner edge of the other wire.

Hint: See the class notes for a very similar calculation, involving the power line over the earth.

7) As a continuation of the previous problem, find the capacitance per unit length of the twin line transmission line.

8) A spherical shell of uniform charge density ρ_{s0} [C/m²] in free space has a radius a. Calculate the stored energy of this shell, using the electric field formula for stored energy. Put your final answer in terms of the total charge Q on the shell.

9) Repeat the above problem using the potential formula for stored energy.

10) A coaxial cable has an inner radius of a and an outer radius of b. Between the two conductors is a dielectic region with a relative permittivity of ε_r. A charge density of ρ_{l0} [C/m] is on the inner conductor (wire), and a charge density of $-\rho_{l0}$ [C/m] is on the outer conductor. Find the electric stored energy per unit length (per meter in the z direction). Use this to find the capacitance per unit length of the coaxial cable. (You can double-check your answer using the class notes, since the capacitance per unit length of the coaxial cable was derived in the class notes using the $C = Q/V$ method.)