ECE3340

Review of Numerical Methods for Fourier Transform Applications

PROF. HAN Q. LE

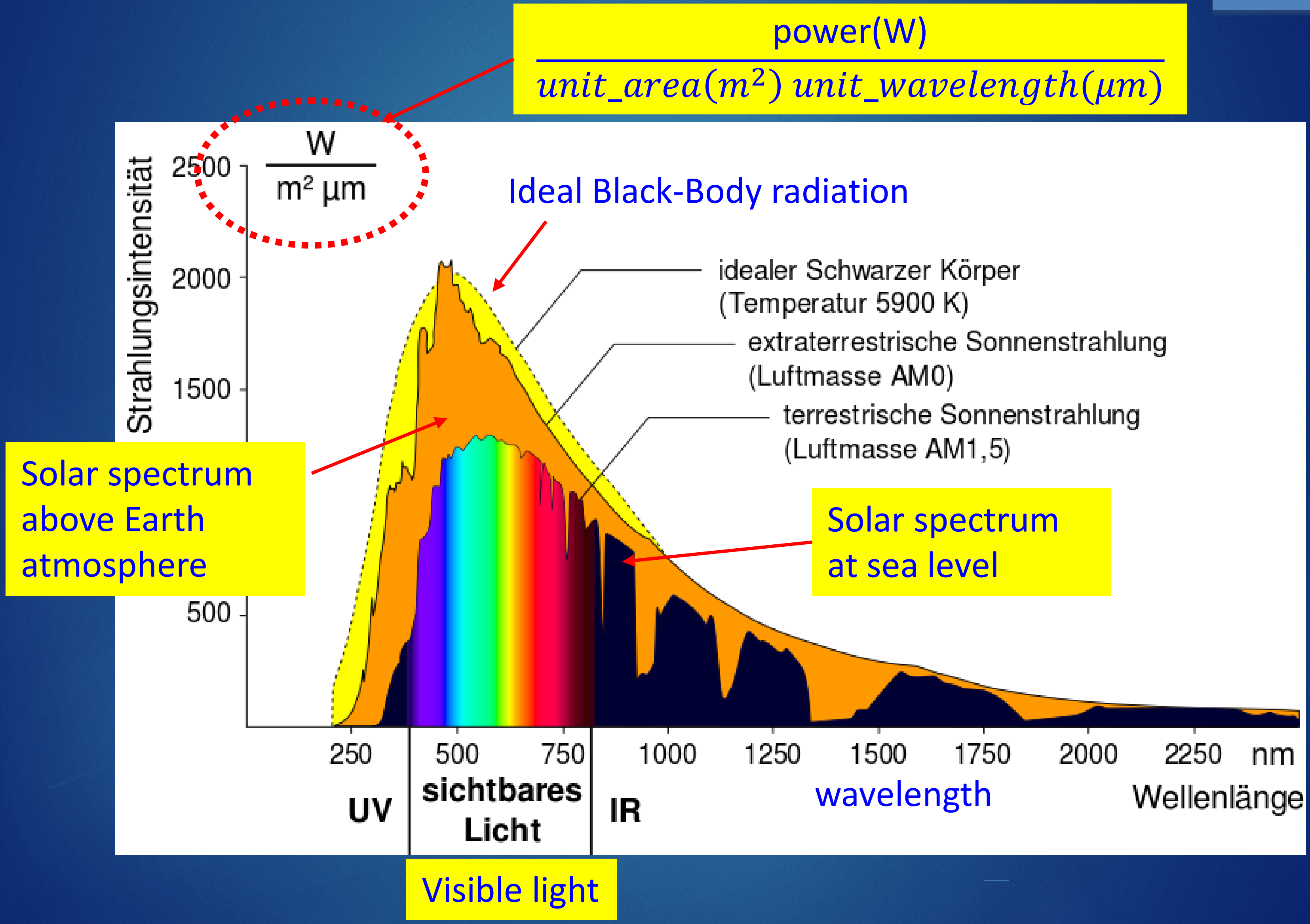
Note: PPT file is the main outline of the chapter topic – associated Mathematica file(s) contain details and assignments

Outline

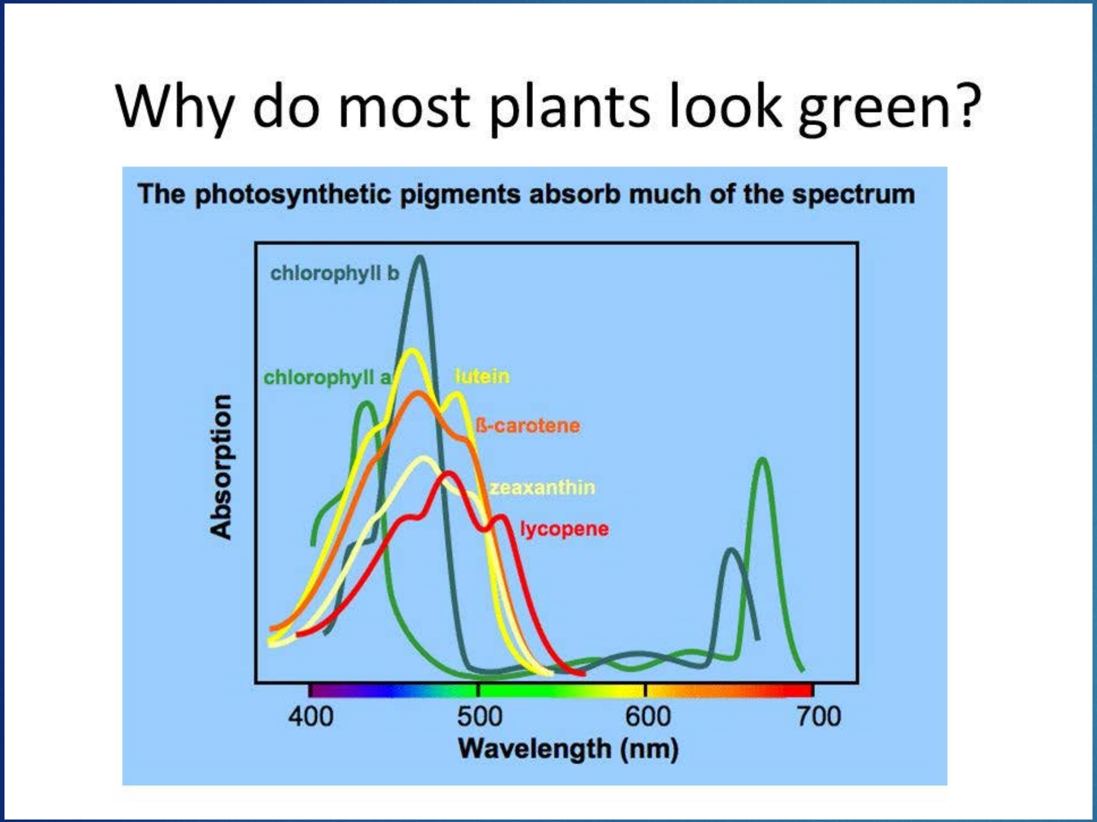
1. Introduction: concept of spectrum and periodic phenomena
2. Review of Fourier transform
3. Review of Fourier analysis
4. Numerical method: FFT
5. Applications in linear time-invariant system and signal processing

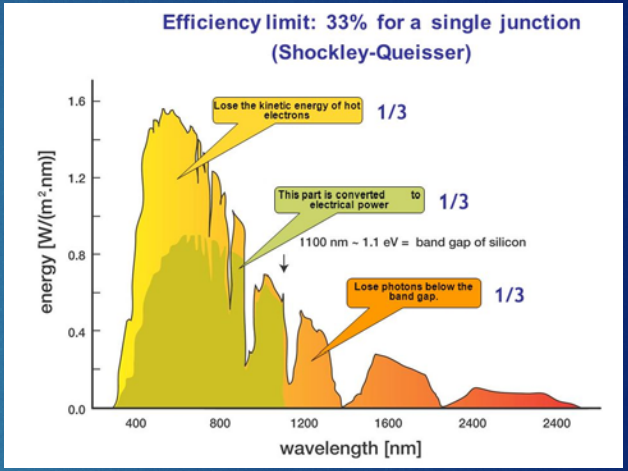
Introduction: spectrum concept, periodic phenomena

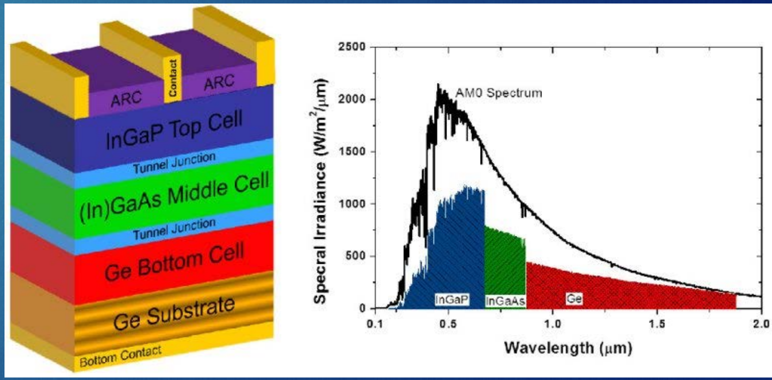
Power from the Sun



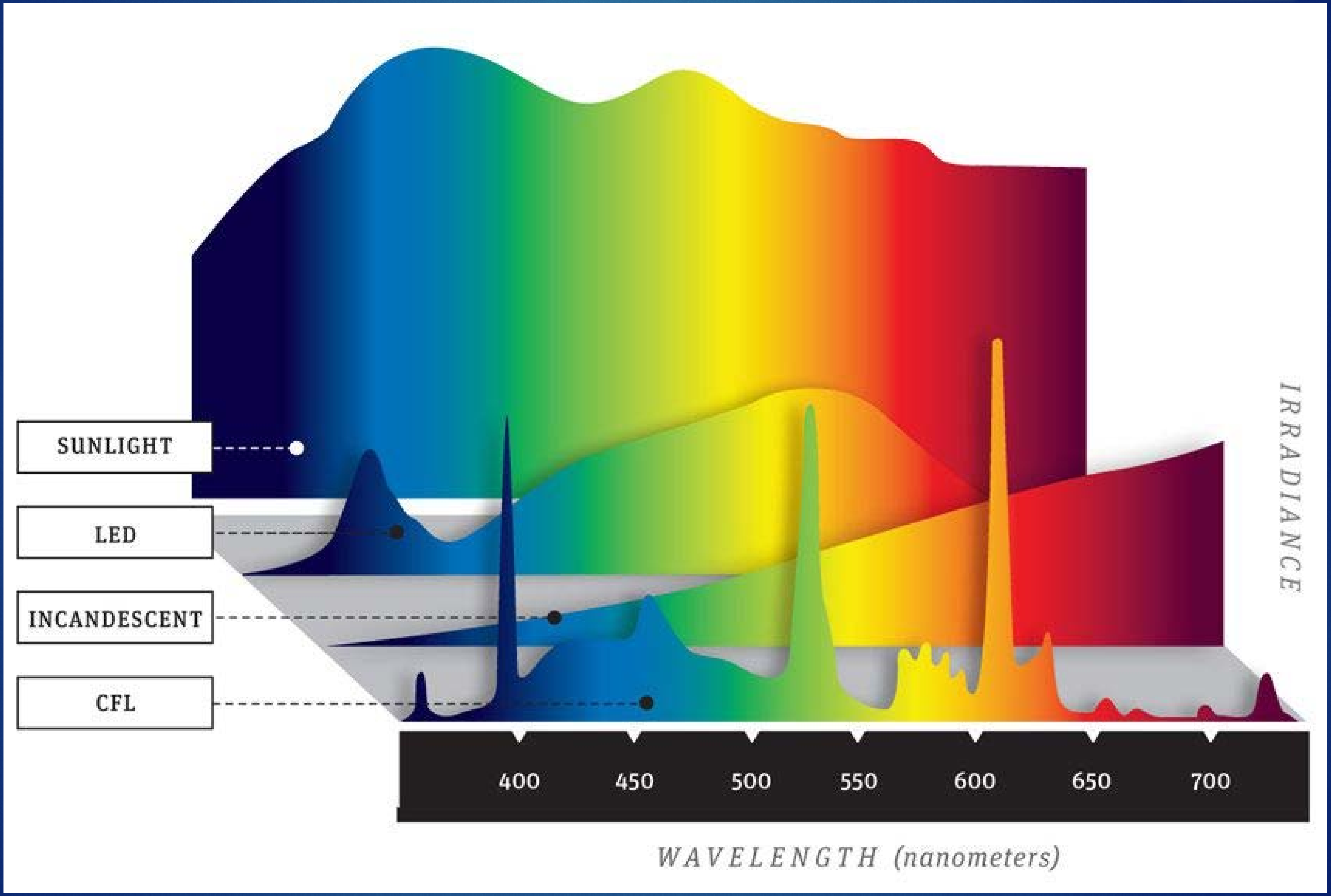
Where does solar spectrum (or solar power spectral density) matter?





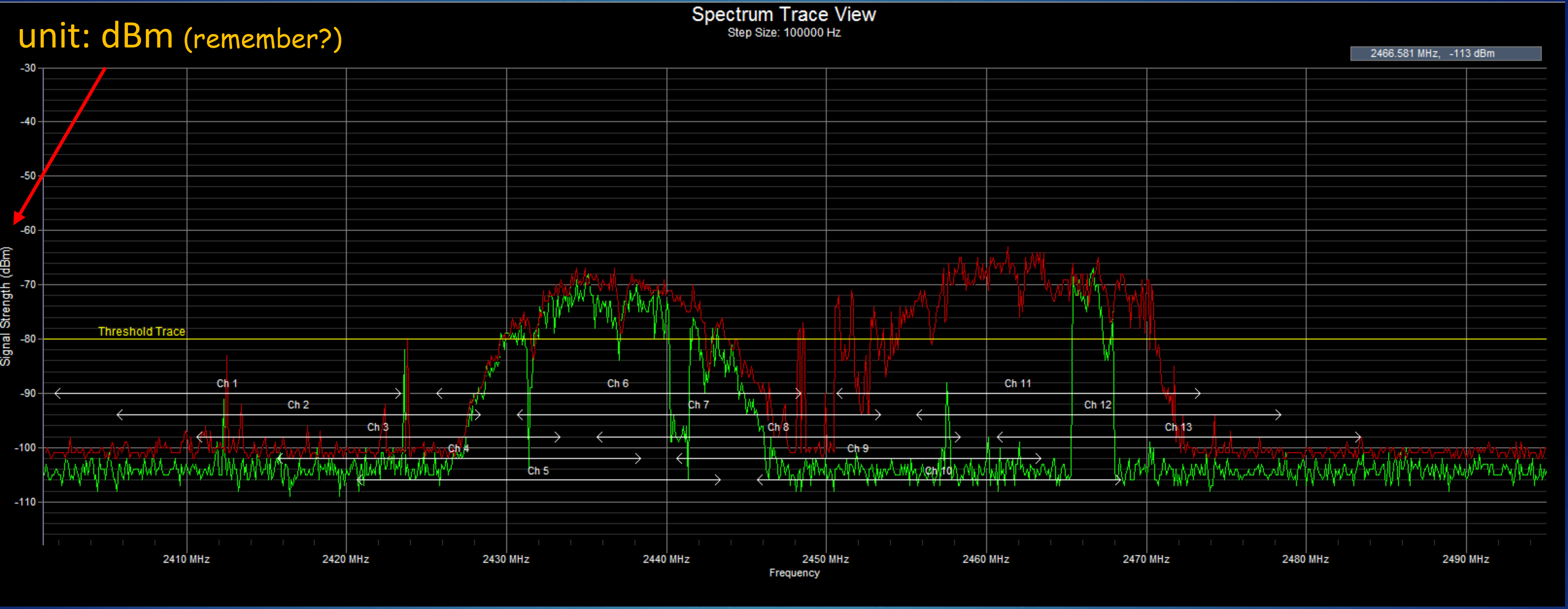


Common lighting spectra



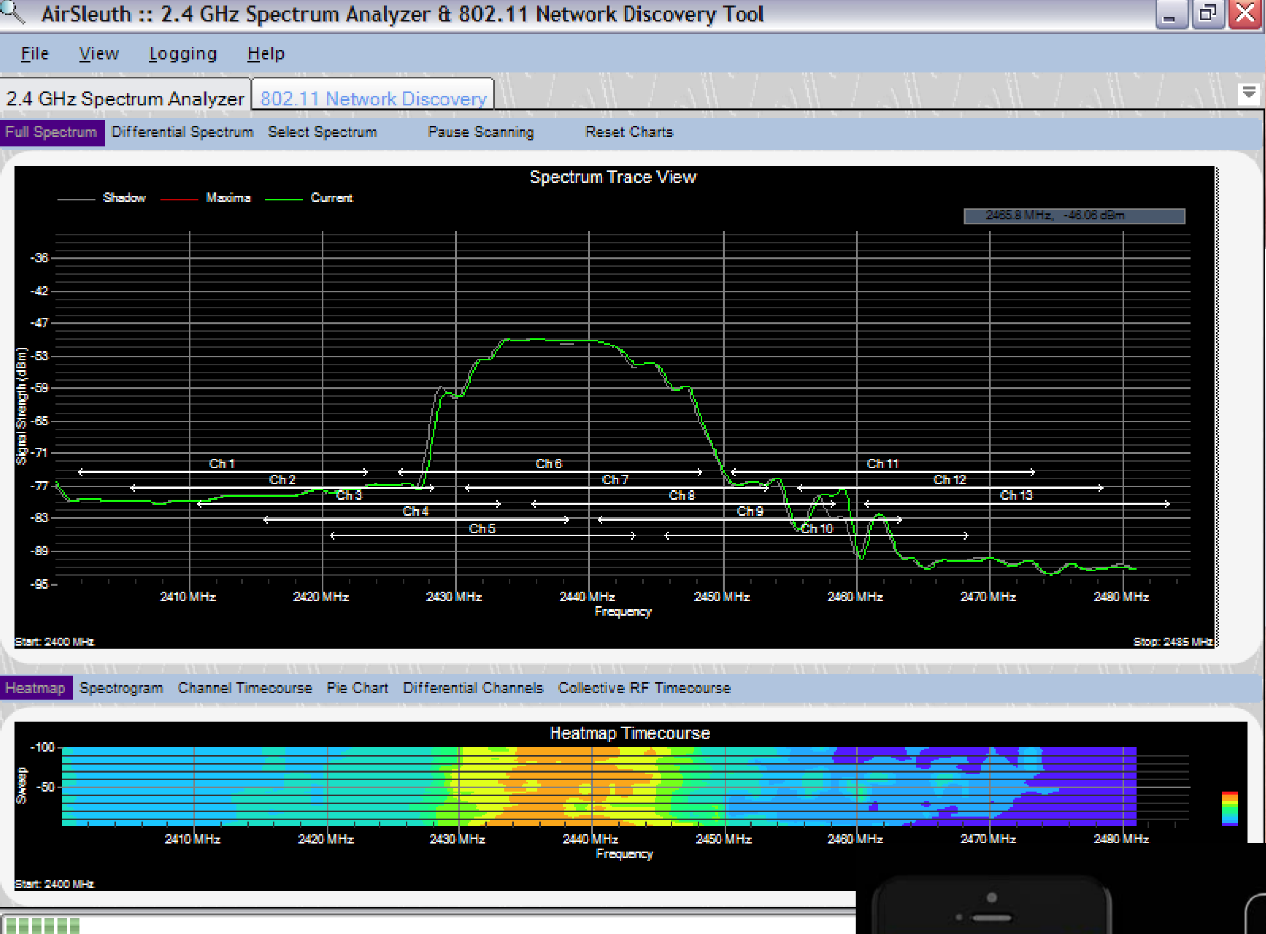
… what, exactly, is a spectrum?

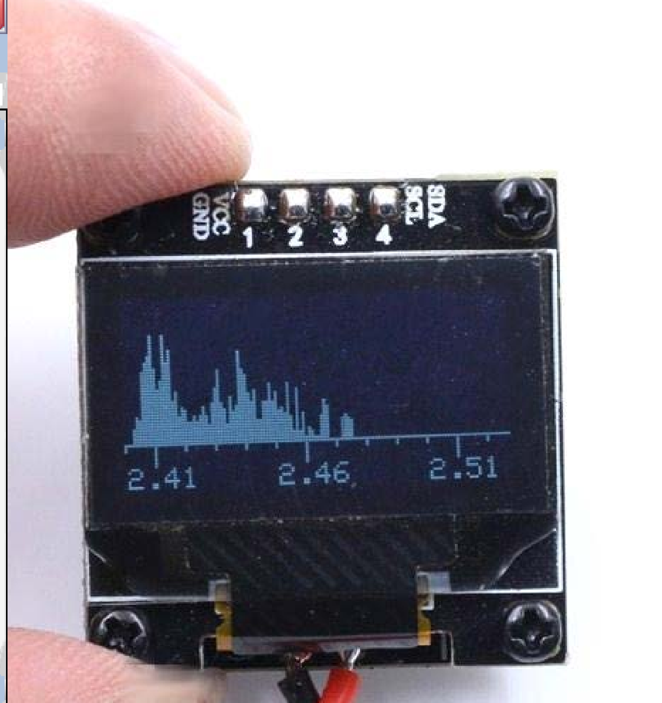
A type spectrum that we can't live without…



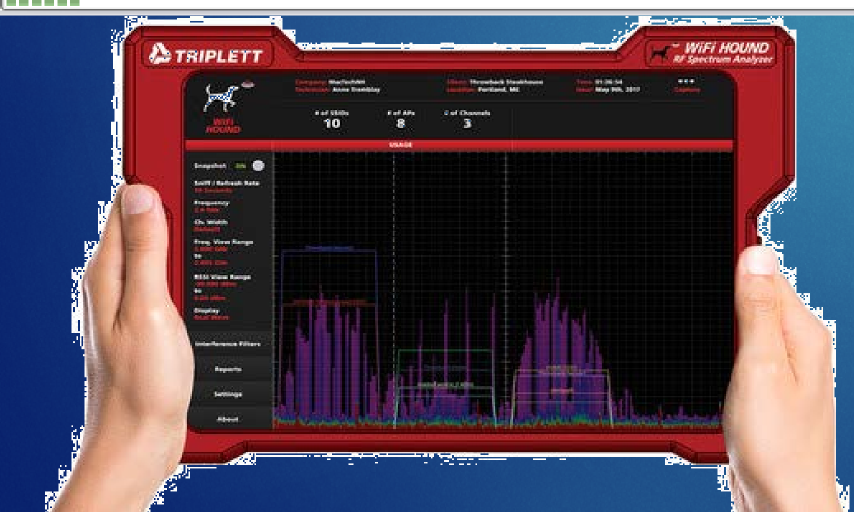
2.4 GHz band

You are depending on it right now in this class…



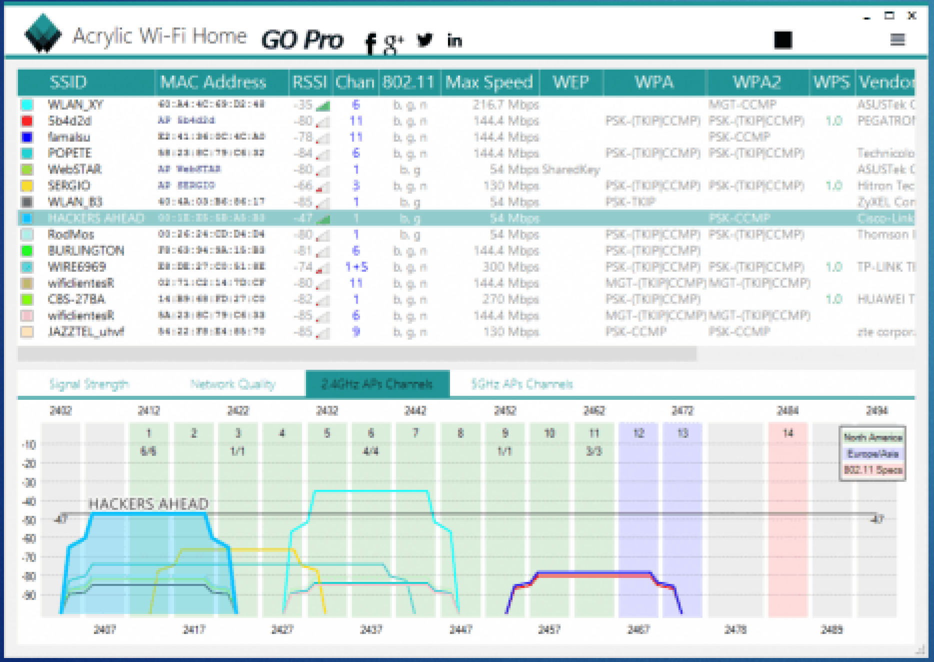


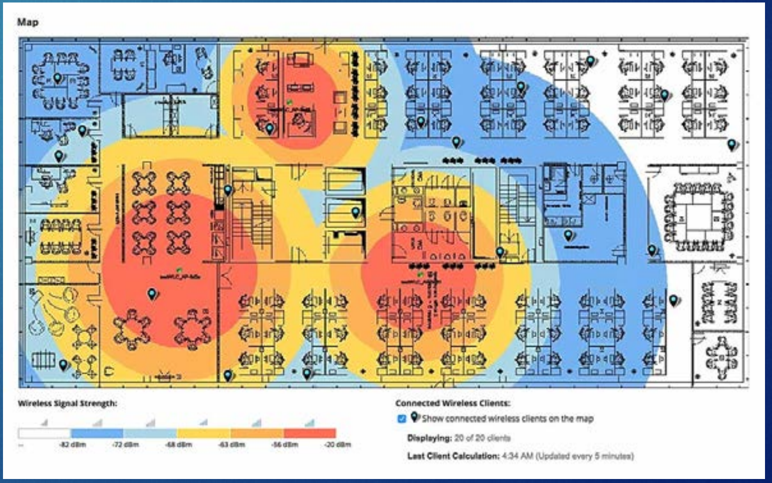
For wifi-sniffing drone



This is a spectrum. Except it is not RF power vs. frequency, but RF power vs. wifi source. (spectrum in the broad sense).

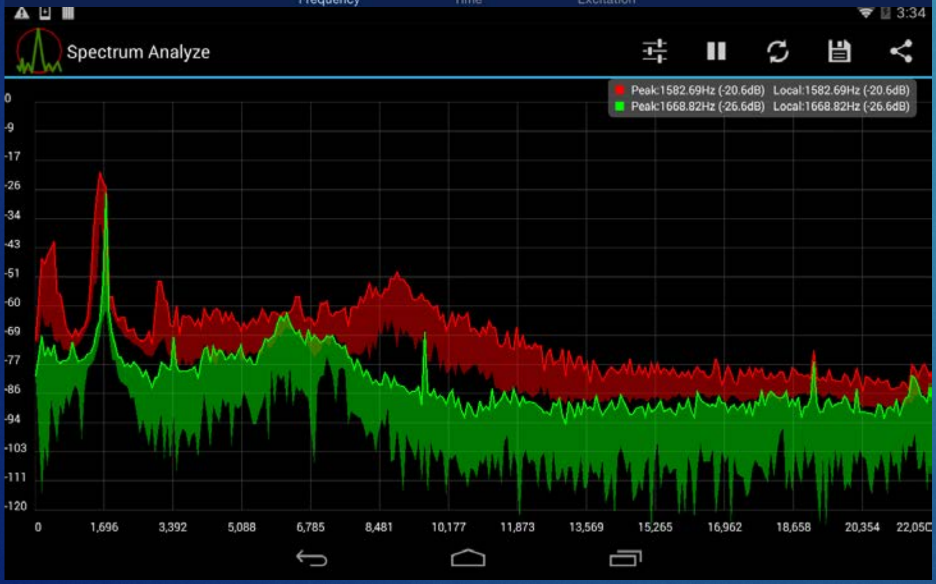
Below are examples of PC-based wifi spectrum analyzer apps (but not as accurate and sensitive as hardware-based RF spectrum analyzer)

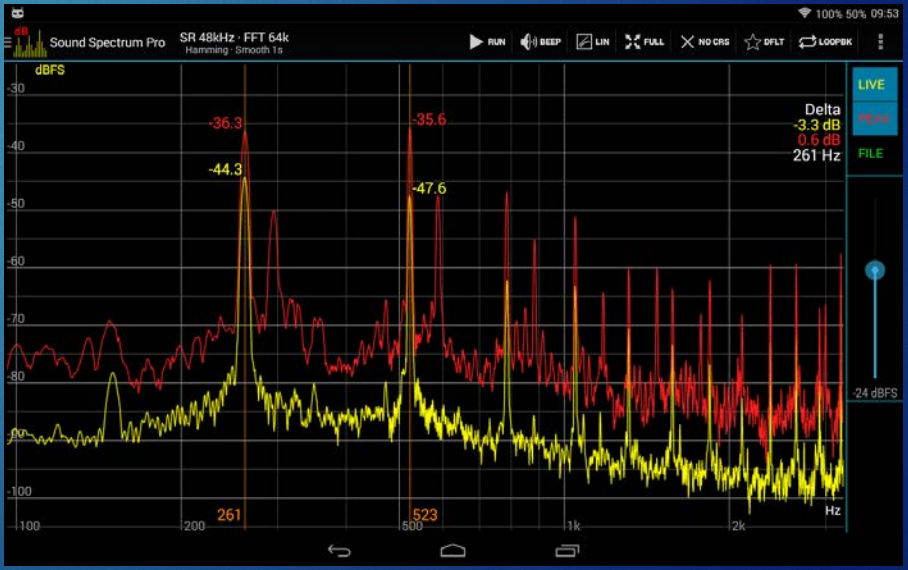




… what else can have spectrum (or spectra)?

Android, iOS apps for audio spectrum analyzer





SCIENTIFIC REPORTS

Article | OPEN | Published: 18 July 2017

Exploring spatial and temporal trends in the soundscape of an ecologically significant embayment

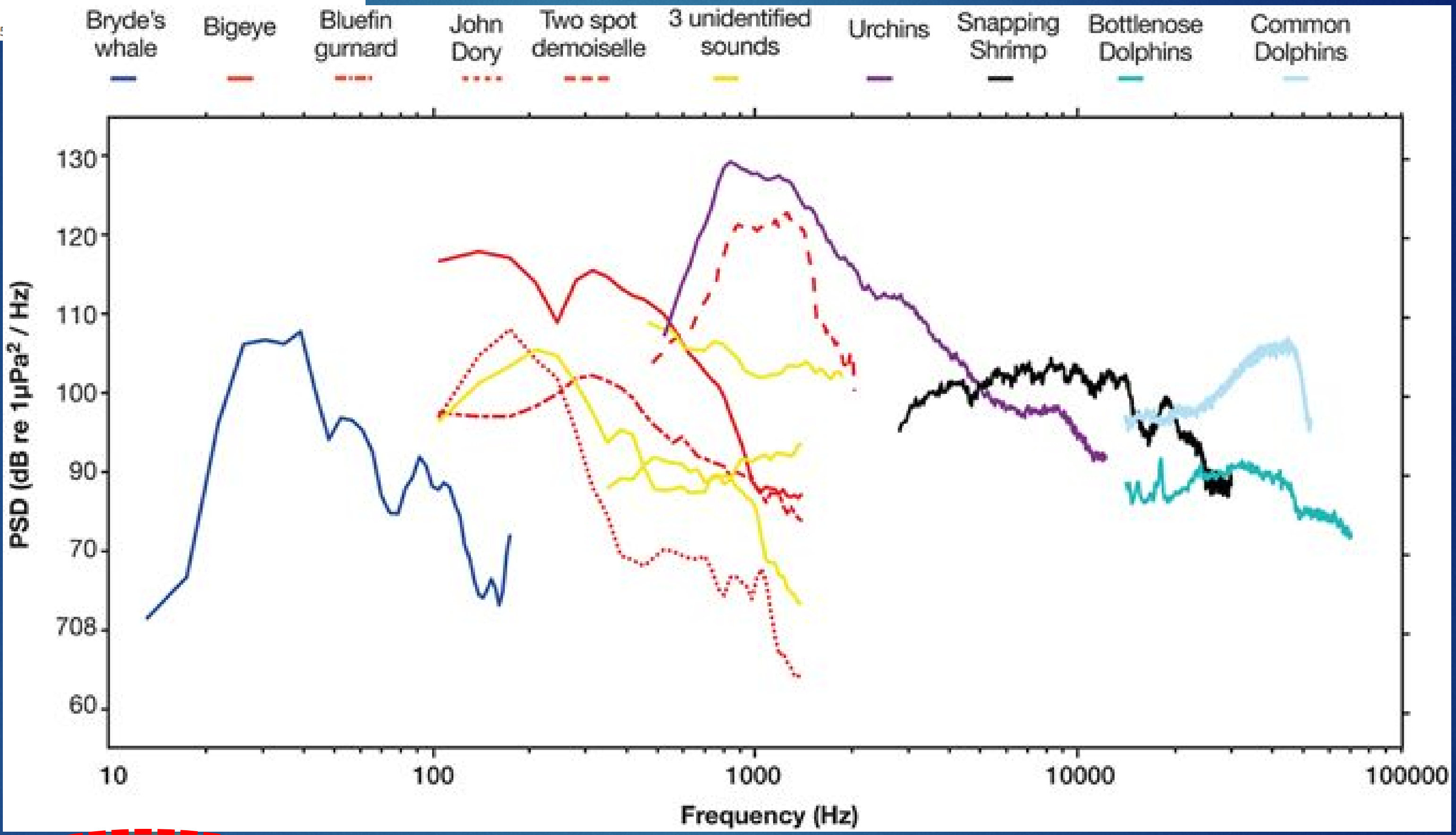
R. L. Putland, R. Constantine & C. A. Radford

Scientific Reports **7**, Article number:

Biophony

keyword

Power Spectral Density



Power spectral density (dB re 1 µPa^{2}Hz^{-1}) for the best example of nine different biological sounds manually identified from recordings taken throughout the Hauraki Gulf.

Marine environment sound spectrum (power spectral density)

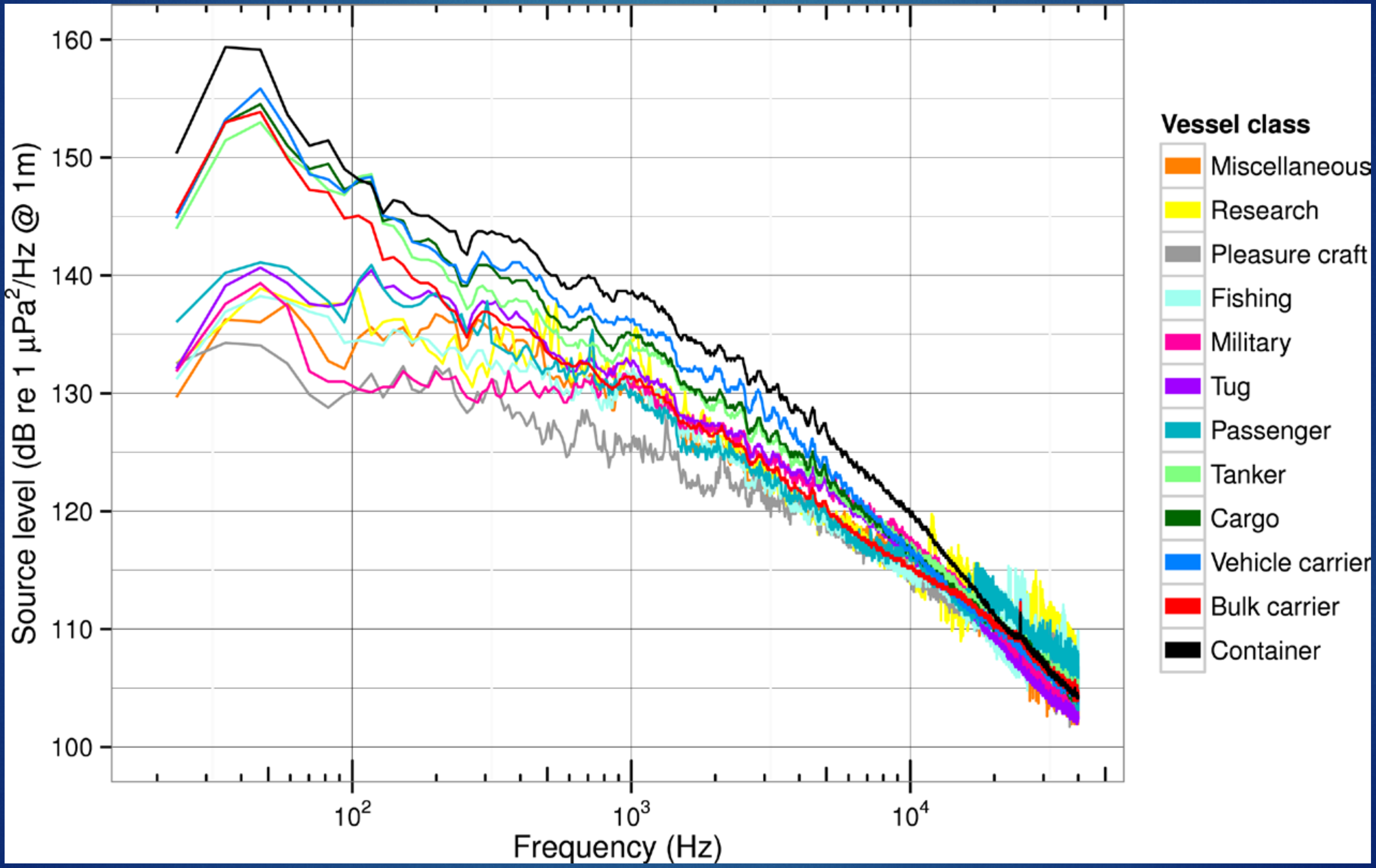
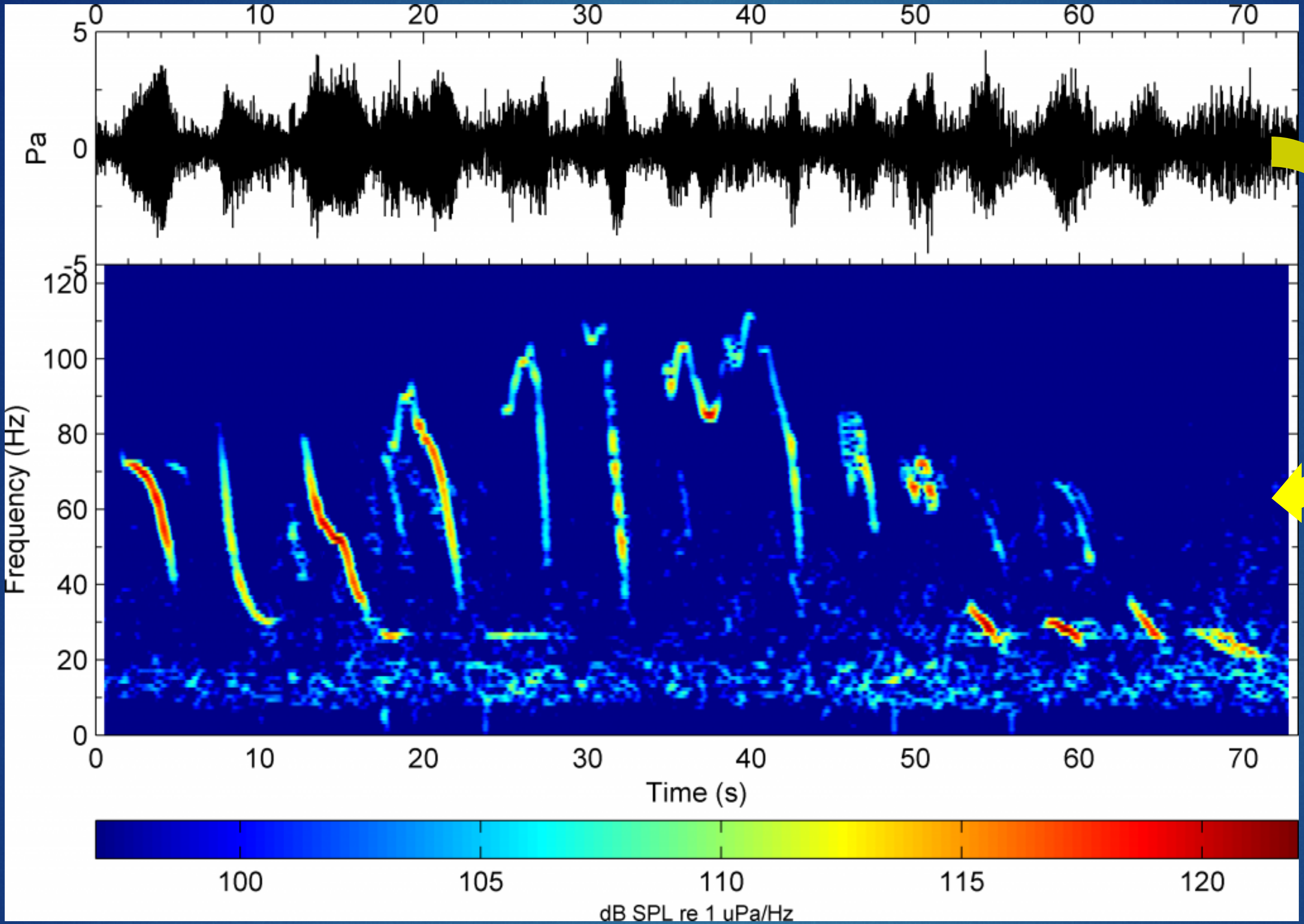


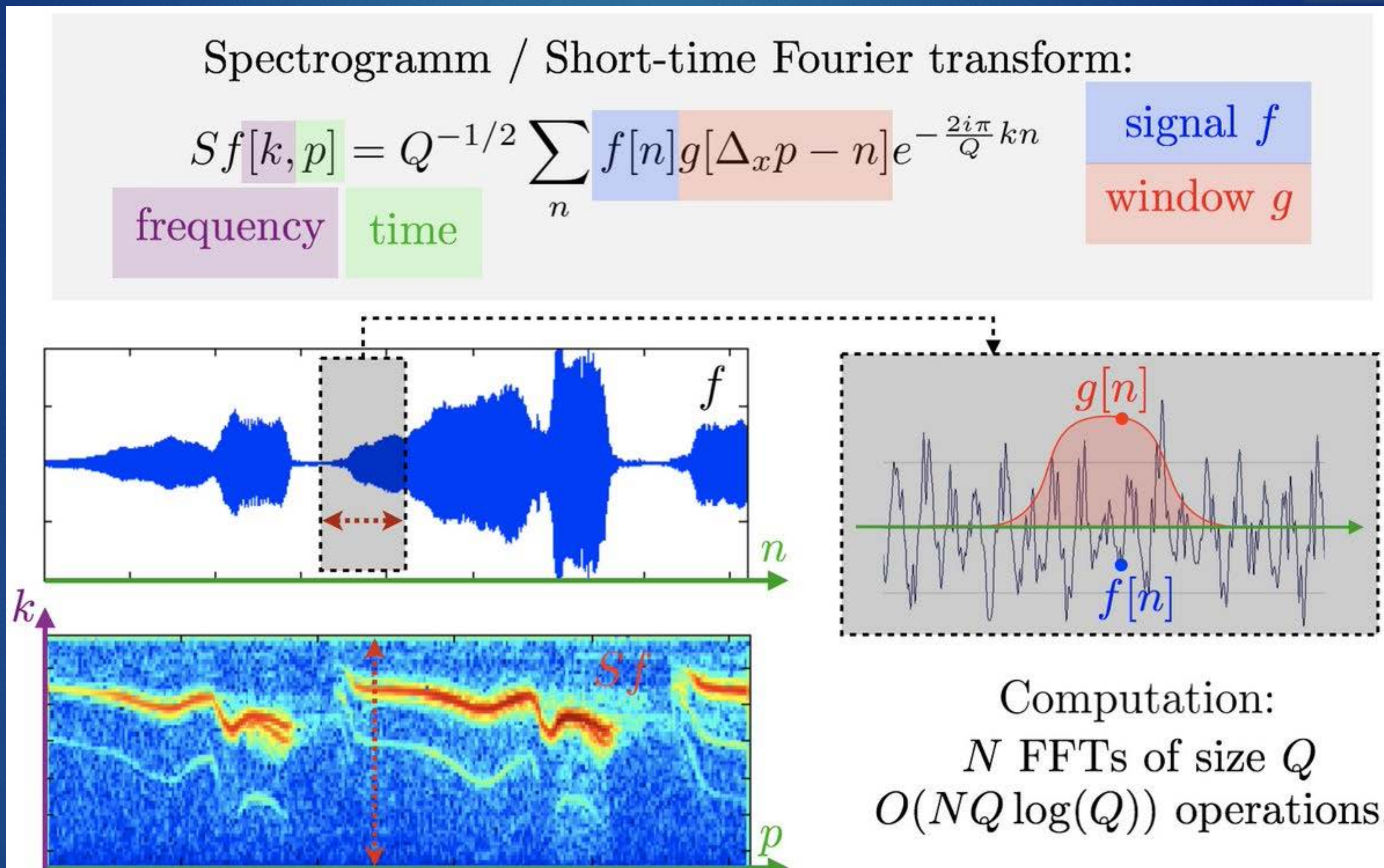
Figure 2: The sonobuoys detected a variety of forms in FM (frequency modulated) calls of Antarctic blue whales within the aggregation. The most common call types are to the left of the spectrogram. (Photo: Brian Miller)

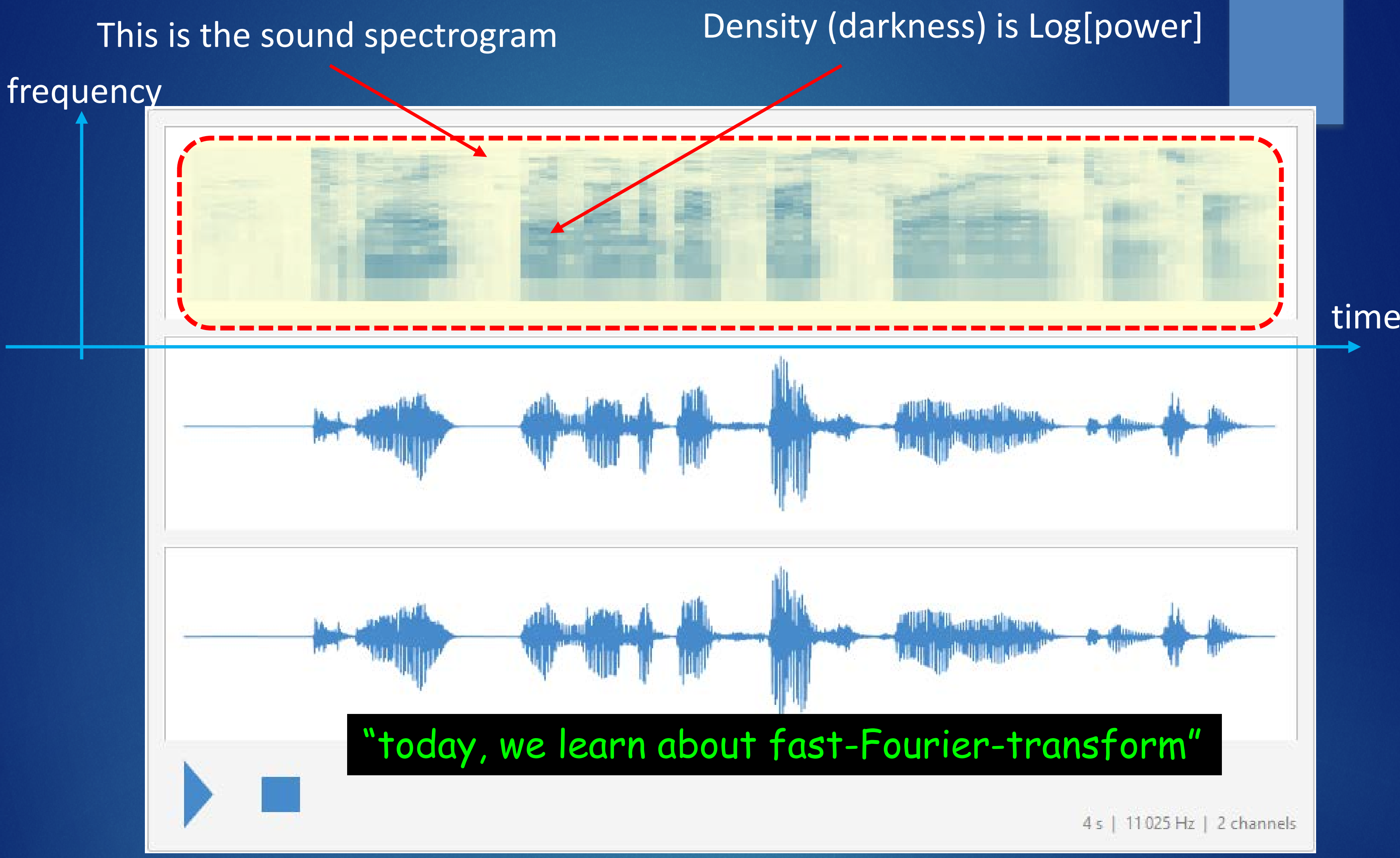


What numerical method allowing using this data to obtain this "spectrogram" ?

http://www.antarctica.gov.au/magazine/2011-2015/issue-28-june-2015/science/acoustic-technology-provides-insights-into-blue-whale-behaviour

A closely related concept of spectrum: spectrogram





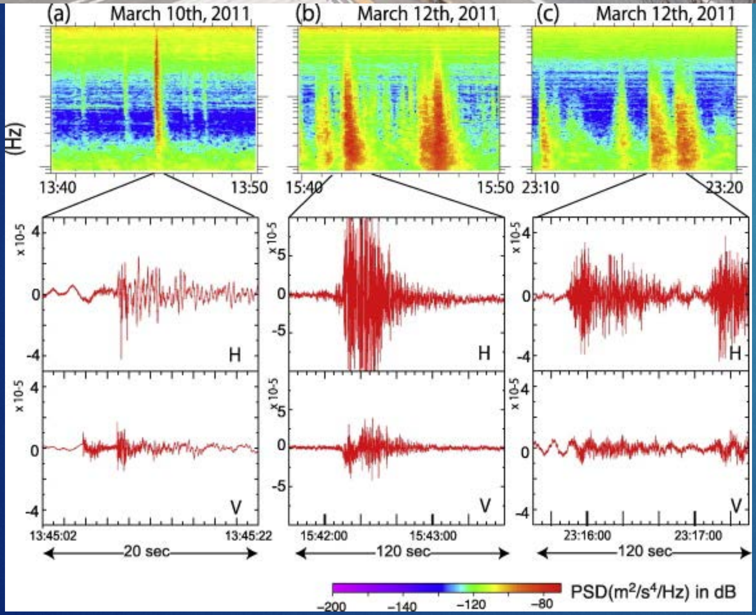
In Mathematica, Fourier[] would generate an array for us to calculate the spectrum sweep it as a function of time. We have a time-dependent spectrum, or spectrogram.

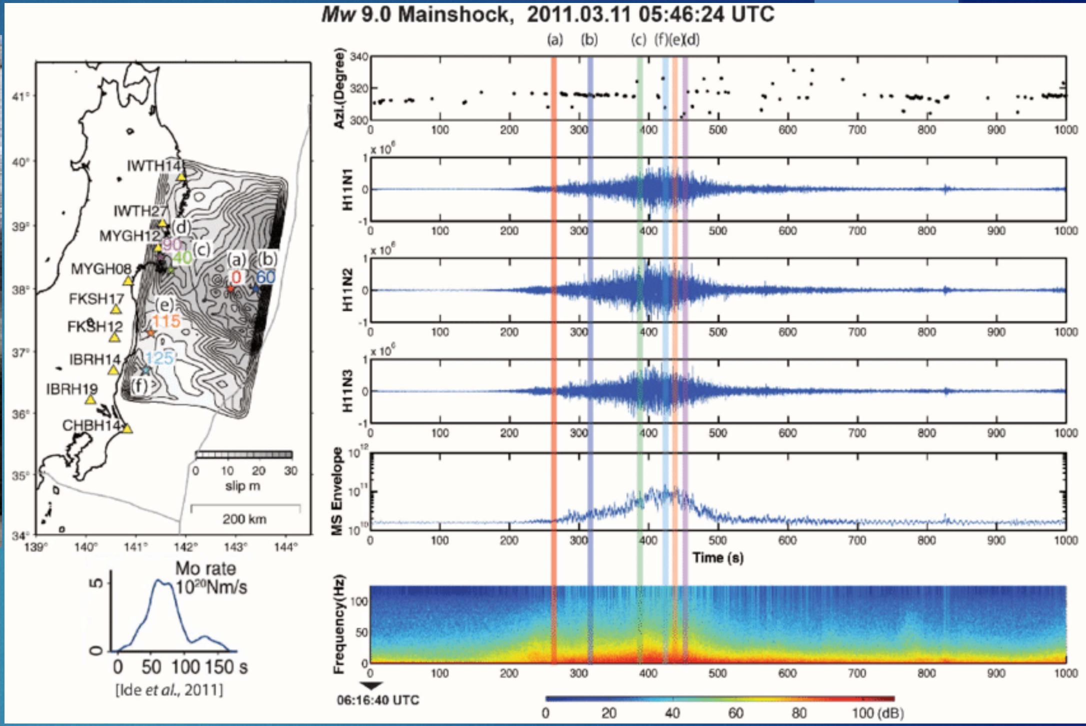
… what else can have spectrum (or spectra)?

any sequential or serial, or ordered structure numerical data:

- time-series or temporal signals
- spatial signal such as images (Fourier optics)


MIYAKO


(a) March 10th, 2011
(b) March 12th, 2011
(c) March 12th, 2011
(Hz)
13:40
13:50
15:40
15:50
23:10
23:20
H
V
13:45:02
13:45:22
20 sec
15:42:00
15:43:00
120 sec
23:16:00
23:17:00
120 sec
PSD(m²/s⁴/Hz) in dB

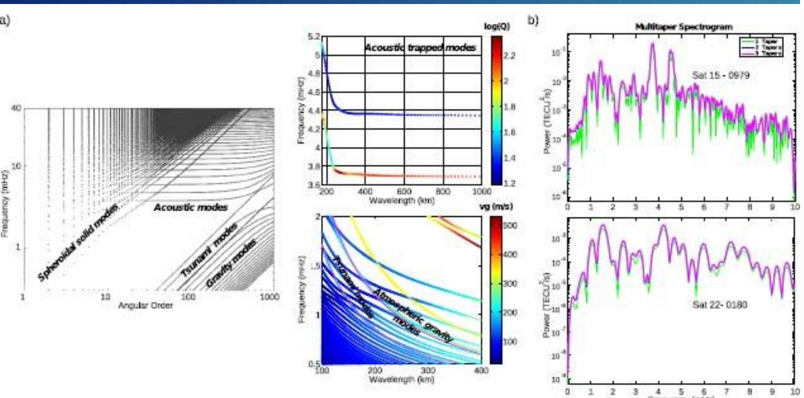

Mw 9.0 Mainshock, 2011.03.11 05:46:24 UTC
IWTH14
IWTH27
MYGH12
MYGH08
FKSH17
FKSH12
IBRH14
IBRH19
CHBH14
slip m
200 km
Mo rate
10²⁰Nm/s
[Ide et al., 2011]
Azi.(Degree)
H11N1
H11N2
H11N3
MS Envelope
Time (s)
Frequency(Hz)
06:16:40 UTC
100 (dB)

Earth, Planets and Space
July 2011, 63:62 | Cite as

The resonant response of the ionosphere imaged after the 2011 off the Pacific coast of Tohoku Earthquake

Authors Authors and affiliations

Lucie M. Rolland, Philippe Lognonné, Elvira Astafyeva, E. Alam Kherani, Naoki Kobayashi, Michèle Mann, Hiroshi Munekane



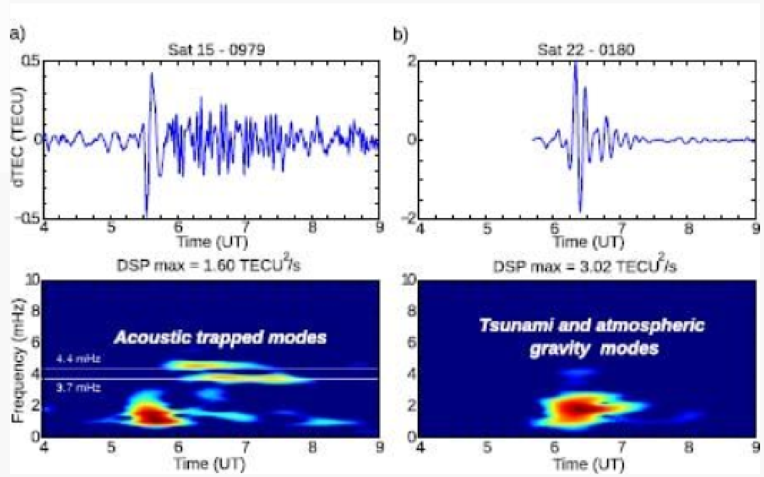
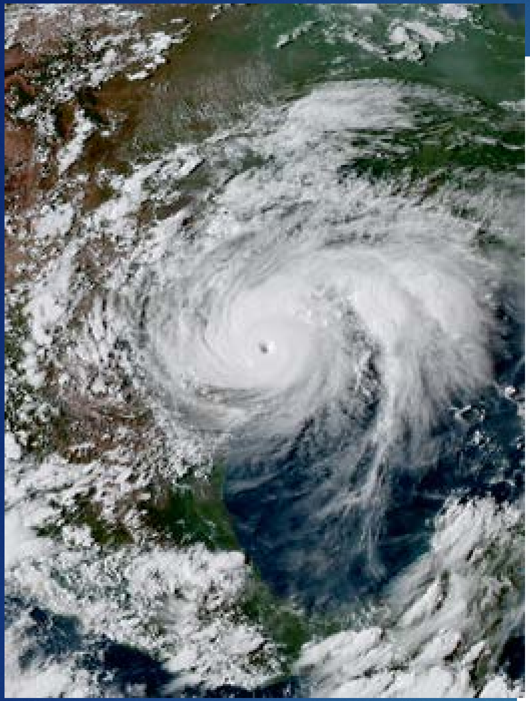
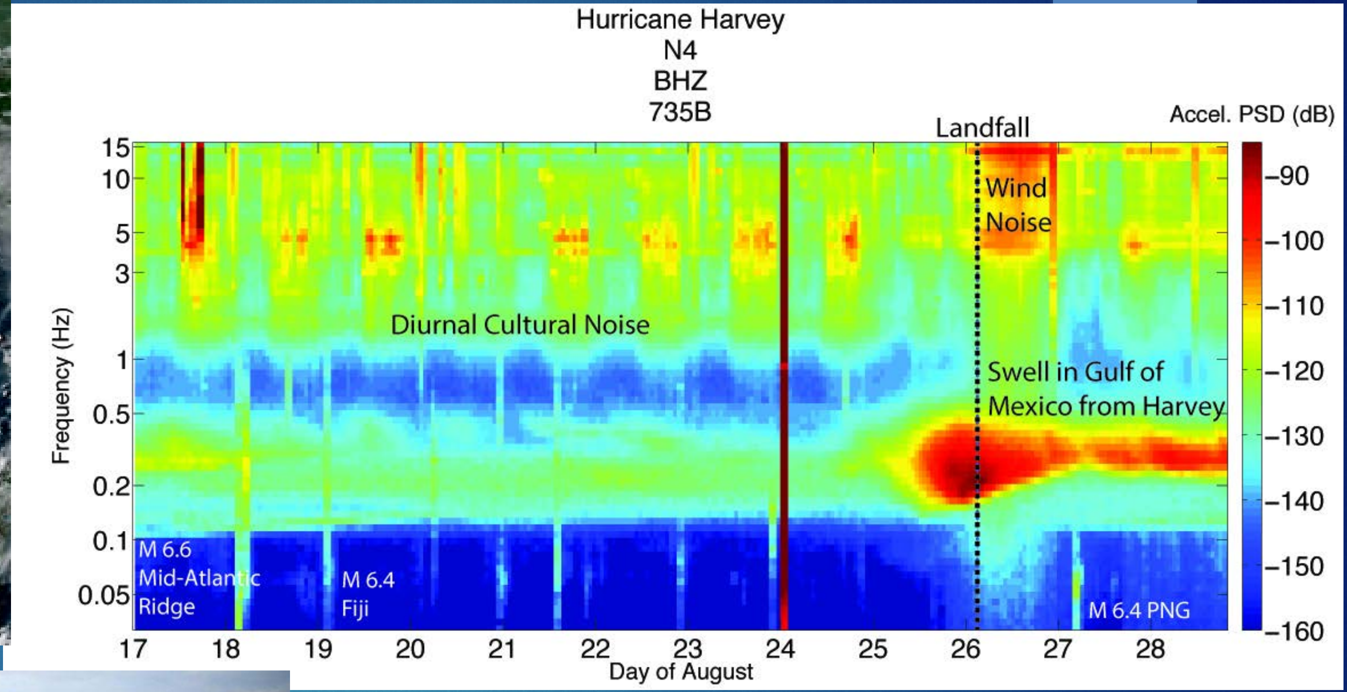


Fig. 3.

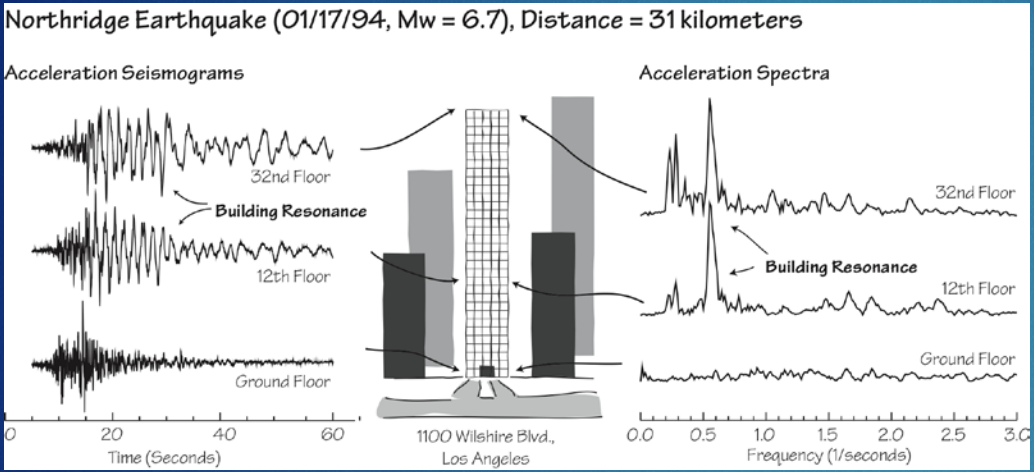
Upper panel: two filtered slant TEC time series. Bottom panel: Corresponding spectrograms. (a) For station 0979 observing satellite 15. It shows the primary gravito-acoustic pulse and two signals that oscillates with frequencies close to the two fundamental acoustic resonance frequencies (${}_0S_{29}$ and ${}_0S_{36}$) at ~3.7 and ~4.4 mHz. (b) For station 0180 observing satellite 22. It shows a signal that oscillates with a dominant frequency of ~1.8 mHz.

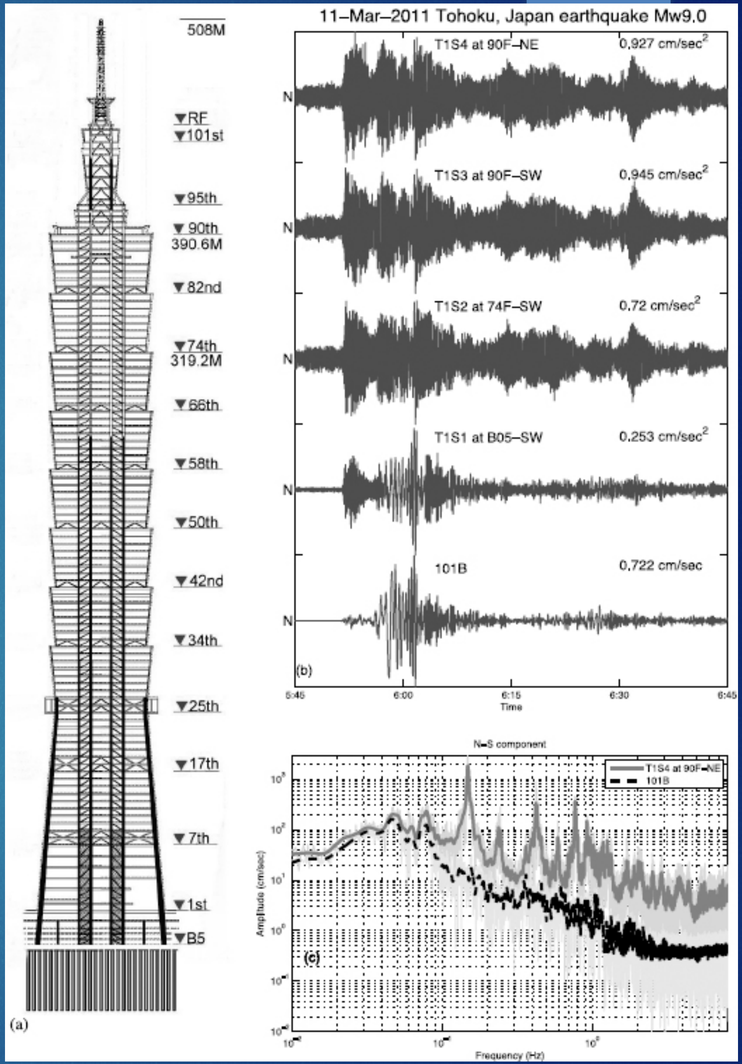
Observation of Earth natural acoustic resonance. We can say the Earth has an extreme deep voice, 3.7 and 4.4 mHz. How long is one period of these resonances?



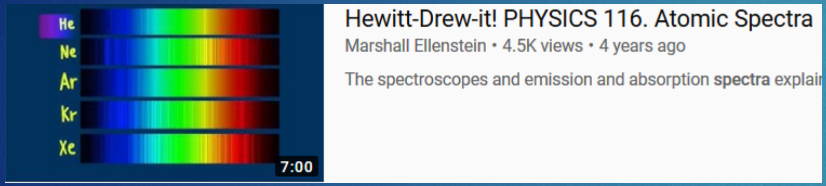

Hurricane Harvey
N4
BHZ
735B
Landfall
Accel. PSD (dB)
Frequency (Hz)
Day of August
Diurnal Cultural Noise
Wind
Noise
Swell in Gulf of
Mexico from Harvey
M 6.6
Mid-Atlantic
Ridge
M 6.4
Fiji
M 6.4 PNG

Spectra can tell a lot about an object natural frequencies and properties

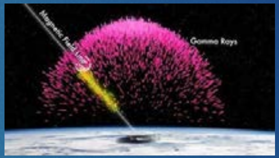


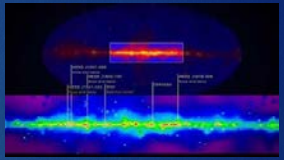


Some examples of resonances of natural frequencies



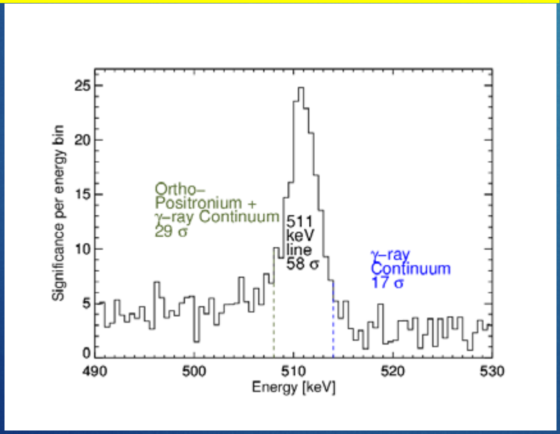
for amusement: [the music code of the Rosslyn Chapel]

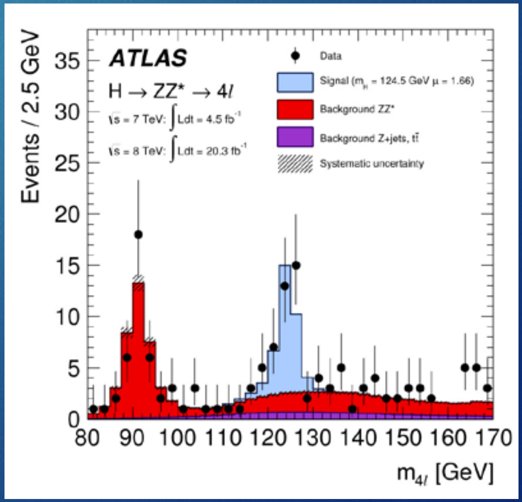




gamma-ray and nuclear spectroscopy

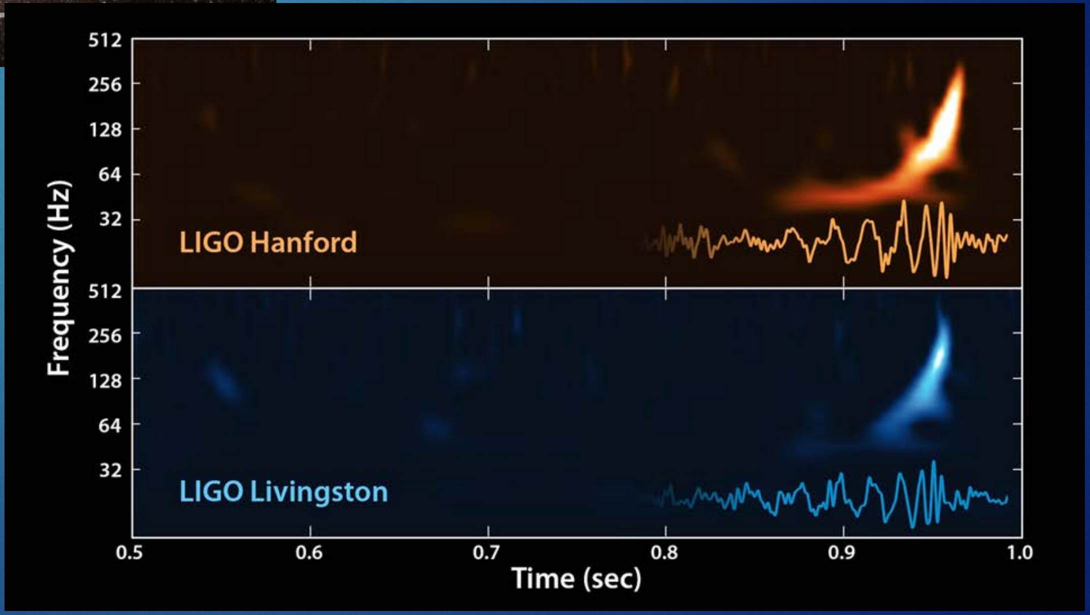
electron-positron annihilation spectral signature from the Milky Way

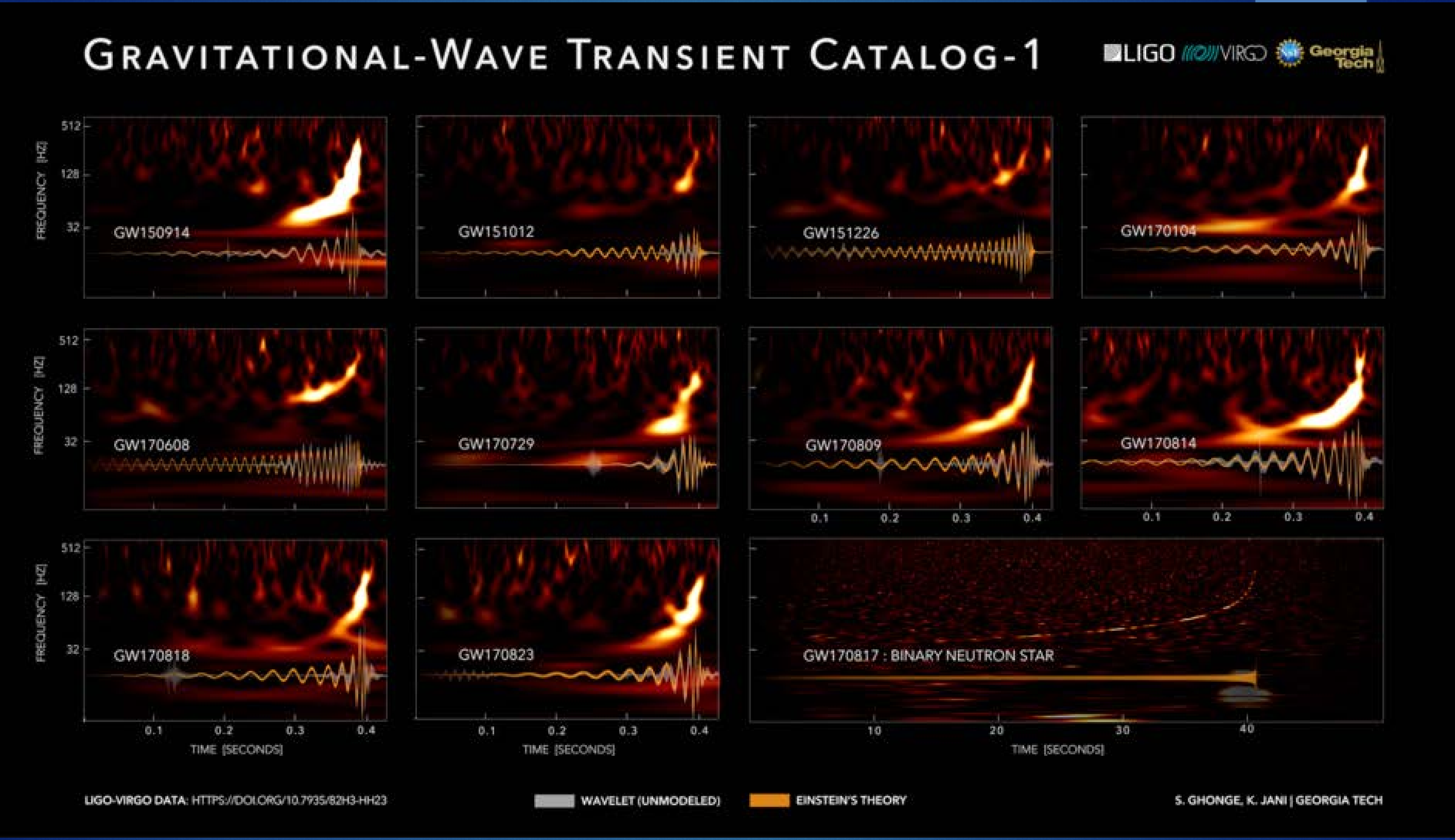




the Higgs boson spectral signature

Gravitational-wave spectra/spectrogram



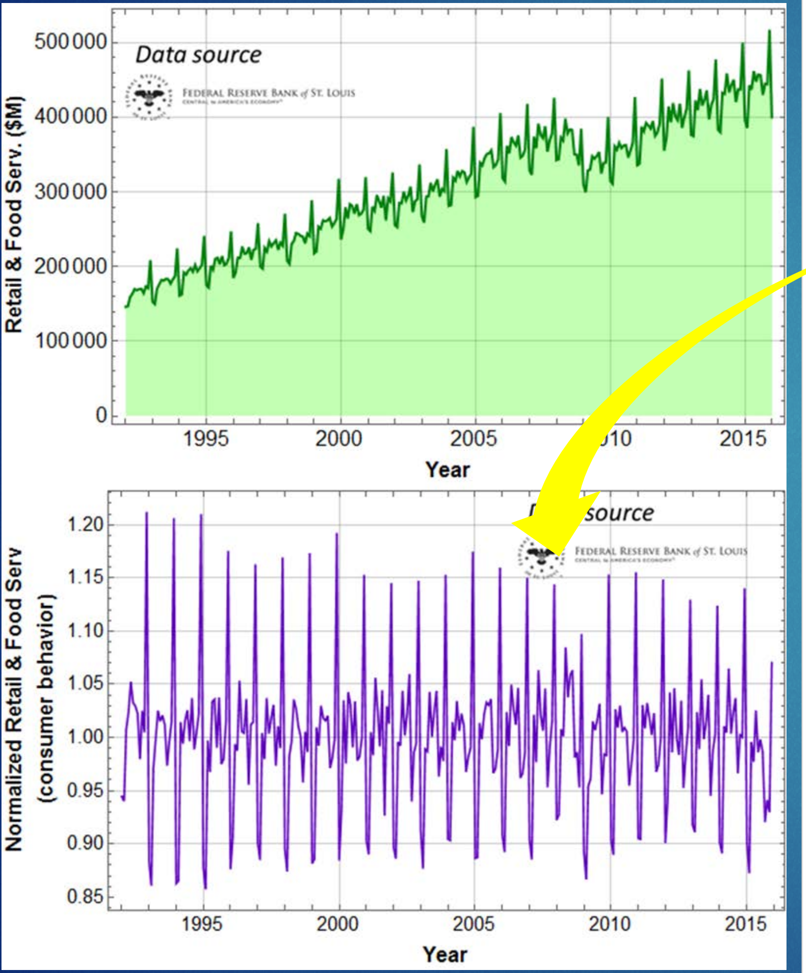

GRAVITATIONAL-WAVE TRANSIENT CATALOG-1
LIGO VIRGO Georgia Tech
FREQUENCY [HZ]
512
128
32
GW150914
GW151012
GW151226
GW170104
GW170608
GW170729
GW170809
GW170814
GW170818
GW170823
GW170817 : BINARY NEUTRON STAR
0.1
0.2
0.3
0.4
10
20
30
40
TIME [SECONDS]
LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23
WAVELET (UNMODELED)
EINSTEIN'S THEORY
S. GHONGE, K. JANI | GEORGIA TECH

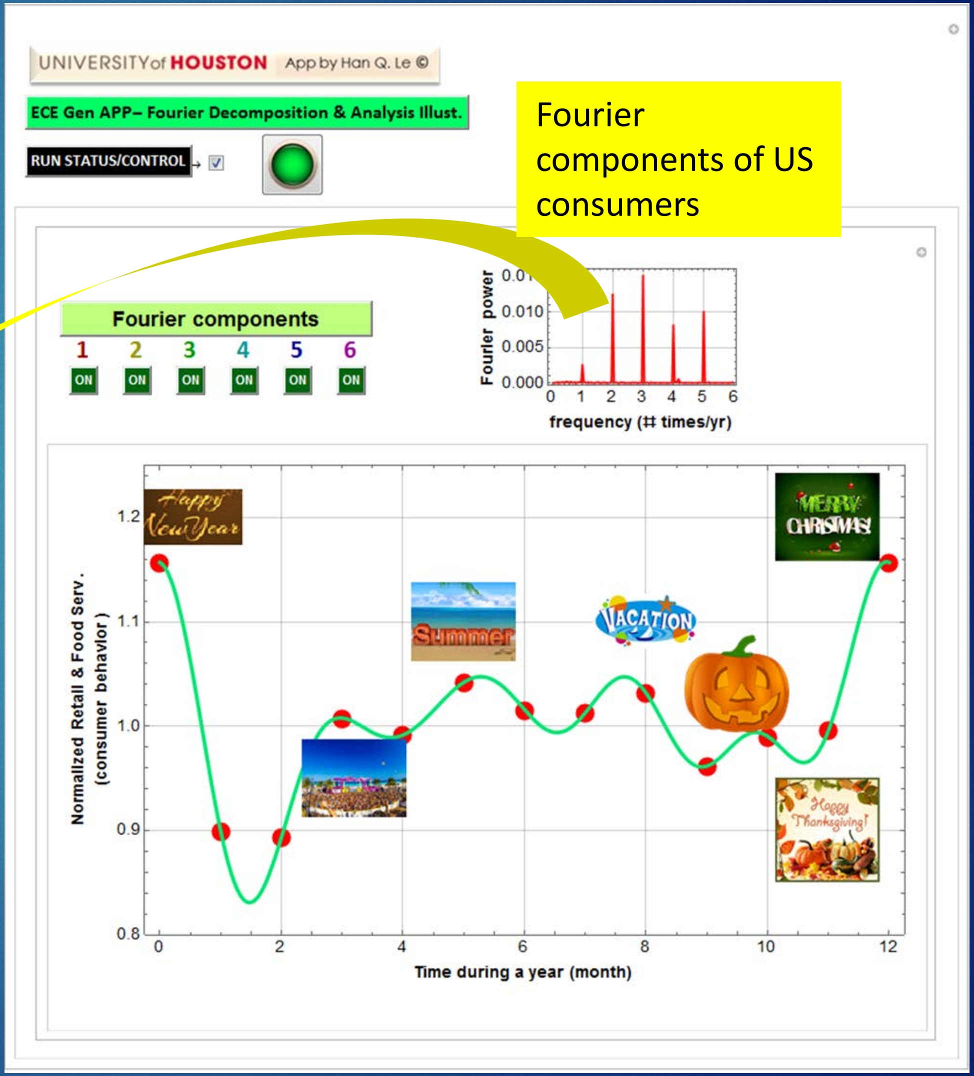
Spectral analysis (producing spectrum and spectrogram) is fundamentally a method of mapping/classification that is very useful for any quantitative science.

Especially for periodic phenomena, Fourier-spectral analysis is a mapping of power/energy/magnitude (P/E/A) vs. frequency category. Example: We identify individual voices of family, friends, acquaintances based on our supervised learning of classification with audio spectrum. (in a household, we can even tell whom even with just a cough or a footstep of the person).

Even for non-periodic phenomena, Fourier spectral analysis can be useful to classify types of randomness such as white noise, 1/f, Brownian noise,… (we will touch upon Itô calculus if we have time).

Example of periodic human socio-economic activities





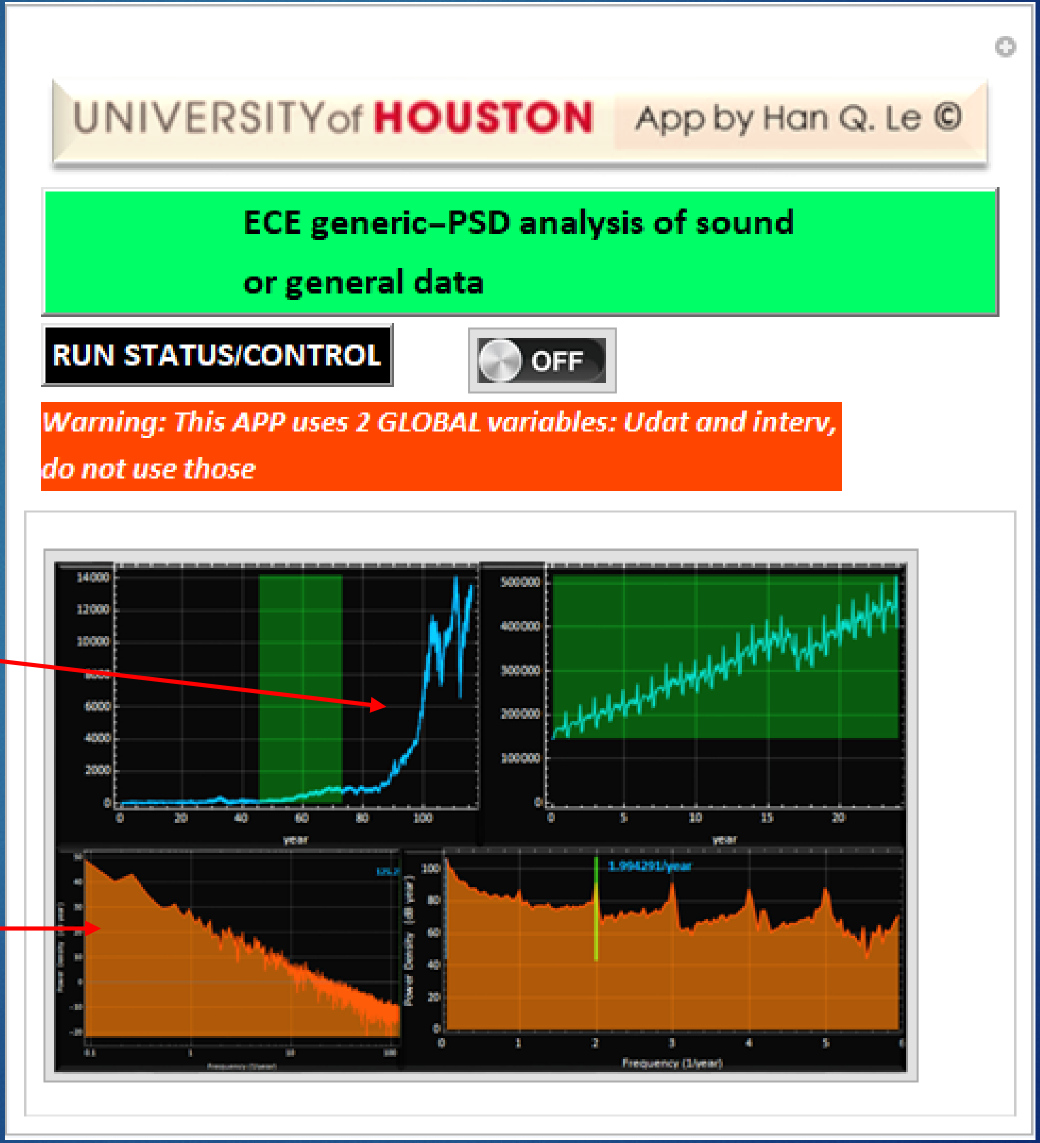
Fourier components of US consumers

Stock market is a Brownian "random down Wall Street"

No periodicity because of market efficiency

Dow-Jones index (1896-2016)

Dow-Jones index power spectral density (Brownian noise)



more review of periodic phenomena

what a spectrum is not…

("spectral range" is often colloquially – or in general language- synonymous with "spectrum." But strictly and technically, do not be confused of spectral range vs. spectrum)

Merriam-Webster

spectrum noun

spec·trum | \ ˈspek-trəm \

plural **spectra** \ ˈspek-trə \ *or* **spectrums**

Definition of *spectrum*

1 a : a continuum of color formed when a beam of white light is dispersed (as by passage through a prism) so that its component wavelengths are arranged in order

b : any of various continua that resemble a color spectrum in consisting of an ordered arrangement by a particular characteristic (such as frequency or energy): such as

(1) : ELECTROMAGNETIC SPECTRUM

(2) : RADIO SPECTRUM

(3) : the range of frequencies of sound waves

(4) : MASS SPECTRUM

c : the representation (such as a plot) of a spectrum

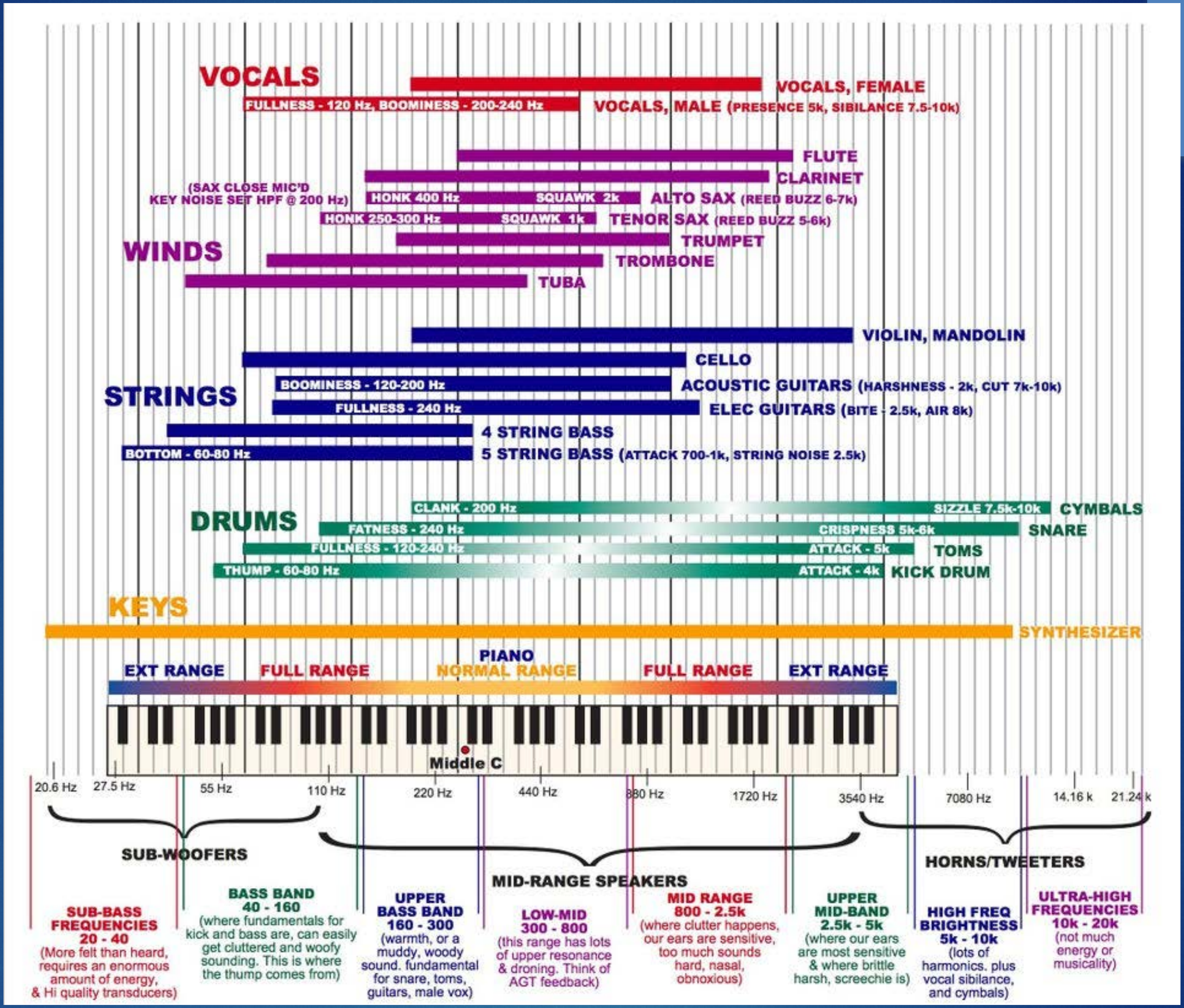
2 a : a continuous sequence or range

// a wide *spectrum* of interests

// opposite ends of the political *spectrum*

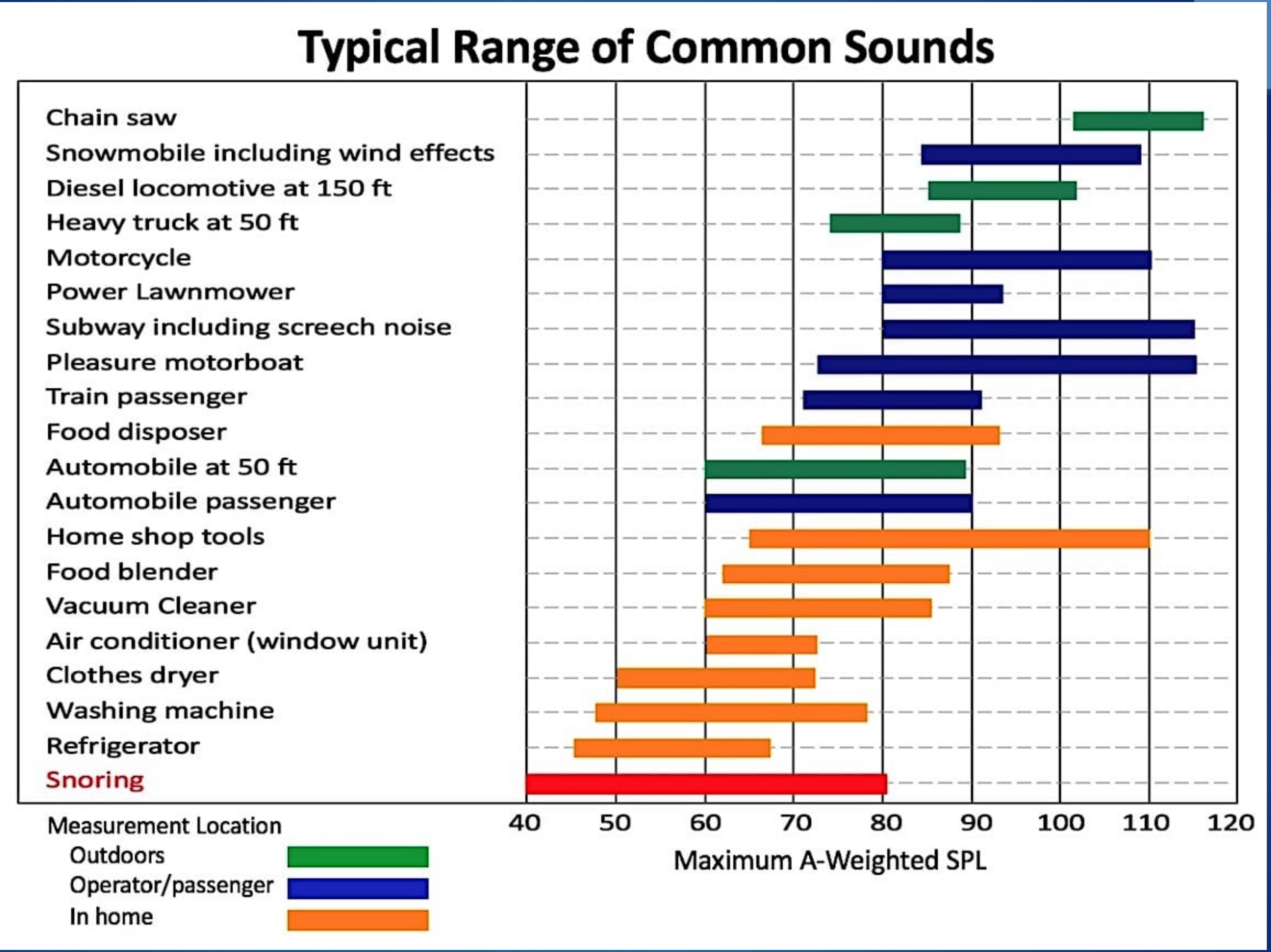
General usage

Technical usage (plot of power/ energy/magnitude vs. frequency)



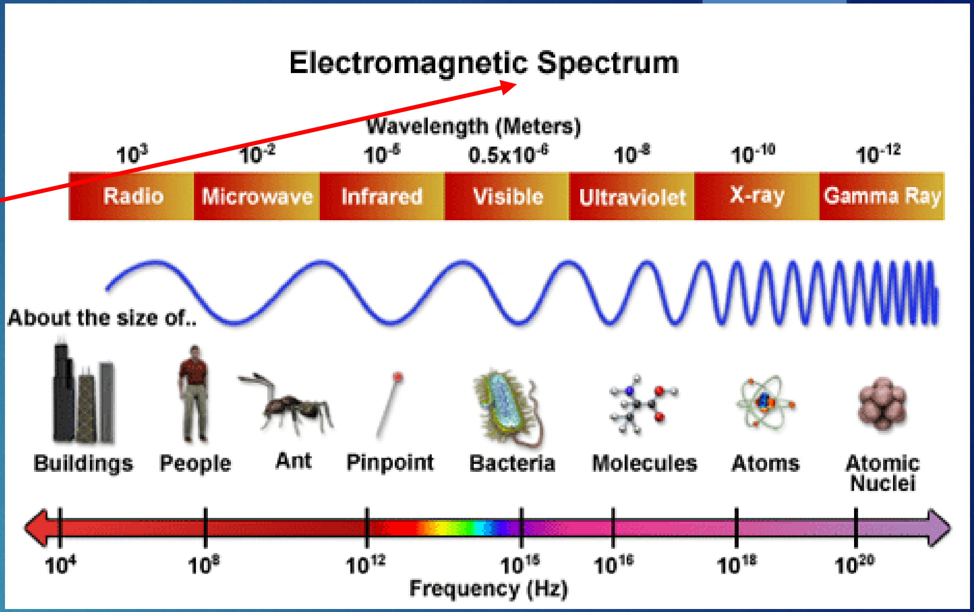
Frequency spectral range is not P/E/A spectrum – it is only the "frequency-axis range" of a P/E/A spectrum

(how loud various things can be)

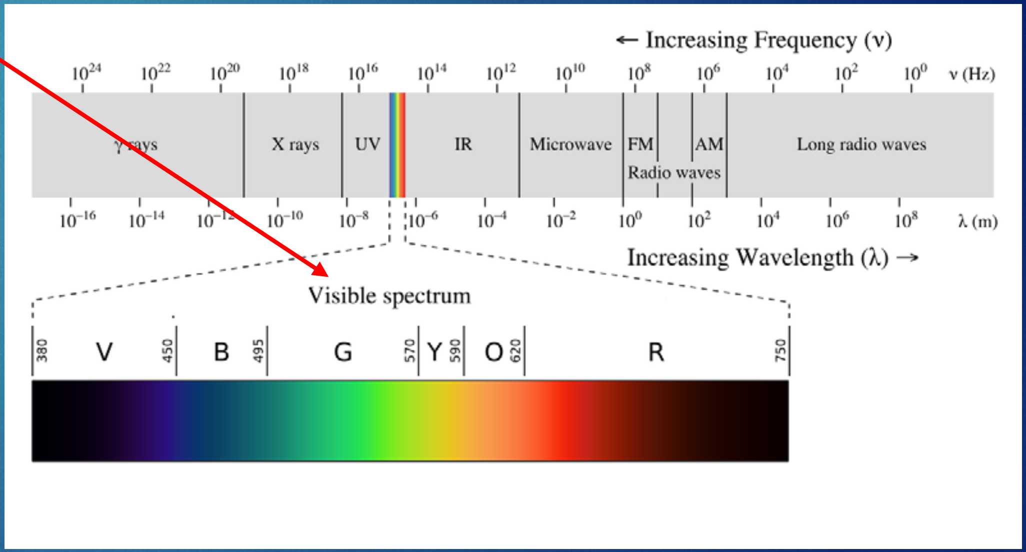


Amplitude range is NOT spectrum

This really means the spectral range of EM waves



This really means the human-eye visible spectral range



This is the reason why we use a more specific and accurate expression: “power/energy/ magnitude spectral density,” and “spectrum” for short.

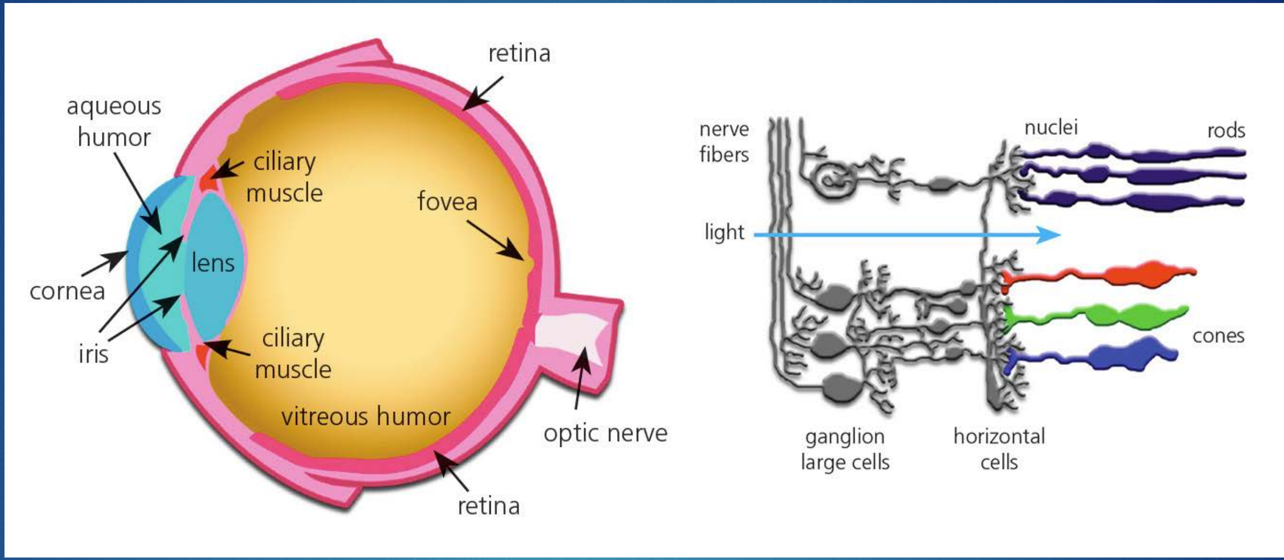
It’s OK to mix general language and technical usage of word “spectrum” as long as we know what it means.

Example: *What is the solar spectrum*? In the technical context, it means solar irradiance spectral density, and not a light wavelength range from far-IR to UV.

How to obtain the spectrum of a signal?

- Use an analog spectrum analyzer (filters, heterodyne detectors, frequency-dispersive devices, … that can separate frequencies)
- If the signal is digitized, do numerical Fourier transform (aka discrete Fourier transform).

Example of analog spectrum analyzer we all have



Awfully low spectral resolution, but it works great!

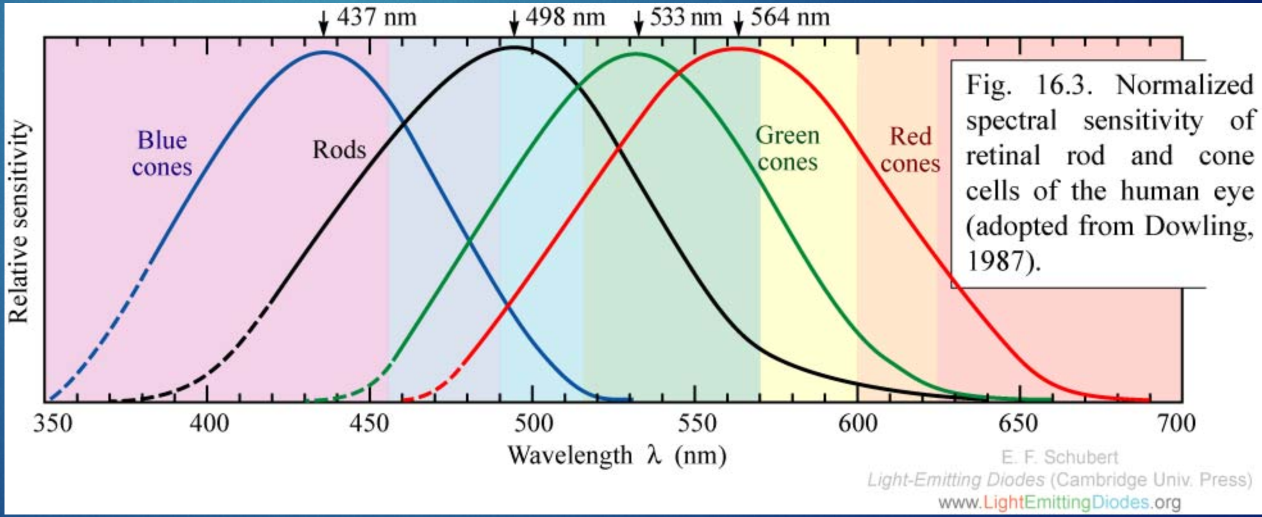
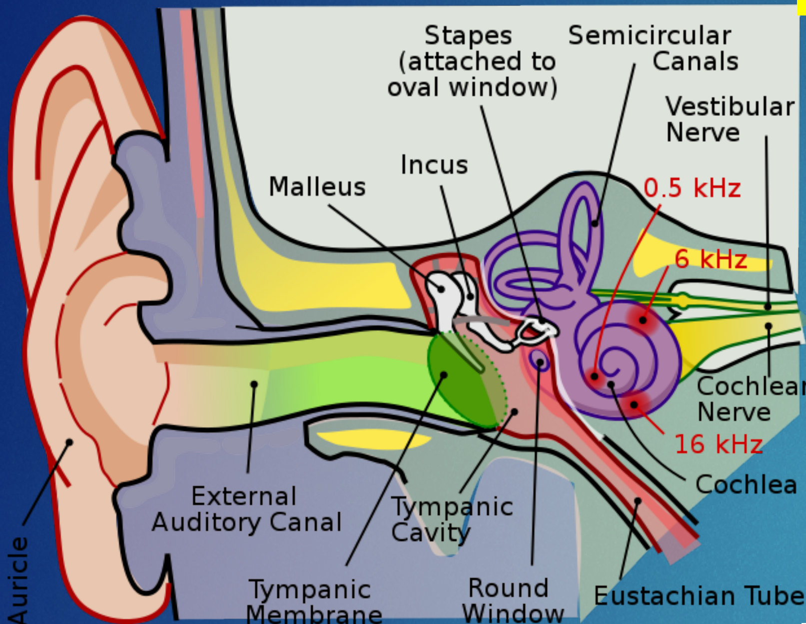
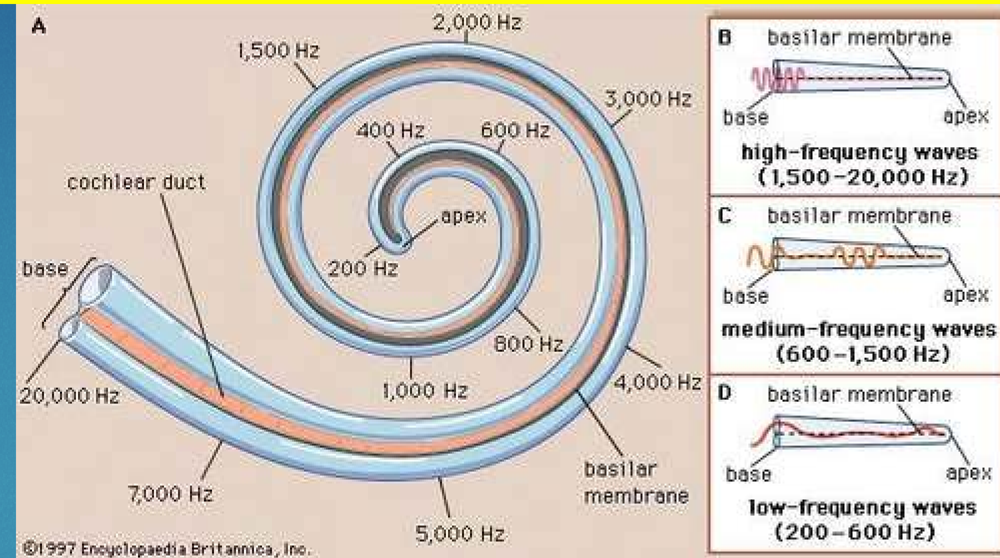


Fig. 16.3. Normalized spectral sensitivity of retinal rod and cone cells of the human eye (adopted from Dowling, 1987).

Example of analog spectrum analyzer we all have



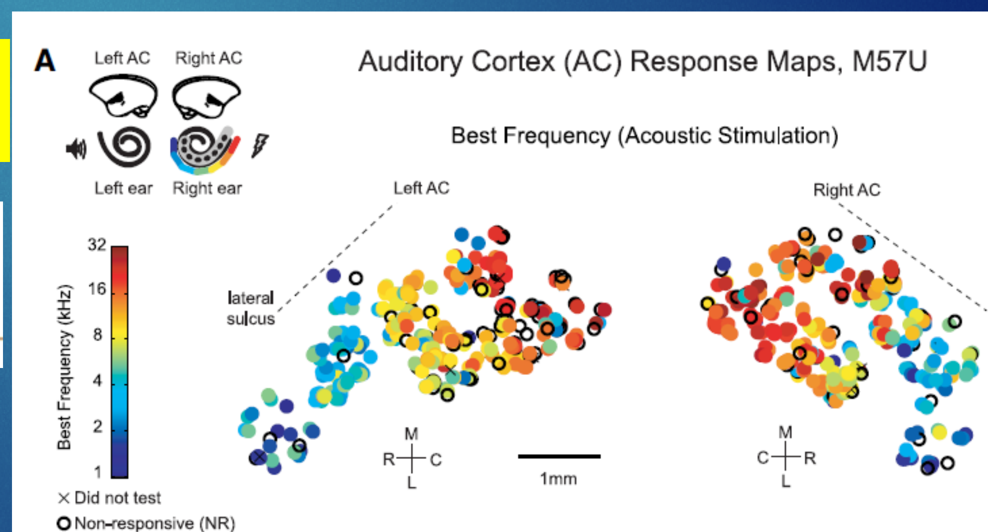
The cochlea maps sensing neurons to log of frequency.



Log of frequency is mapped ~ linearly to the auditory cortex


Selective Neuronal Activation by Cochlear Implant Stimulation in Auditory Cortex of Awake Primate

Luke A. Johnson,[1] Charles C. Della Santina,[1,2] and Xiaoqin Wang[1]
Departments of [1]Biomedical Engineering and [2]Otolaryngology-Head and Neck Surgery, Johns Hopkins University School of Medicine, Baltimore, Marylan- 21025



How to obtain the spectrum of a signal?

- Use an analog spectral analyzer (filters, heterodyne detectors, frequency-dispersive devices that can separate frequencies)
- If the signal is digitized, do numerical Fourier transform (aka discrete Fourier transform).

This is what we do in this class

We define the power-spectral-density (PSD) function of a signal s(t) as:

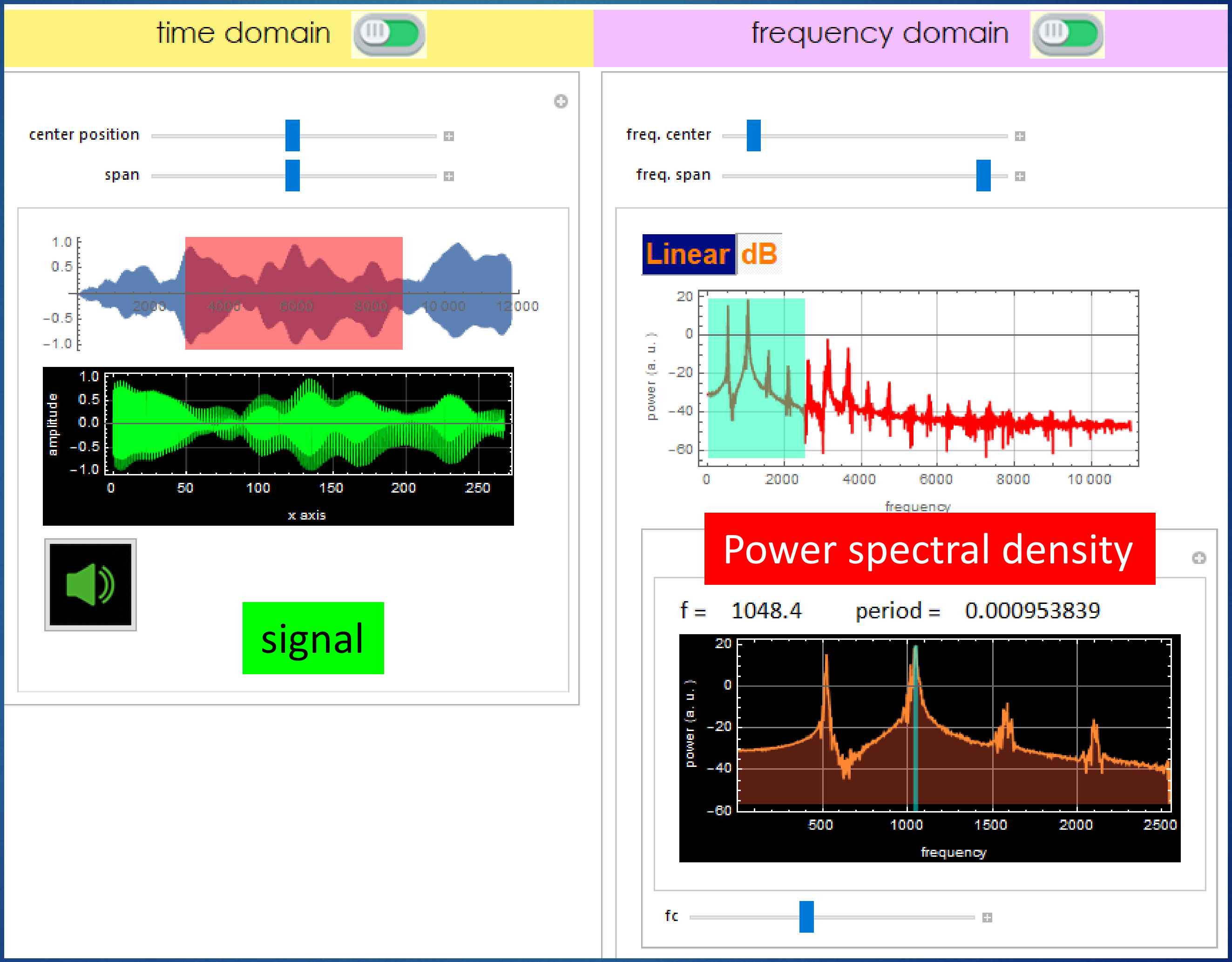
$$P(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_0^T s(t)\, e^{i\,2\,\pi\, f\, t}\, dt \right|^2 \qquad \left(\frac{s\text{−unit}^2}{\text{Hz}}\right)$$

If it is an energy signal, i. e. finite in time, then the ESD is:

$$E(f) = \left| \int_{-\infty}^{\infty} s(t)\, e^{i\,2\,\pi\, f\, t}\, dt \right|^2 \qquad \left(\frac{s\text{−unit}^2 \times \text{sec}}{\text{Hz}}\right)$$

Power or energy here is not the real physical power or energy; they are used only as analogous concepts to the real ones.

Now, we are ready to try some exercises (open and run the app)



Conceptual relationship between spectrum and spectral response:

- spectrum of a signal coming from a source
- spectral response of an object given a stimulus

To be continued to lecture part 2