ECE3340

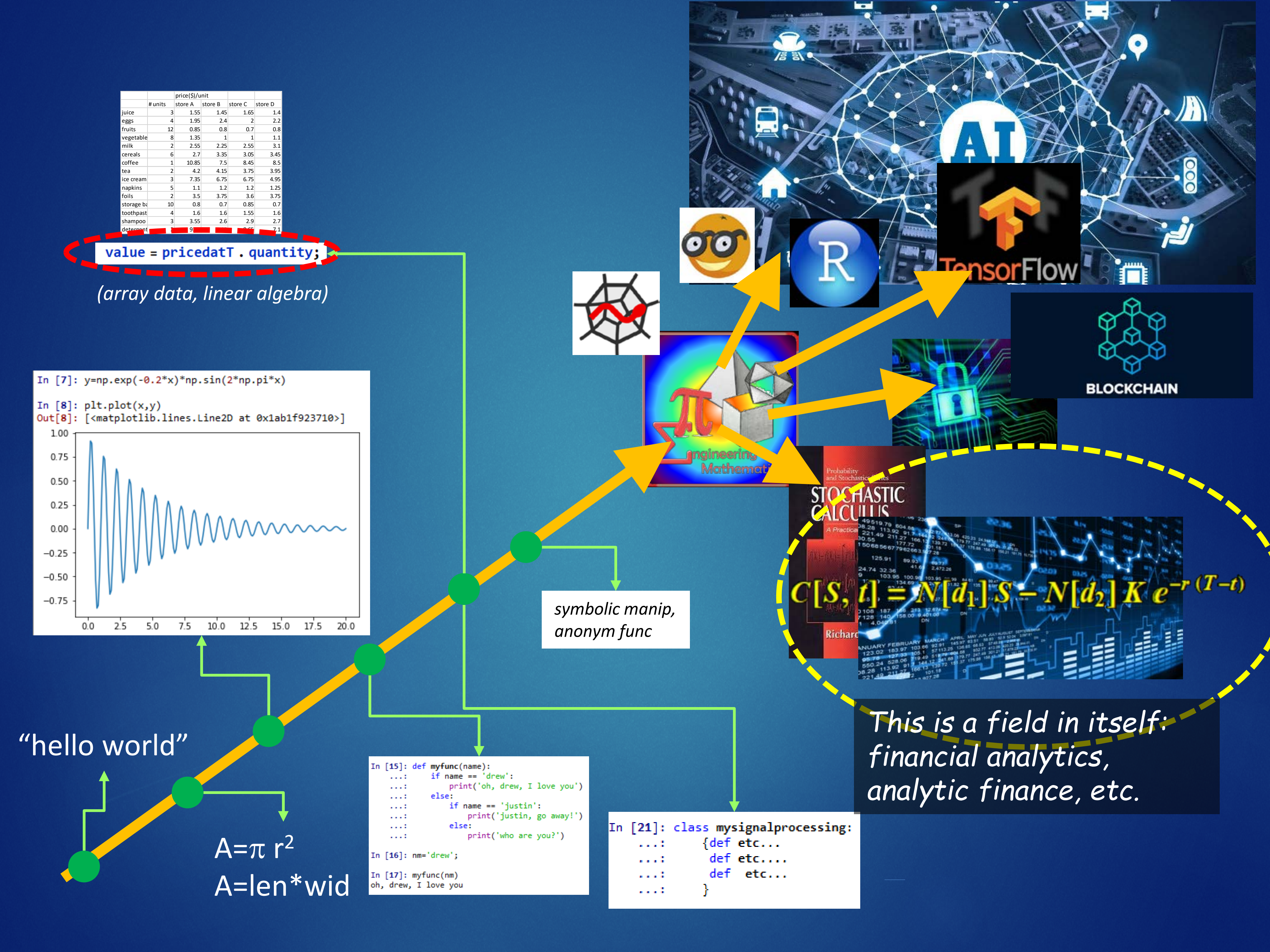
Introduction to Stochastic Processes and Numerical Methods

PROF. HAN Q. LE

Note: PPT file contains only the main outline of the chapter topics – Use associated Mathematica file(s) that contain details and assignments for in-depth learning

Overview

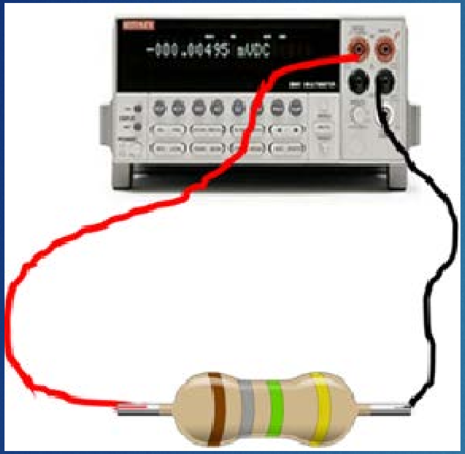
- Concepts introduction: noises and random events
- Descriptive statistics, probability
  - Distribution functions, probability density func (pdf), cumulative density func (cdf). Multivariate.
  - Examples and applications
- Numerical simulations and Monte Carlo
- Bayes' theorem and intro to Bayesian decision theory
  - Review cluster classifications, ROC concepts, Kalman's filter
- Introduction to stochastic calculus (*if have time*)
  - Ito calculus & finance applications: Black-Scholes model

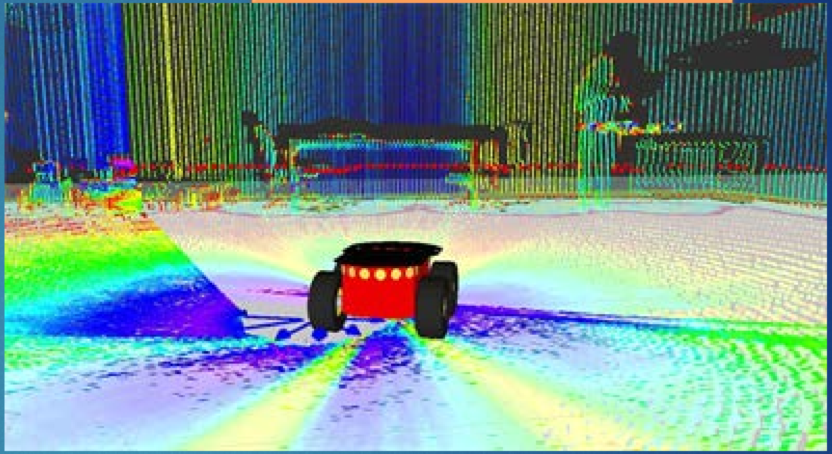


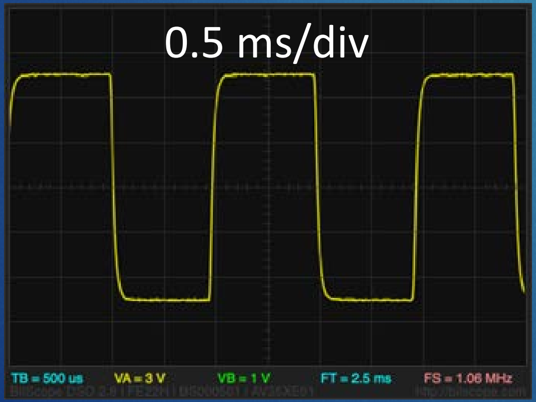
		price($)/unit			
	# units	store A	store B	store C	store D
juice	3	1.55	1.45	1.65	1.4
eggs	4	1.95	2.4	2	2.2
fruits	12	0.85	0.8	0.7	0.8
vegetable	8	1.35	1	1	1.1
milk	2	2.55	2.25	2.55	3.1
cereals	6	2.7	3.35	3.05	3.45
coffee	1	10.85	7.5	8.45	8.5
tea	2	4.2	4.15	3.75	3.95
ice cream	3	7.35	6.75	6.75	4.95
napkins	5	1.1	1.2	1.2	1.25
foils	2	3.5	3.75	3.6	3.75
storage ba	10	0.8	0.7	0.85	0.7
toothpast	4	1.6	1.6	1.55	1.6
shampoo	3	3.55	2.6	2.9	2.7
detergent	[illegible]	[illegible]	[illegible]	[illegible]	7.1

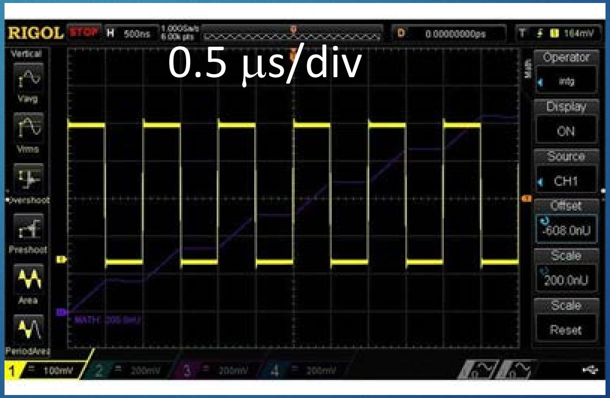
link to Mathematica lecture file

- Electronic noises









link to Mathematica lecture file

- For example, which of these two statements, **a** or **b** in each case below i

Case 1

a- According to our design, after pushing the start button, at time t=5 minutes, our drone will be at {x,y,z}={15 meters, 20 meters, 6 meters} or approximately somewhere pretty close there.

b- we can predict with 90% confidence that the position of our robot at time t=5 minutes to be within 0.1 m in every dimension of {x,y,z}={15 meters, 20 meters, 6 meters} (alternatively, we can say: {15 meters, 20 meters, 6 meters} ± 0.1 m)

- Another example, case 2:

a- We expect to have approximately 12,500 customers to visit our store on Black Friday.

b- We predict with 90% confidence that the number of shoppers on Black Friday to be 12,500 ± 600 people.

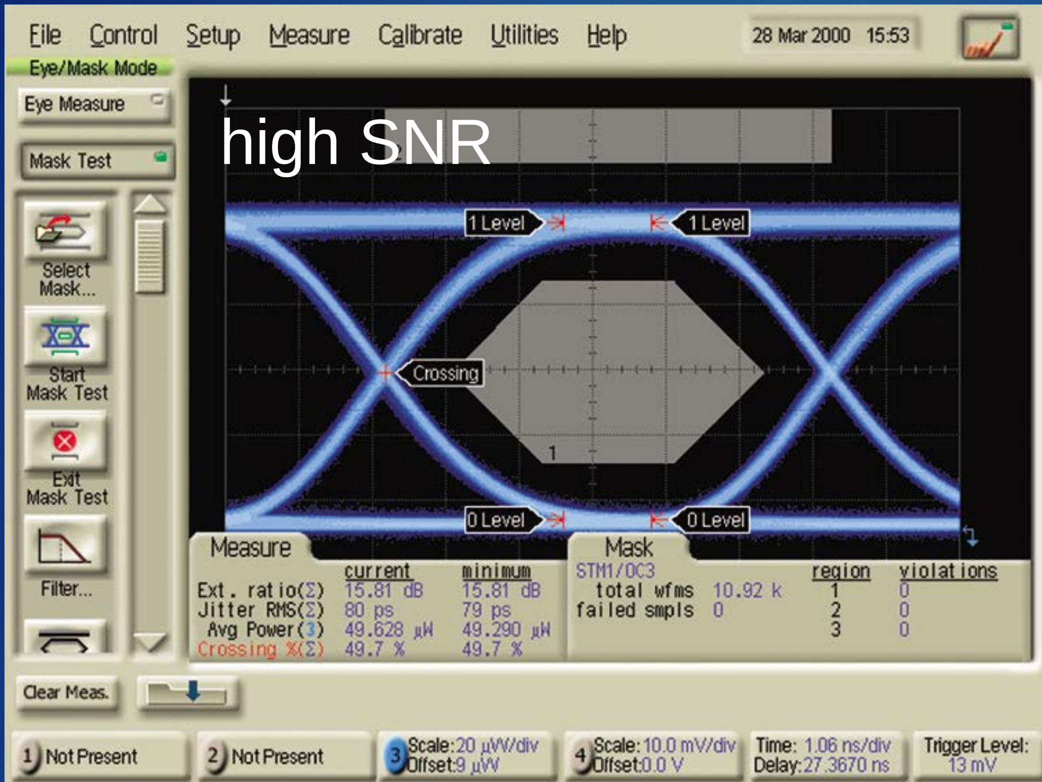
- Another example, case 3:

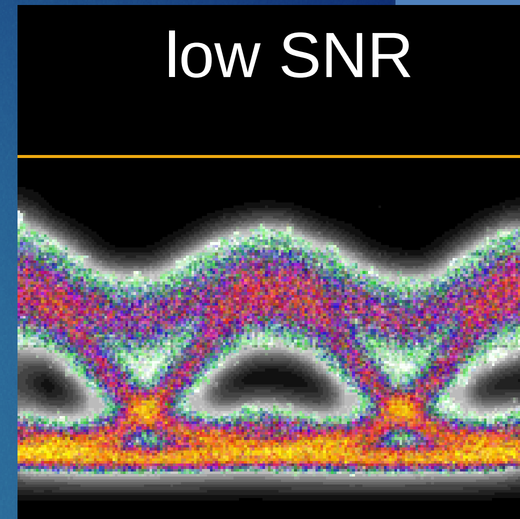
a- Our super quantum computer can predict that the market will crash by 20% or more within the next 3 months with a confidence of... 3.1415926535%.

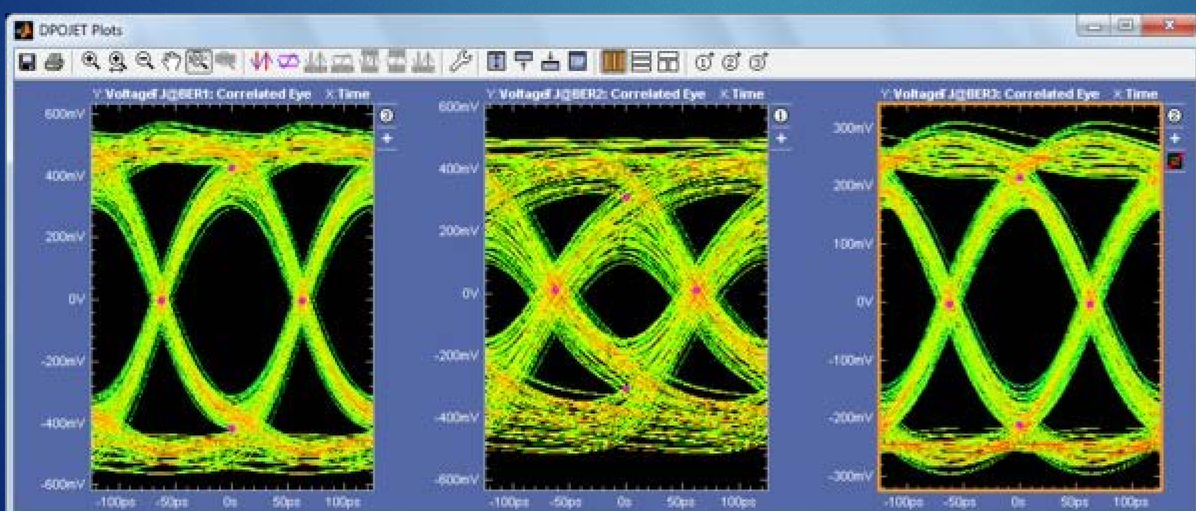
b- We predict that the market can go up if the economy improves and the Fed holds the rate steady; but the market can crash if the World crisis gets worse. On the other, the market can also stay the same as it is if everything stays the same.

Note: **b** statement is no joke. Market prognosticators say that every single day on TV.


And GOD Said
I can't hear you. Bad signal to noise ratio

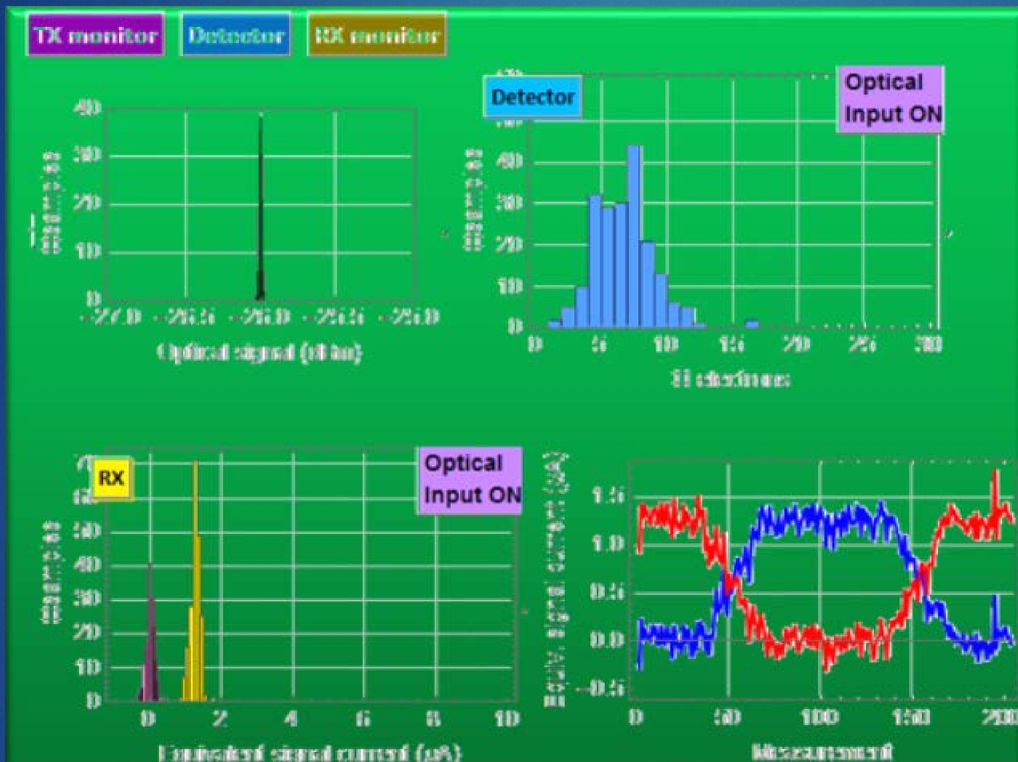

File Control Setup Measure Calibrate Utilities Help
28 Mar 2000 15:53
Eye/Mask Mode
Eye Measure
Mask Test
Select Mask...
Start Mask Test
Exit Mask Test
Filter...
high SNR
1 Level
1 Level
Crossing
1
0 Level
0 Level
Measure
current minimum
Ext. ratio(Σ) 15.81 dB 15.81 dB
Jitter RMS(Σ) 80 ps 79 ps
Avg Power(3) 49.628 µW 49.290 µW
Crossing %(Σ) 49.7 % 49.7 %
Mask
STM1/OC3
total wfms 10.92 k
failed smpls 0
region violations
1 0
2 0
3 0
Clear Meas.
1 Not Present
2 Not Present
3 Scale: 20 µW/div Offset: 9 µW
4 Scale: 10.0 mV/div Offset: 0.0 V
Time: 1.06 ns/div Delay: 27.3670 ns
Trigger Level: 13 mV


low SNR


DPOJET Plots
Voltage J@BER1: Correlated Eye X Time
Voltage J@BER2: Correlated Eye X Time
Voltage J@BER3: Correlated Eye X Time
600mV
400mV
200mV
0V
-200mV
-400mV
-600mV
300mV
200mV
100mV
0V
-100mV
-200mV
-300mV
-100ps -50ps 0s 50ps 100ps

Effects of noises on communication system

Example of Monte Carlo simulation of a telecom link



We'll go through this in the later part of the chapter

Topics

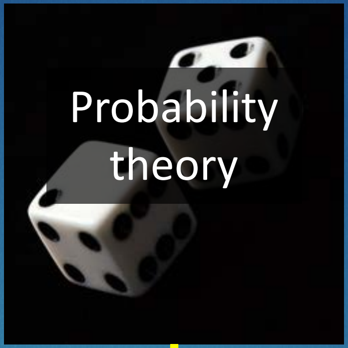
note: main lecture notes in Mathematica .nb file

- Concepts introduction: noises and random events
- Descriptive statistics, probability
  - Distribution functions, probability density func (pdf), cumulative density func (cdf). Multivariate.
  - Examples and applications
- Numerical simulations and Monte Carlo

part 1: these are most important

- Bayes' theorem and intro to Bayesian decision theory
  - Review cluster classifications, ROC concepts, Kalman's filter
- Introduction to stochastic calculus (*if have time*)
  - Ito calculus & finance applications: Black-Scholes model

part 2: how much we cover will depend on time



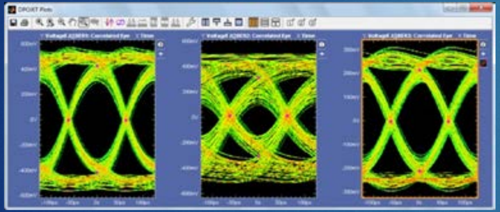
scientific engineering

statistics data science

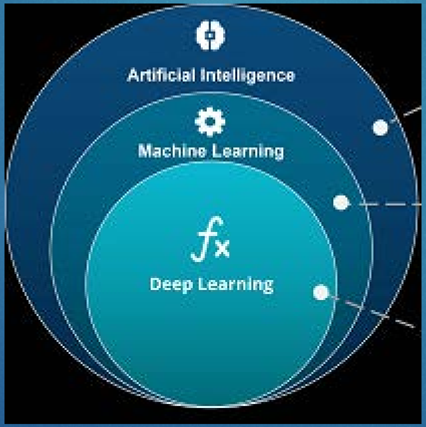
game theory (econ.) stochastic calculus/ finance analytics

cryptography/ information science

stochastic physical processes



signal processing, control communications



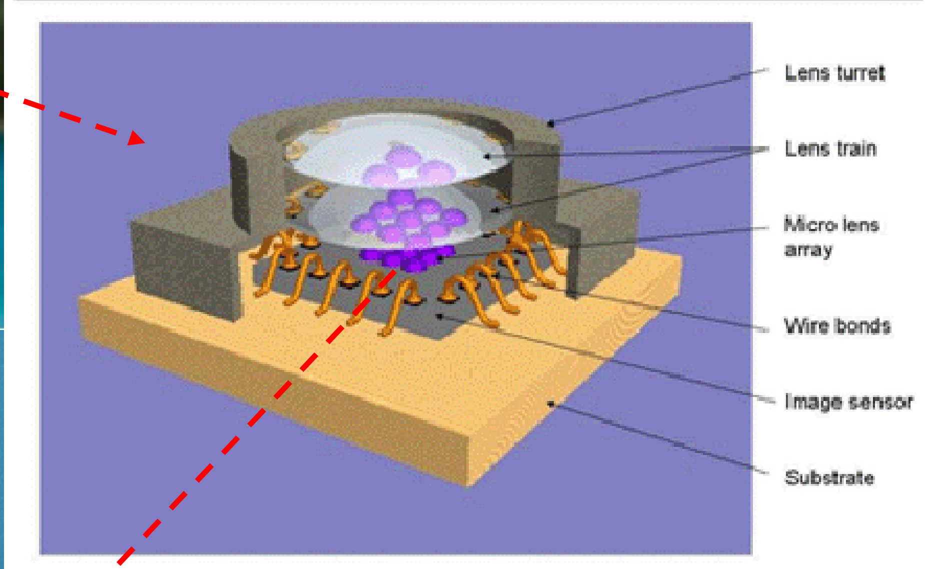
… random events, probability, statistics and all that

It's about "chance"

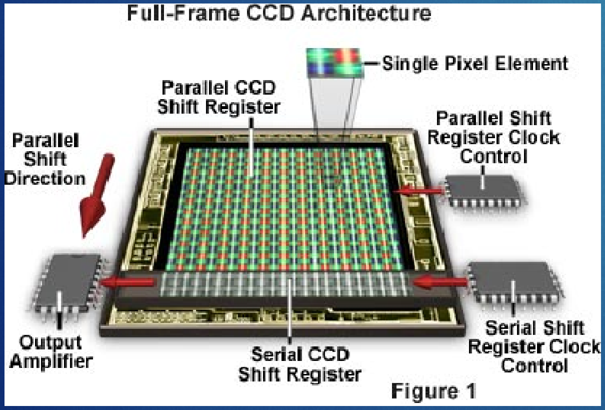
- What is the chance of snake eyes?
- What is the chance of black jack, or straight flush?
- What is the chance of winning the next Powerball?
- What is the chance of getting hit by lightning?
- What is the chance the market will go up or down?
- What is the chance of shower/snow/windy tomorrow?
- …

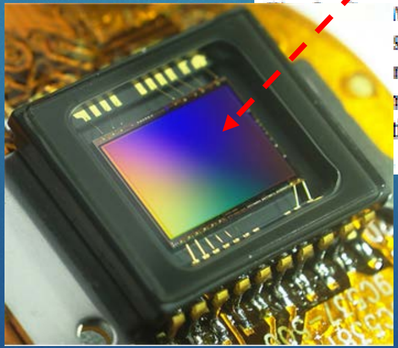
The following are guidance to follow the lecture in Mathematica .nb file – link will not work and you must download the file

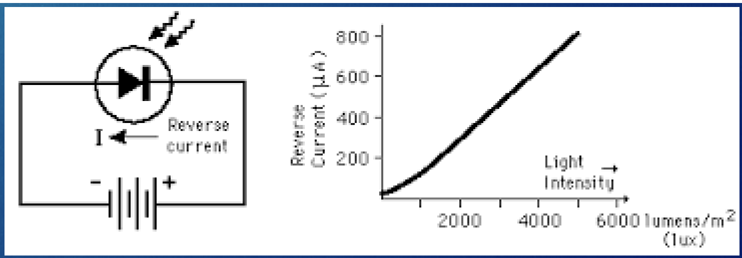
Illustration of discrete random events

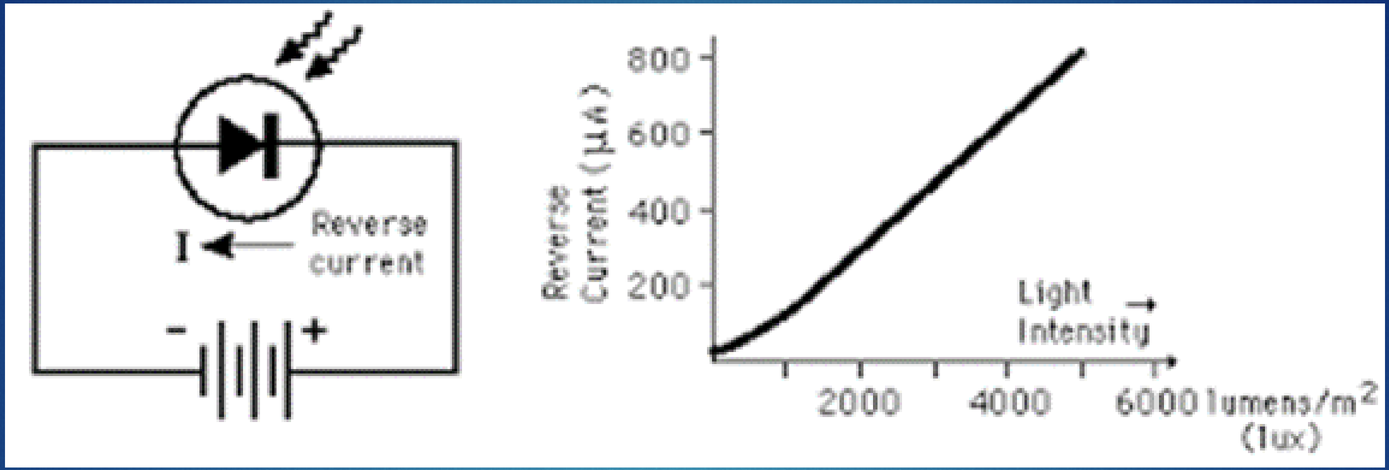


way drawing of a solid state camera module. The imager die is mounted on strate and connected to it by wire bonds. Over the imager is placed the lens mounted in a turret. The turret is held in the correct location with respect to mager by the lens housing, which attaches to the substrate. The optically tive area of the imager is covered with an array of micro lenses, one per Source: Tessera.

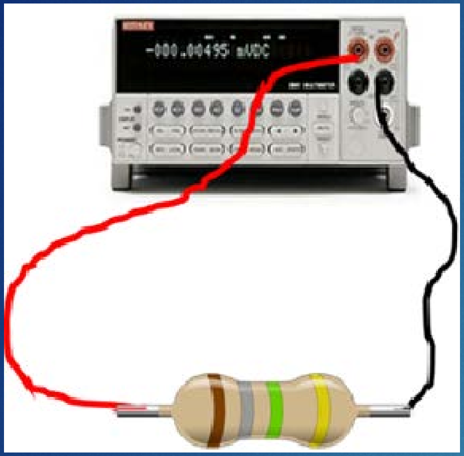


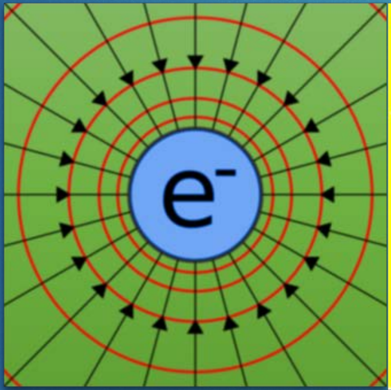


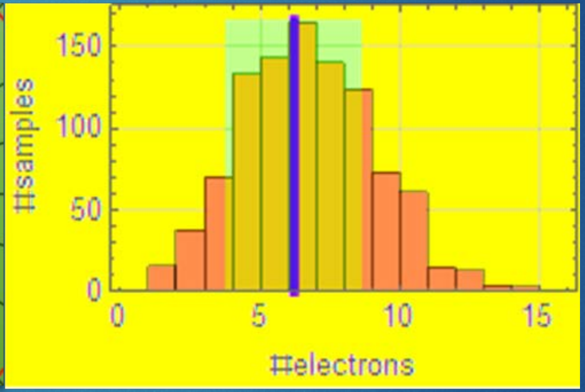




Let's say light power is constant, the expected current is 10 nA.
We measure current at a rate 10 GHz, i. e. time interval=0.1 ns
(we make many measurements, 10 billions measurement per sec).
what do we expect the values of the current?


-000.00495 mVDC


e-


#samples
150
100
50
0
0
5
10
15
#electrons

https://www.youtube.com/watch?v=IdTxGJjA4Jw

3.6 Shot noise calculation

What is the implication of the discovery of the electron?

The average number of electrons per measurement is:

$$n_{ave} = i_0 \Delta t / e$$

where e is the charge. The number of electrons per unit interval obeys Poisson distribution:

$$P[n] = \frac{n_{ave}{}^{n}}{n!} e^{-n_{ave}}$$

Exercise: calculate in class work

What is the noise fluctuation?

$$\sigma_{sn} = \sqrt{n_{ave}} \frac{e}{\Delta t}$$. Notice that:

$$\sigma_{sn}{}^{2} = \frac{n_{ave} e^2}{\Delta t^2} = i_0 \frac{e}{\Delta t} = i_0 e 2 B$$

Or: $$\overline{i^2_{noise}} = 2 e i_0 B$$

(remember that $1/\Delta t = 2B$ Nyquist); then PSD of this distribution (number of electrons in Δt interval) is:

$$\frac{\sigma_{sn}{}^{2}}{2 B} = \frac{i_0 e 2 B}{2 B} = i_0 e$$

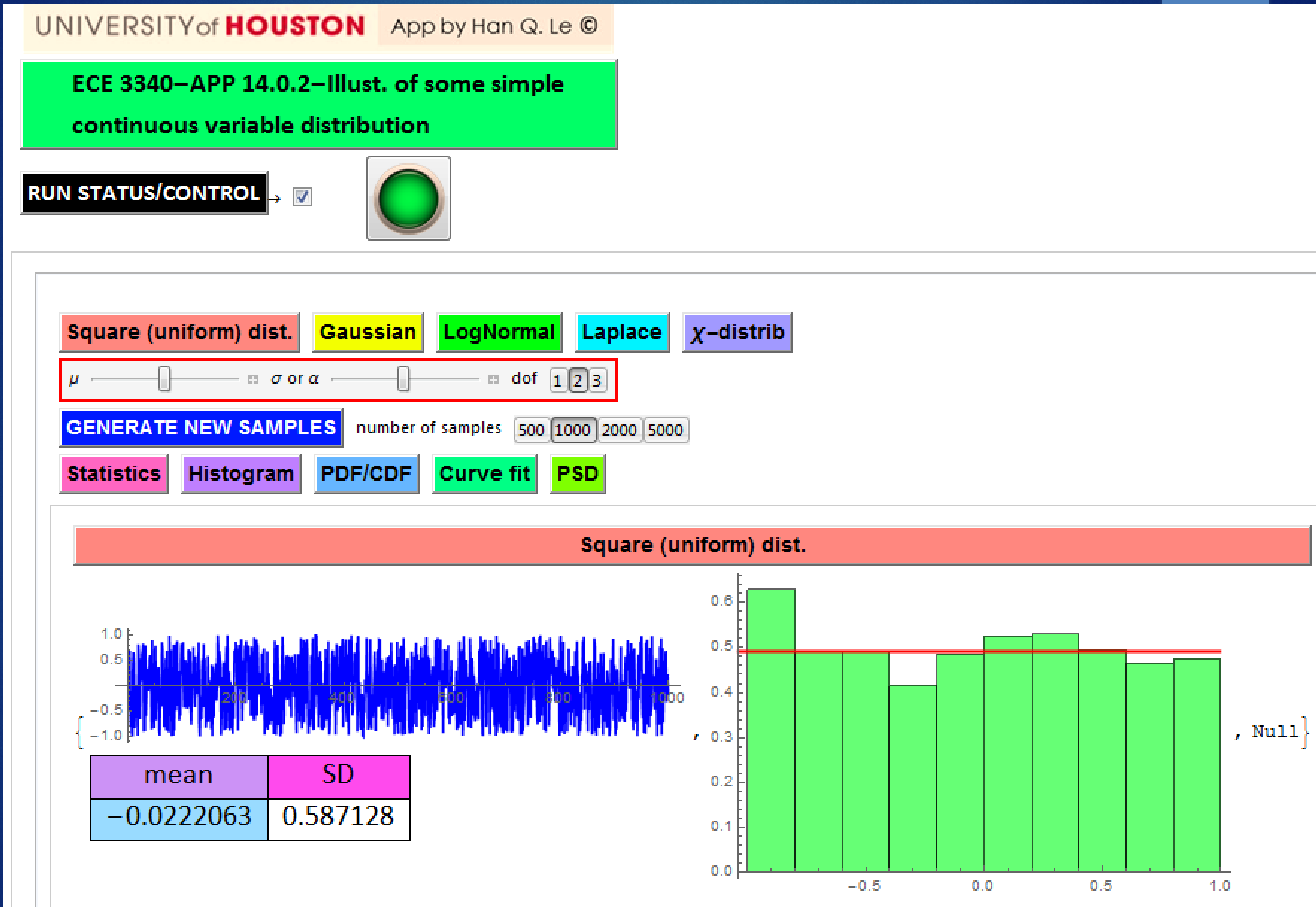
Statistical measure concepts

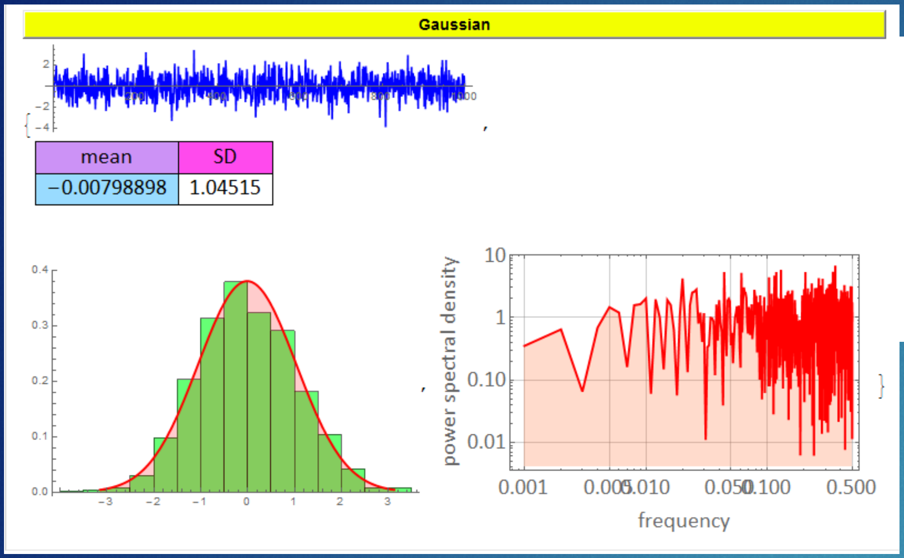
Mean
Standard of deviation

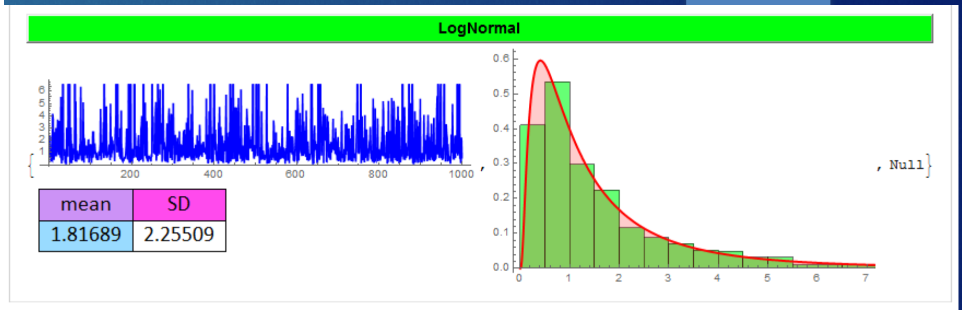
Histogram
Probability density function
Cumulative density function

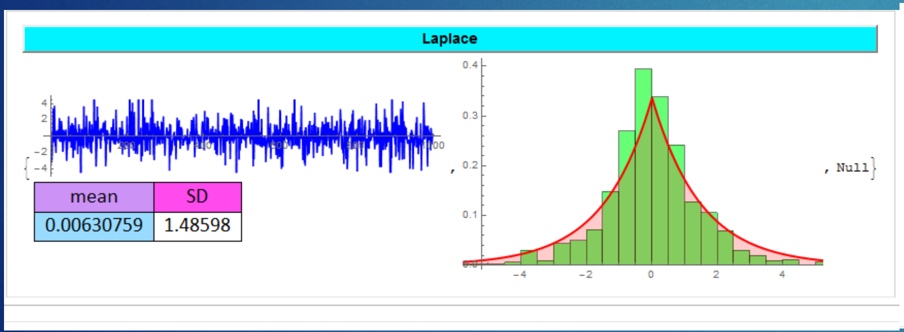
Power spectral density of sample population

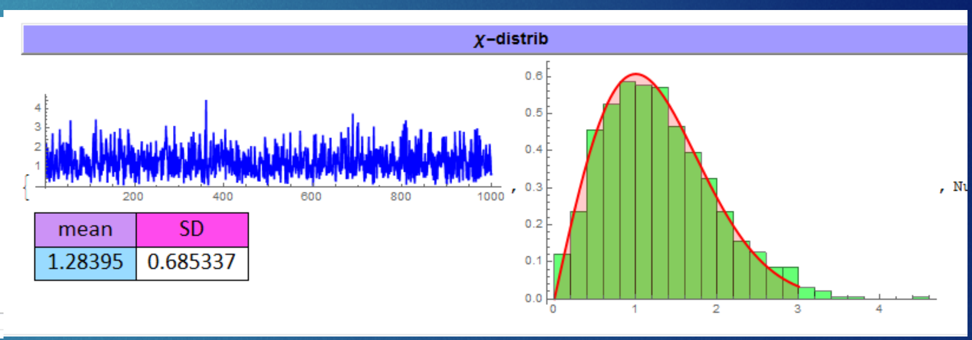
Please go through all examples in the APP


UNIVERSITY of HOUSTON
App by Han Q. Le ©
ECE 3340–APP 14.0.2–Illust. of some simple continuous variable distribution
RUN STATUS/CONTROL
Square (uniform) dist.
Gaussian
LogNormal
Laplace
χ–distrib
μ
σ or α
dof 1 2 3
GENERATE NEW SAMPLES
number of samples 500 1000 2000 5000
Statistics
Histogram
PDF/CDF
Curve fit
PSD
Square (uniform) dist.
mean
SD
−0.0222063
0.587128
, Null

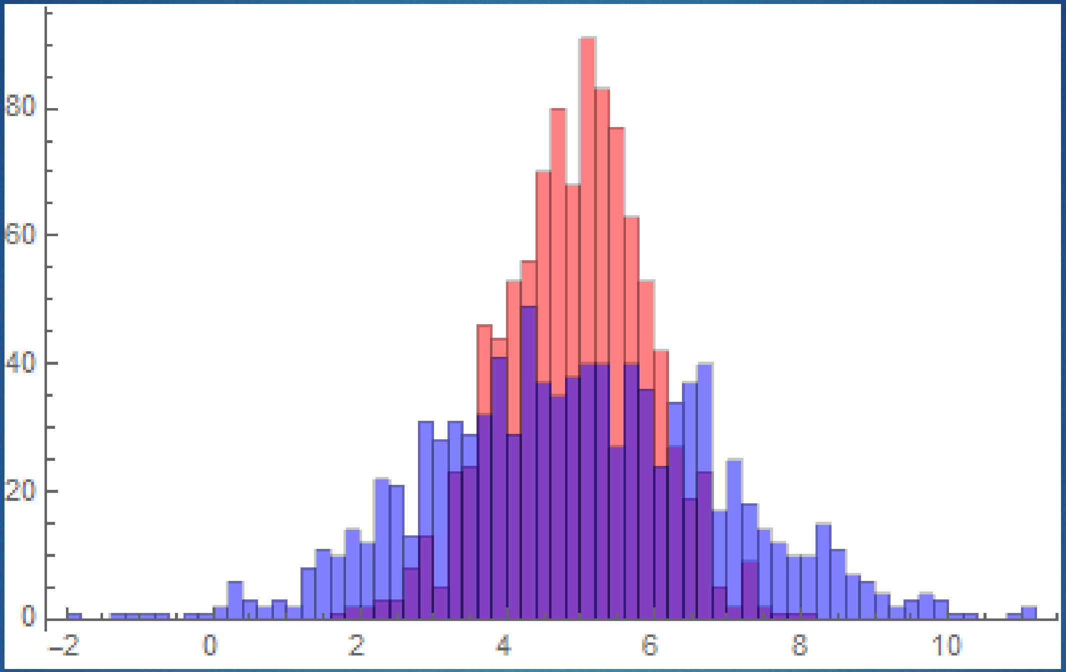

Gaussian
mean | SD
−0.00798898 | 1.04515
power spectral density
frequency


LogNormal
mean | SD
1.81689 | 2.25509
, Null

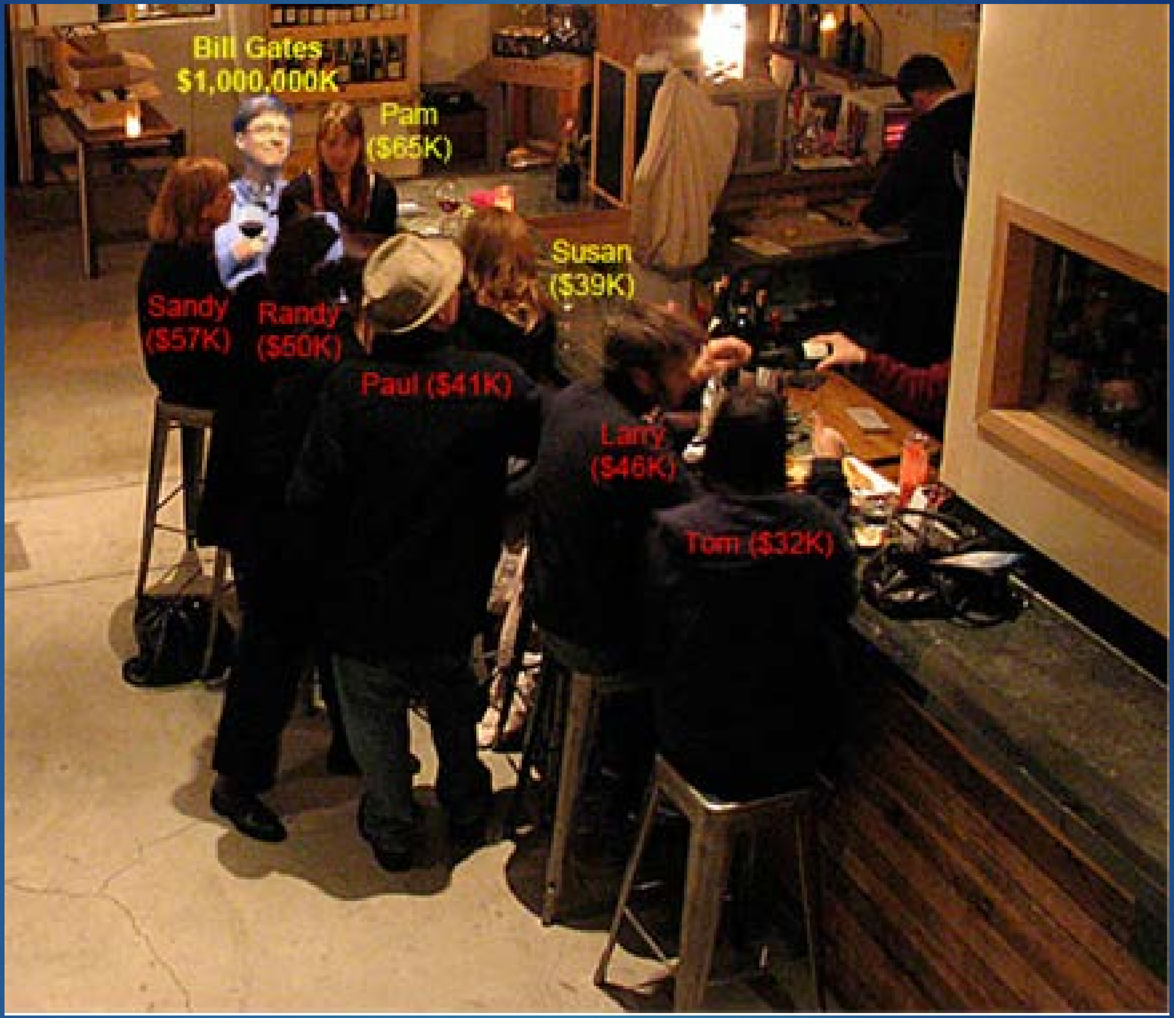

Laplace
mean | SD
0.00630759 | 1.48598
, Null


χ−distrib
mean | SD
1.28395 | 0.685337

Populations with different variance (or standard of deviation)

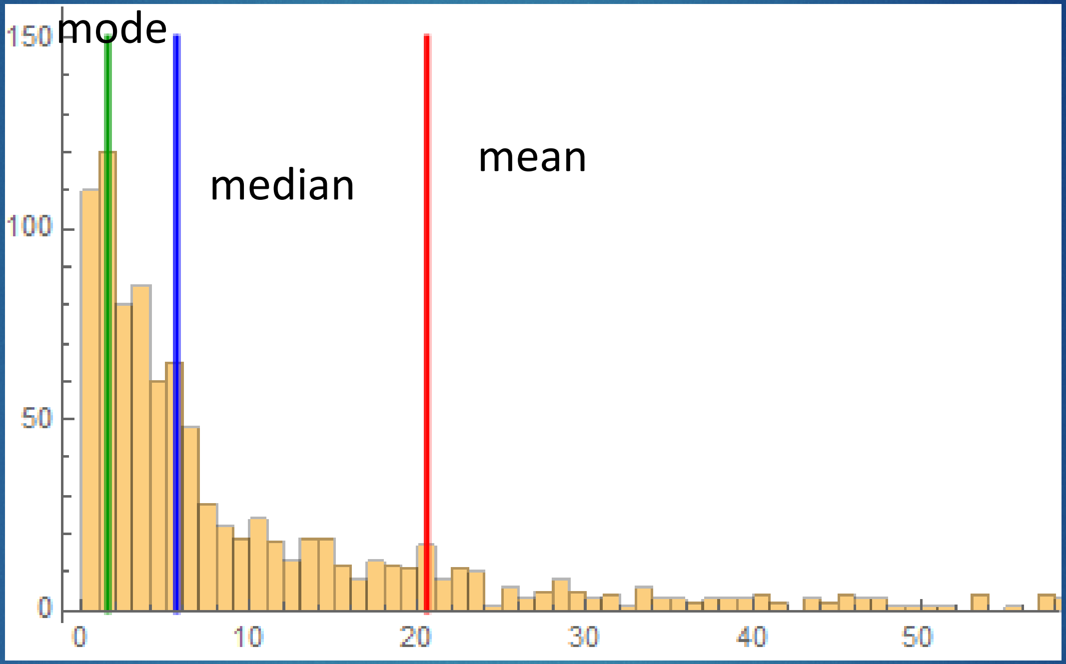


Suppose you measure the same physical quantity with two instruments. You make several measurements. One instrument gives you the red distribution, the other, blue. What can you conclude about the instrument and how do you compare them quantitatively?

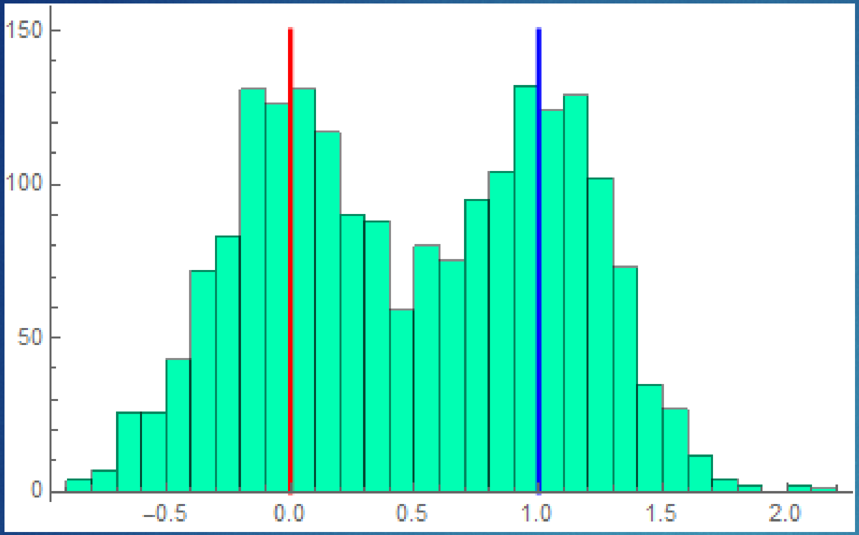


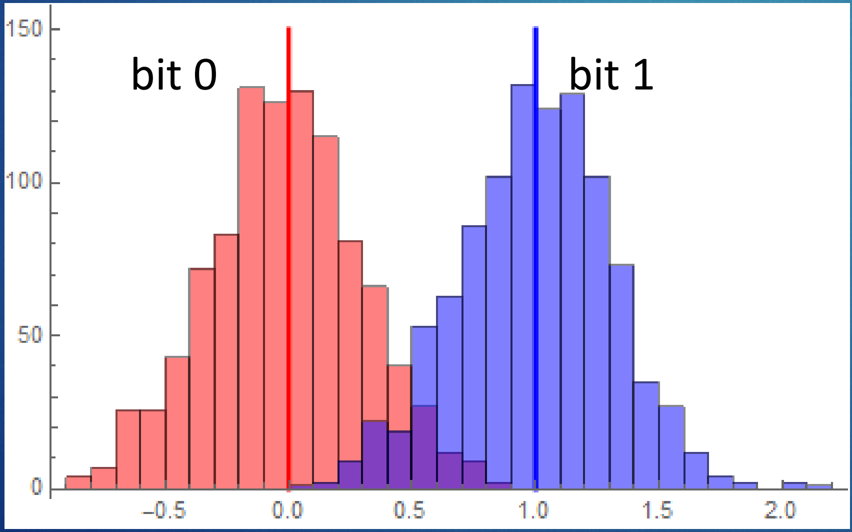
Bill Gates walked into a bar, and the average income of everyone in the bar is $multi-millions....

LogNormal distribution



example of bimodal distribution





You have the data in green. The origin of the data is in blue and red, but of course you don't know which data points come from blue and which from red.
How do you split your green data into bit 1 and bit 0 and how do you estimate the error probability?

Numerical methods and illustrations of statistical measures

- Using computer-generated random variables
- Simulation of random processes based on known distribution
- Essential for Monte Carlo simulation that will be discussed later
- Specific computer exercises

Some common discrete variable distribution function

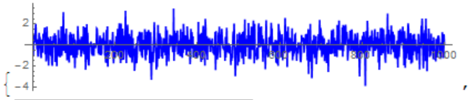
- Bernoulli
- Binomial distribution
- Poisson distribution
- Application of Poisson distribution in ECE: shot noise

To be continued in part 2

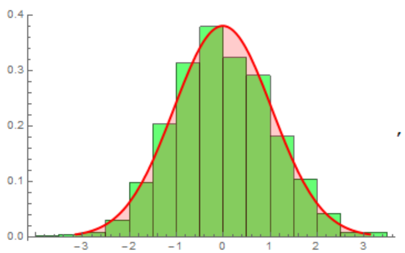
Some common continuous variable distribution function

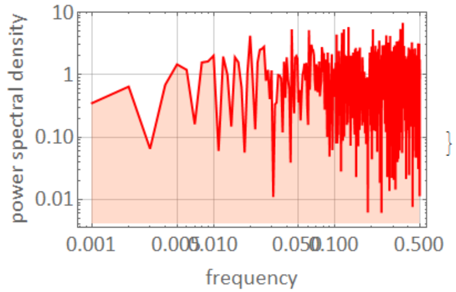
- Uniform (for computer simulation)
- Normal (Gaussian) distribution
- LogNormal distribution
- Laplace distribution
- χ-distribution

Gaussian

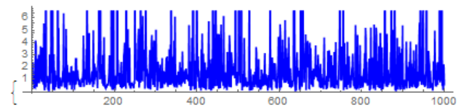


mean	SD
−0.00798898	1.04515

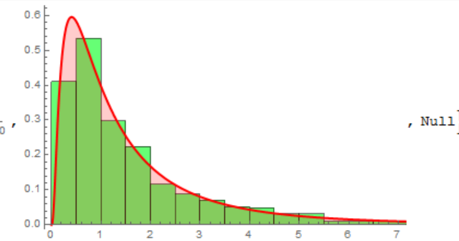




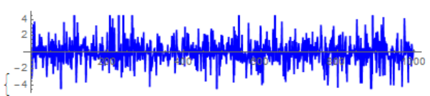
LogNormal



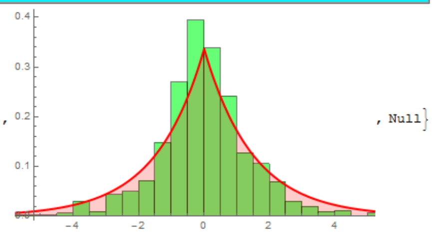
mean	SD
1.81689	2.25509



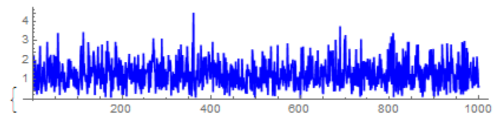
Laplace



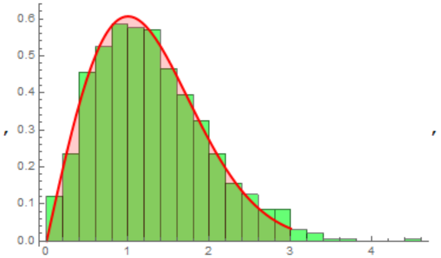
mean	SD
0.00630759	1.48598



χ−distrib



mean	SD
1.28395	0.685337

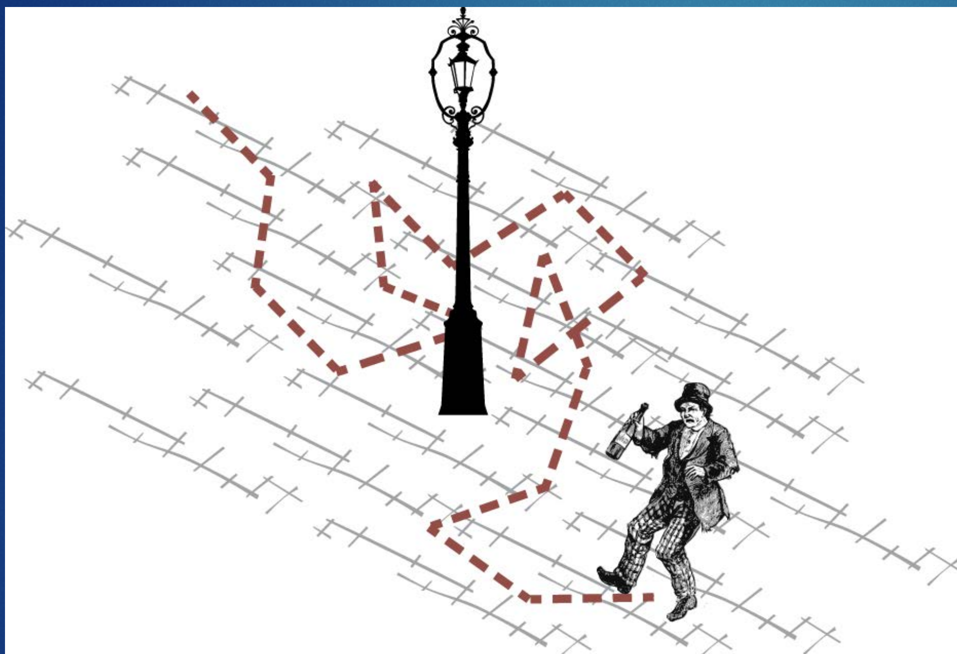


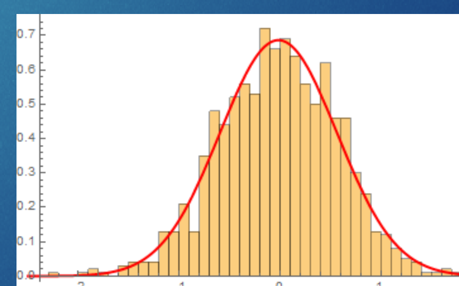
Normal distribution

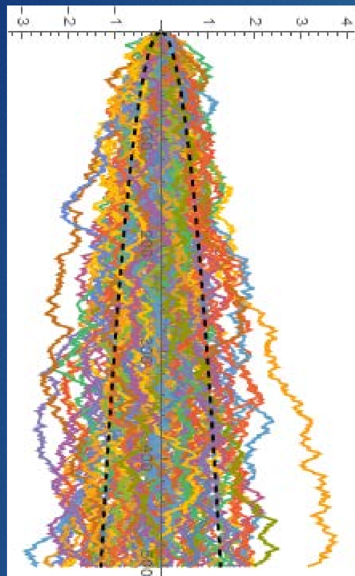
- Central limit theorem
- Commonly used for additive white gaussian noise (AWGN) simulation
- Sometimes, outlier data problem

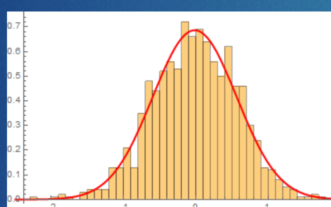
In class numerical exercise on operations of normal distribution variables

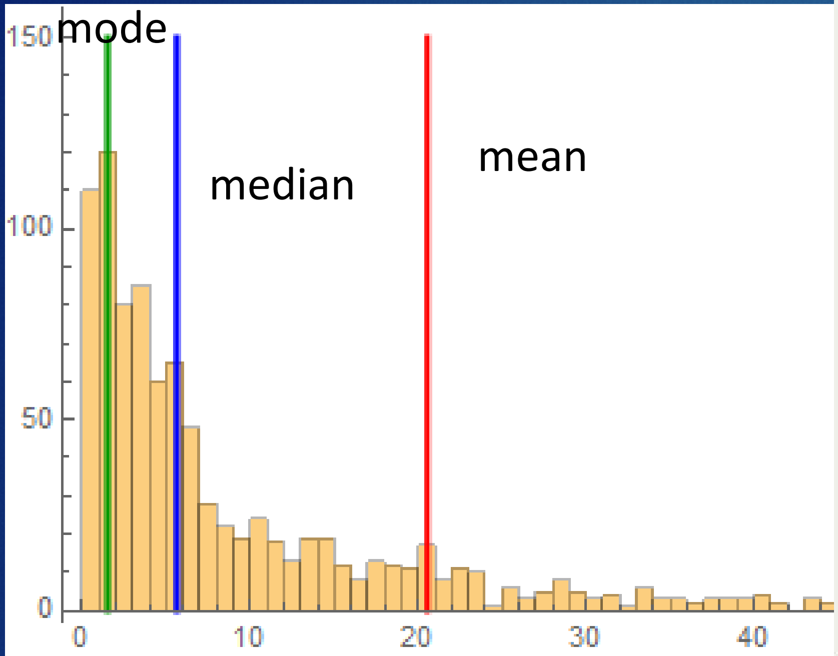
Classwork on drunkard random walk
and Brownian motion



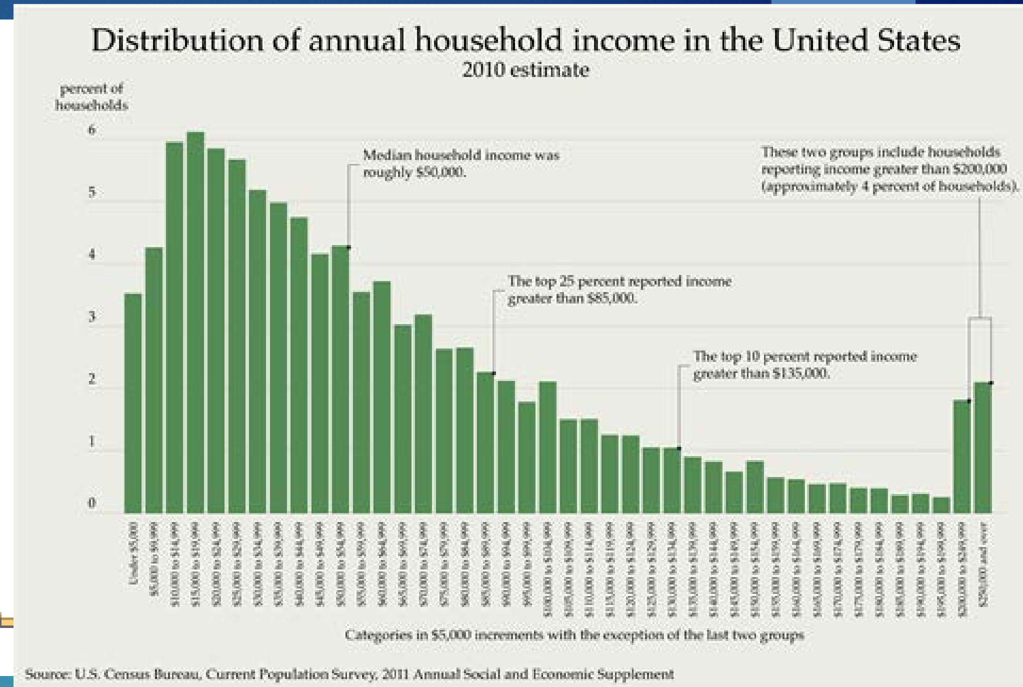




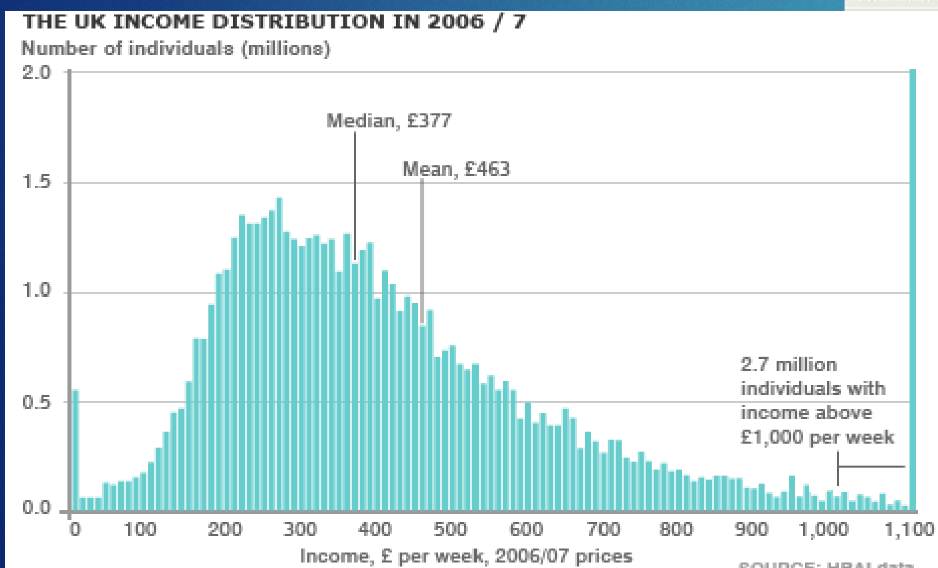




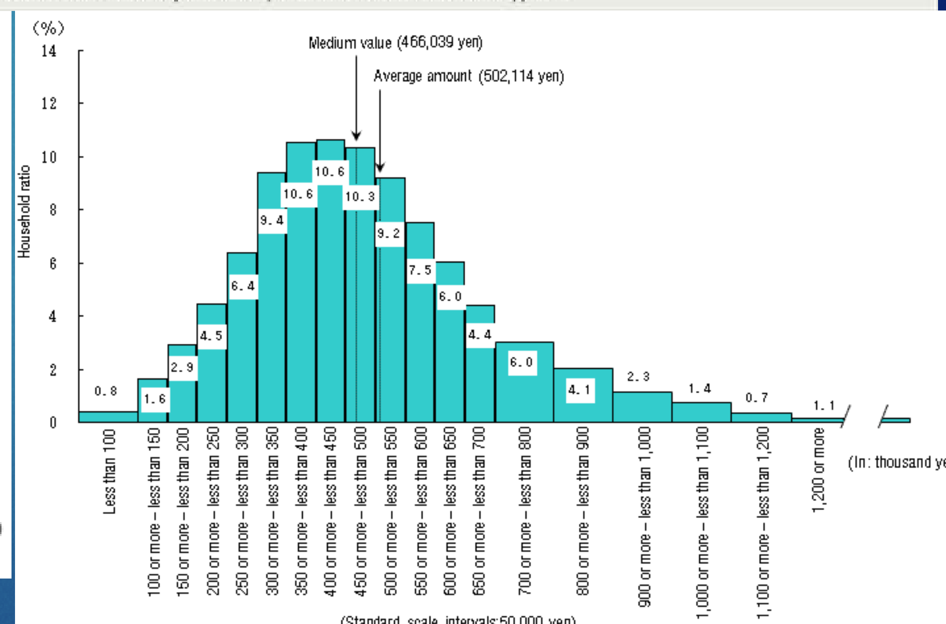
mode
median
mean
150
100
50
0
0
10
20
30
40



Distribution of annual household income in the United States
2010 estimate
percent of households
Median household income was roughly $50,000.
These two groups include households reporting income greater than $200,000 (approximately 4 percent of households).
The top 25 percent reported income greater than $85,000.
The top 10 percent reported income greater than $135,000.
Categories in $5,000 increments with the exception of the last two groups
Source: U.S. Census Bureau, Current Population Survey, 2011 Annual Social and Economic Supplement

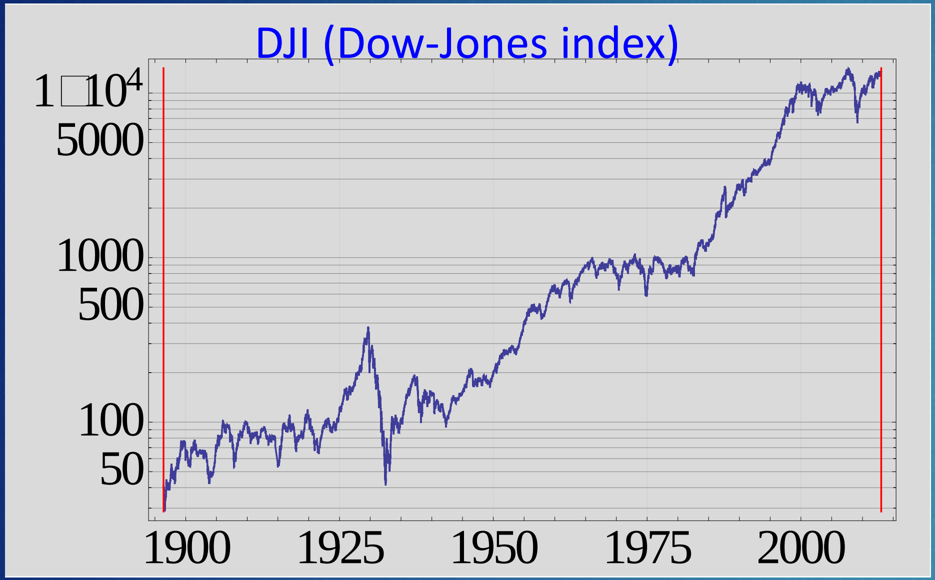


THE UK INCOME DISTRIBUTION IN 2006 / 7
Number of individuals (millions)
Median, £377
Mean, £463
2.7 million individuals with income above £1,000 per week
Income, £ per week, 2006/07 prices
SOURCE: HBAI data

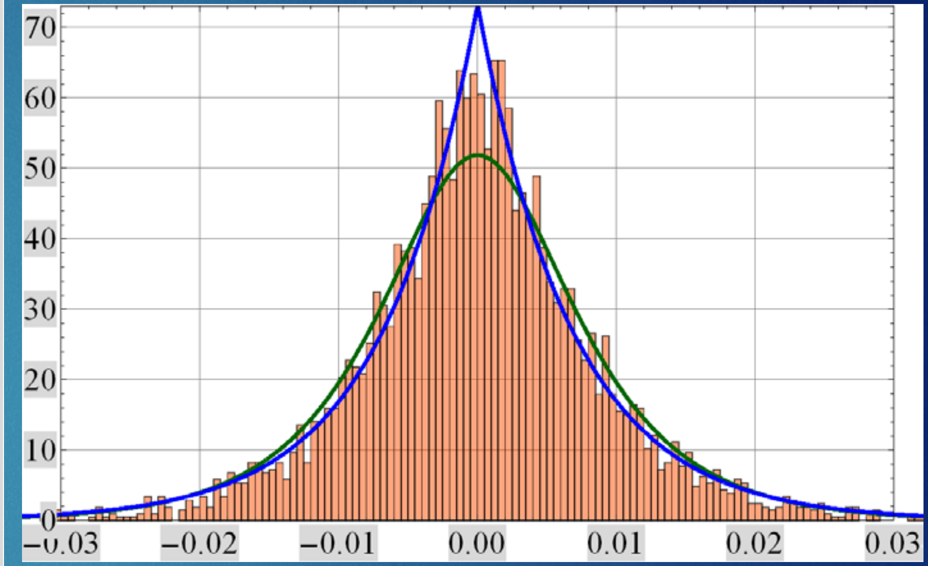


(%)
Medium value (466,039 yen)
Average amount (502,114 yen)
Household ratio
Less than 100
100 or more – less than 150
150 or more – less than 200
200 or more – less than 250
250 or more – less than 300
300 or more – less than 350
350 or more – less than 400
400 or more – less than 450
450 or more – less than 500
500 or more – less than 550
550 or more – less than 600
600 or more – less than 650
650 or more – less than 700
700 or more – less than 800
800 or more – less than 900
900 or more – less than 1,000
1,000 or more – less than 1,100
1,100 or more – less than 1,200
1,200 or more
(In: thousand yen)
(Standard scale intervals:50,000 yen)

Laplace distribution



Distribution of market index day-to-day change



Chi, Chi-Square, t-Dist

- A class of distribution functions useful for variance analysis, Euclidean distance between normal variables

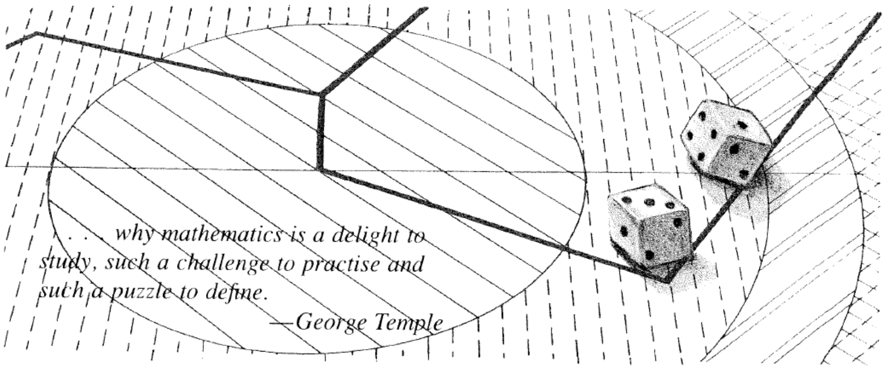
Topics

- Concepts introduction: random events, noises
- Descriptive statistics, probability
- Distribution functions, probability df (pdf), cumulative df (cdf)
- Numerical simulation and Monte Carlo
- Bayes' theorem and intro to Bayesian decision theory

A historical review for those who are interested

THE BEGINNING *of the* MONTE CARLO METHOD

by N. Metropolis



The year was 1945. Two earth-shaking events took place: the successful test at Alamogordo and the building of the first electronic computer. Their combined impact was to modify qualitatively the nature of global interactions between Russia and the West. No less perturbative were the changes wrought in all of academic research and in applied science. On a less grand scale these events brought about a renascence of a mathematical technique known to the old guard as statistical sampling; in its new surroundings and owing to its nature, there was no denying its new name of the Monte Carlo method.

This essay attempts to describe the details that led to this renascence and the roles played by the various actors. It is appropriate that it appears in an issue dedicated to Stan Ulam.

Some Background

Most of us have grown so blasé about computer developments and capabilities —even some that are spectacular—that it is difficult to believe or imagine there was a time when we suffered the noisy, painstakingly slow, electromechanical devices that chomped away on punched cards. Their saving grace was that they continued working around the clock, except for maintenance and occasional repair (such as removing a dust particle from a relay gap). But these machines helped enormously with the routine, relatively simple calculations that led to Hiroshima.

The ENIAC. During this wartime period, a team of scientists, engineers, and technicians was working furiously on the first electronic computer—the ENIAC—at the University of Pennsylvania in Philadelphia. Their mentors were Physicist First Class John Mauchly and Brilliant Engineer Presper Eckert. Mauchly, familiar with Geiger counters in physics laboratories, had realized that if electronic circuits could count, then they could do arithmetic and hence solve, *inter alia*, difference equations—at almost incredible speeds! When he'd seen a seemingly limitless array of women cranking out firing tables with desk calculators, he'd been inspired to propose to the Ballistics Research Laboratory at Aberdeen that an electronic computer be built to deal with these calculations.

John von Neumann, Professor of Mathematics at the Institute for Advanced Study, was a consultant to Aberdeen and to Los Alamos. For a whole host of reasons, he had become seriously interested in the thermonuclear problem being spawned at that time in Los Alamos by a friendly fellow-Hungarian scientist, Edward Teller, and his group. Johnny (as he was affectionately called) let it be known that construction of the ENIAC was nearing completion, and he wondered whether Stan Frankel and I would be interested in preparing a preliminary computational model of a thermonuclear reaction for the ENIAC. He felt he could convince the authorities at Aberdeen that our problem could provide a more exhaustive test of the computer than mere firing-table computations. (The designers of the ENIAC had wisely provided for the capability of much more ambitious versions of firing tables than were being arduously computed by hand, not to mention other quite different applications.) Our response to von Neumann's suggestion was enthusiastic, and his heuristic arguments were accepted by the authorities at Aberdeen.

In March, 1945, Johnny, Frankel, and I visited the Moore School of Electrical Engineering at the University of Pennsylvania for an advance glimpse of the ENIAC. We were impressed. Its physical size was overwhelming—some 18,000 double triode vacuum tubes in a system with 500,000 solder joints. No one ever had such a wonderful toy!

The staff was dedicated and enthusiastic; the friendly cooperation is still remembered. The prevailing spirit was akin to that in Los Alamos. What a pity that a war seems necessary to launch such revolutionary scientific endeavors. The components used in the ENIAC were joint-army-navy (JAN) rejects. This fact not only emphasizes the genius of Eckert and Mauchly and their staff, but also suggests that the ENIAC was technically realizable even before we entered the war in December, 1941.

After becoming saturated with indoctrination about the general and detailed structure of the ENIAC, Frankel and I returned to Los Alamos to work on a model that was realistically calculable. (There was a small interlude at Alamogordo!) The war ended before we completed our set of problems, but it was agreed that we continue working. Anthony Turkevich joined the team and contributed substantially to all aspects of the work. Moreover, the uncertainty of the first phase of the postwar Los Alamos period prompted Edward Teller to urge us not only to complete the thermonuclear computations but to document and provide a critical review of the results.

The Spark. The review of the ENIAC results was held in the spring of 1946 at Los Alamos. In addition to Edward Teller, the principals included Enrico Fermi, John von Neumann, and the Director, Norris Bradbury. Stanley Frankel,
Anthony
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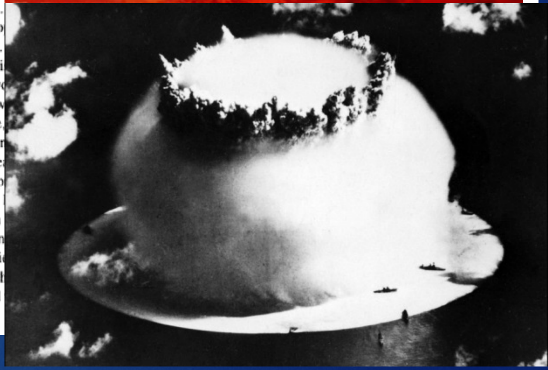


Classwork: random simulation of a simple even-odd game

ODD OR EVEN

yourbet

new round reset

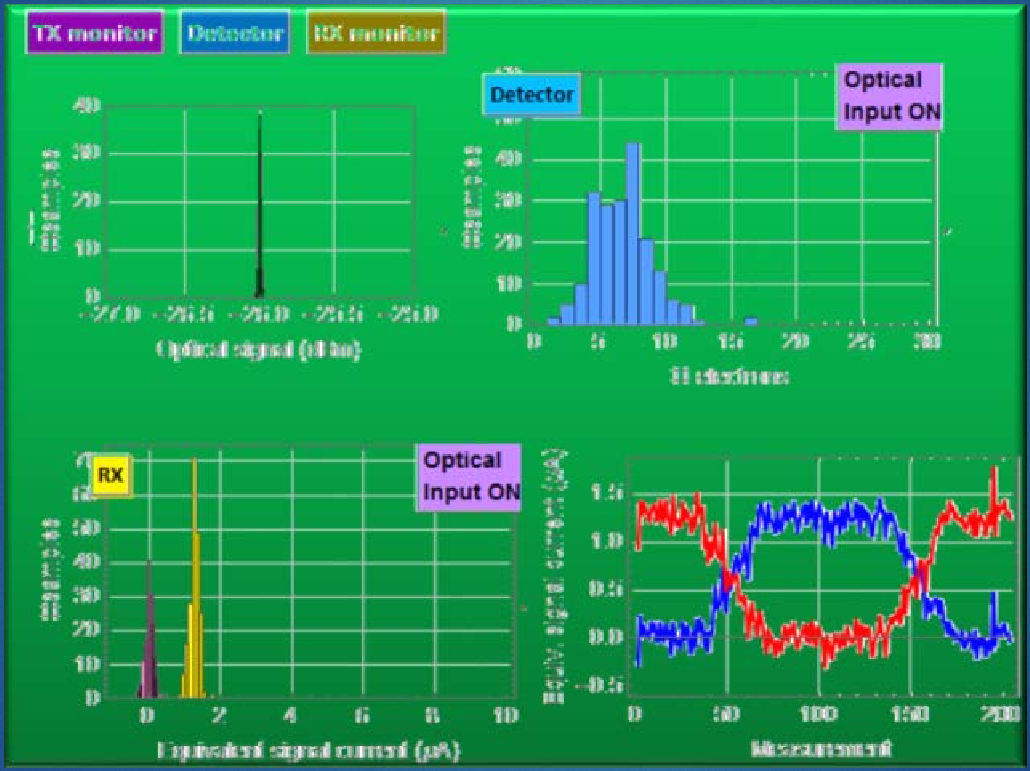
player	2	1	2	1	2	2	2	1	1	2
house	2	1	2	1	2	2	1	1	1	2
w/l	4	2	4	2	4	4	−3	2	2	4

# round	house	player	player accumulate
4	−25	25	25

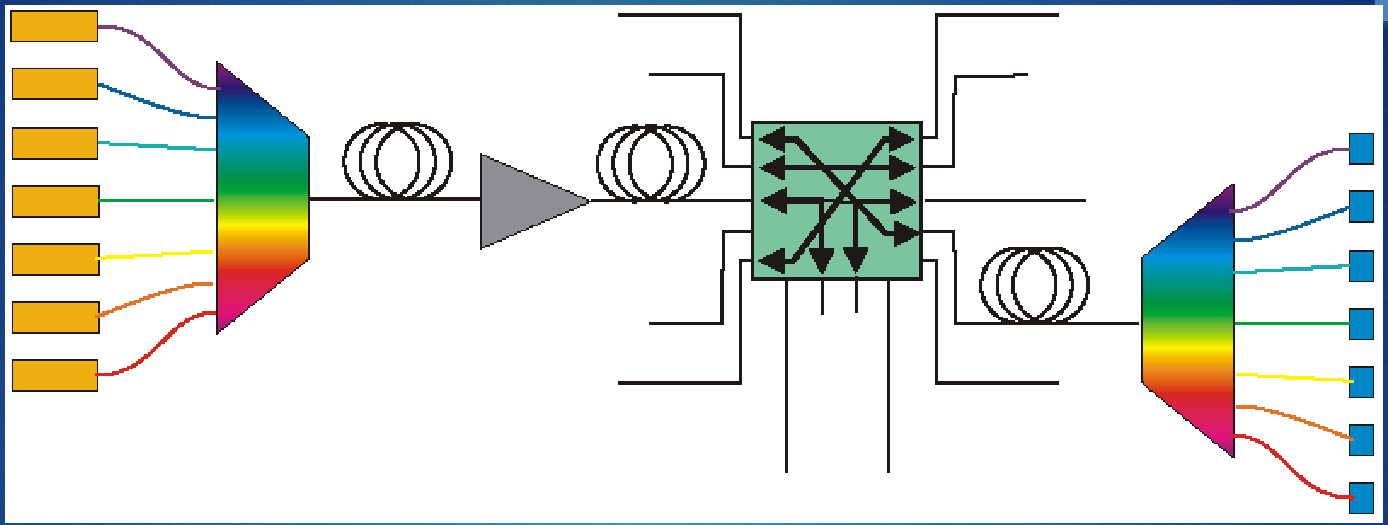
The use of simulation involving random processes to find an answer to some problem.

Example of Monte Carlo simulation

- Basic illustration of an optical communication link



Optical/DWDM networking technology



Transmitter

•Laser
-DFB, DBR, VCSEL
-Tunable, fiber

Modulator
-Electro-optic
-Electroab-sorption

WDMux

•TF filters
•Fiber Bragg G
•Array wave-guide grating
•Diffraction G
•Other gratings

Fiber

•Convent. fiber
•DSF, NZDSF
•Improved fiber

Optical amplifier

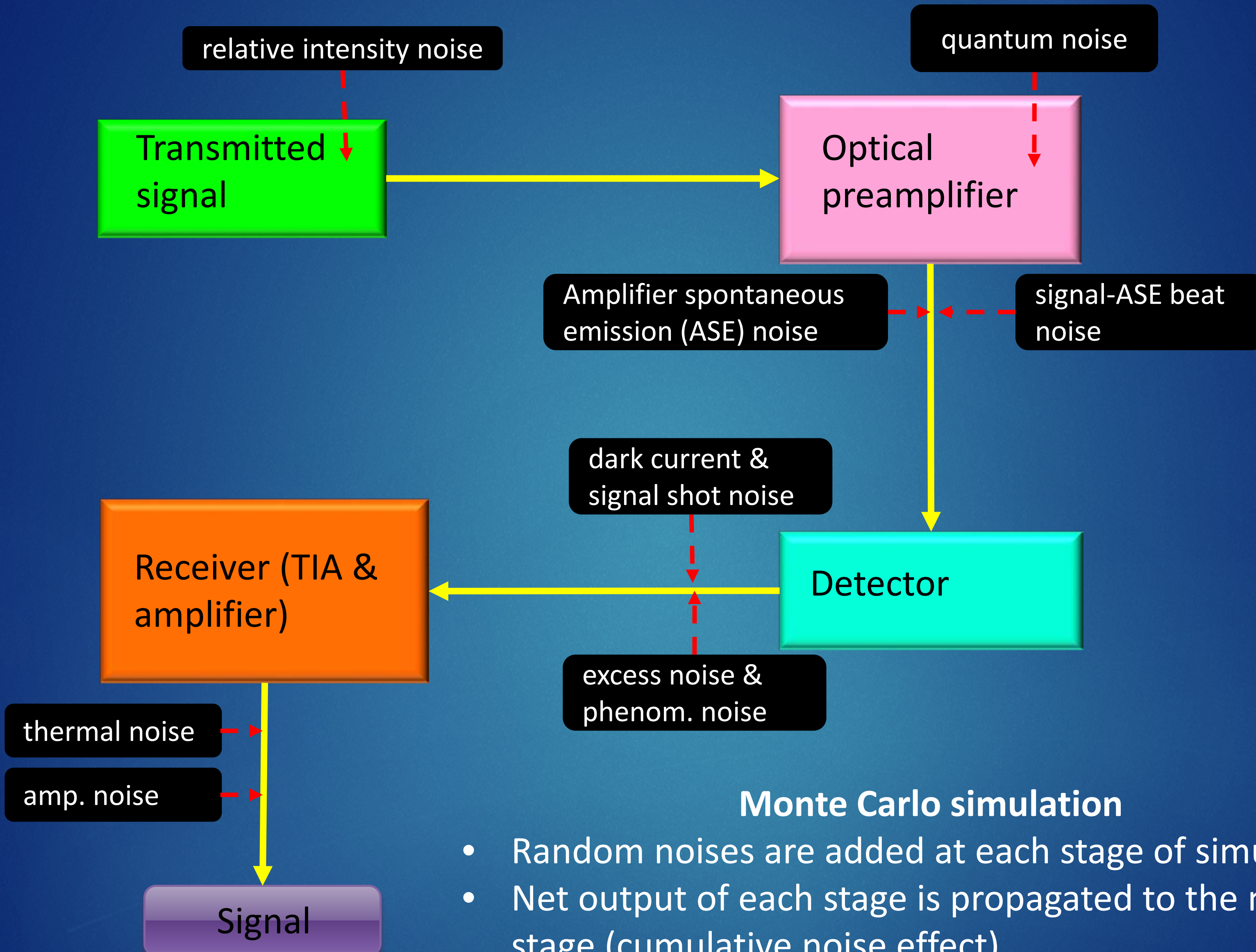
•Erbium-doped Fib. Amp (EDFA)
•Semicond. (SOA)
•Others (Raman)

Optical switch

•Path switch
•Add/Drop mux
• λ-router
•Cross connect
•Couplers
•circulators

Receiver

•Ultrafast PD

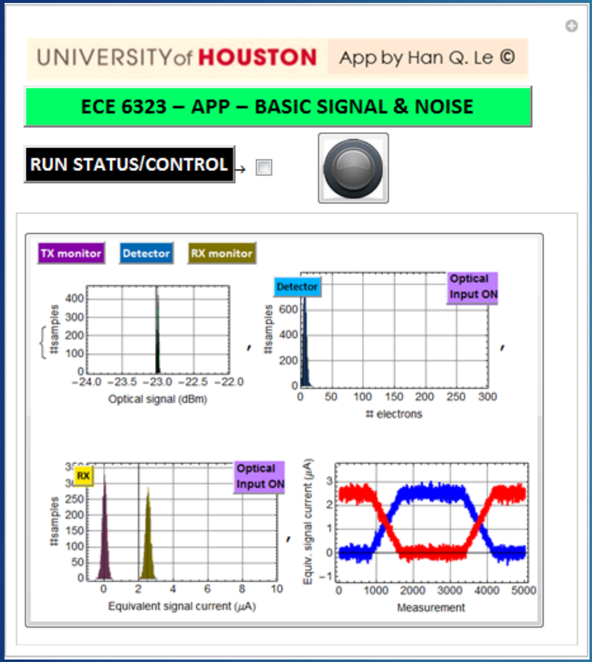


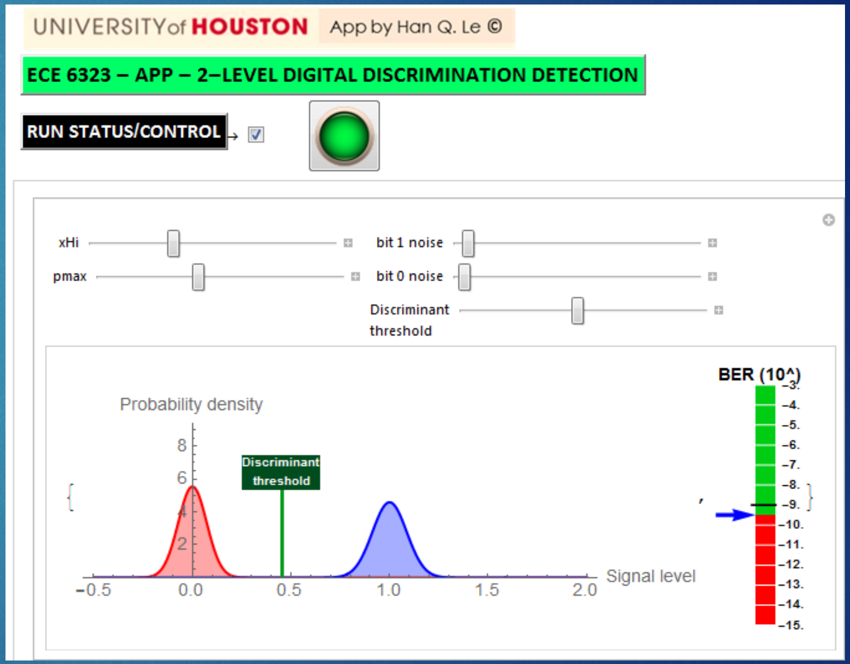
Monte Carlo simulation

- Random noises are added at each stage of simulation
- Net output of each stage is propagated to the next stage (cumulative noise effect)

Example application

Simulation of Bit Error Rate (BER) of a communication link



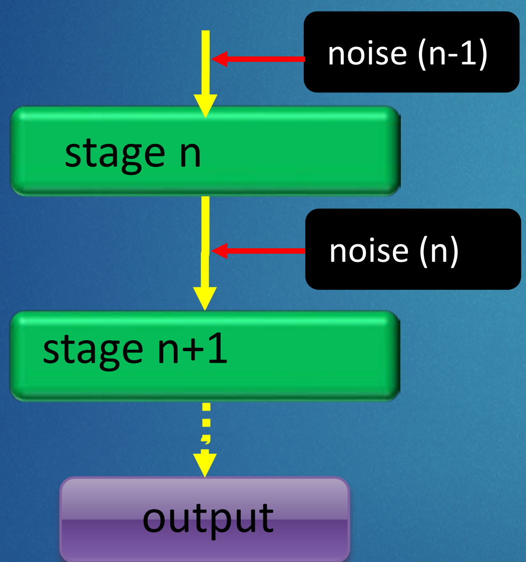


BER rate calculation is based on actual simulation statistics, not from noise model.

Distinction between simulation and model calculation

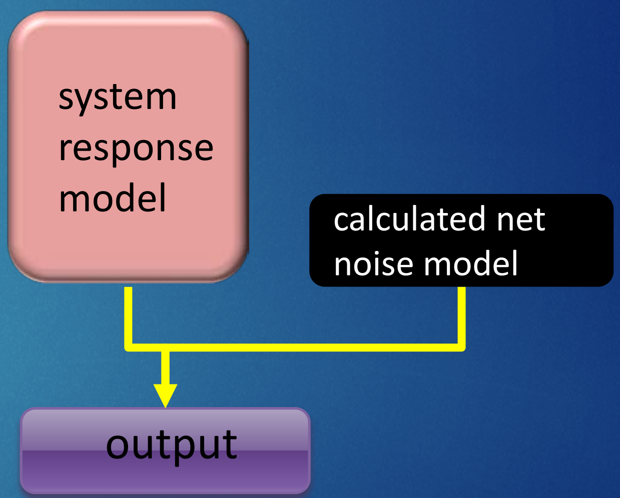
- The example of communication link APP discussed previously is a simulation

Simulation



Can handle realistic and complicated noises: e. g. accumulated 1/f, resonance, different stochastic processes: too complex to model analytically and accurately

Calculation



Practical only when noise model is sufficiently simple (e. g. AWG) to be analytically determined.

Does it matter? model vs. simulation: Yes

- Analogy: calculation is analogous to obtaining the descriptive statistical value of a model. Simulation is like obtaining the statistics of a pseudo-measurement.
- If the model is correct, both should produce agreement.
- However, sometimes, the model is difficult to ascertain or verified. Simulation can verify or <u>even show errors of the model</u> (surprise can happen)

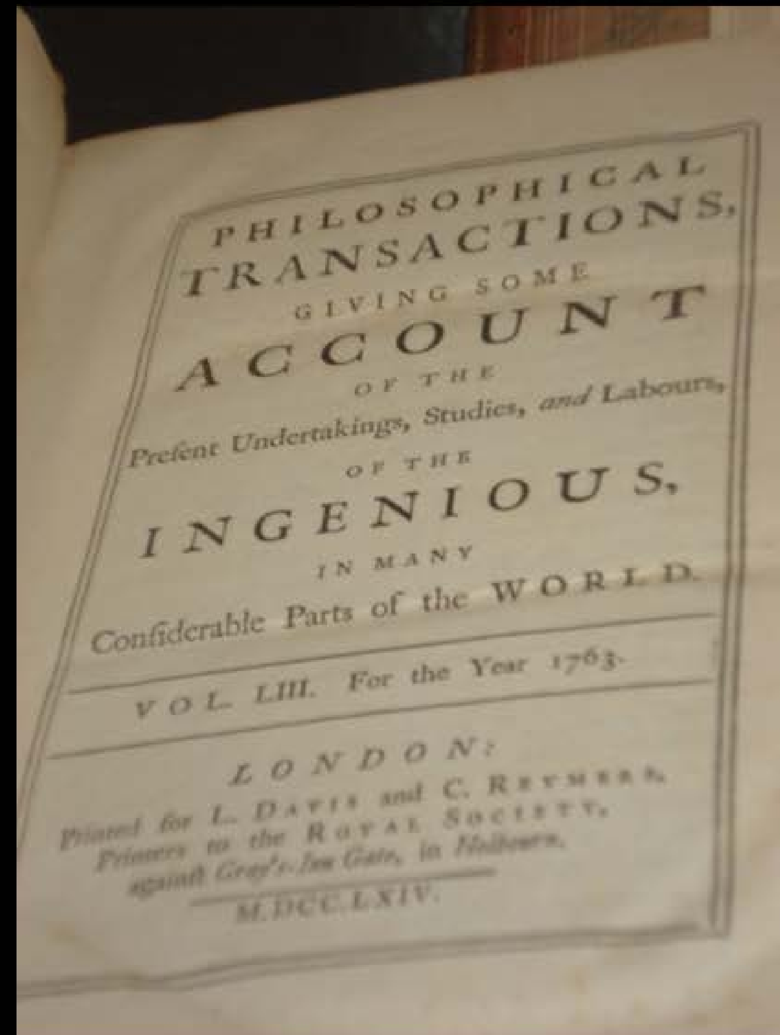
If you are not sure of the prediction of your stochastic model, use Monte Carlo simulation to test it.

A note on other Monte Carlo method application

- There is no single, defined recipe or definition of what Monte Carlo method is, except for the common essential feature that it involves random event generation (*see here for a historical review*)
- How it is done, is determined by applications – *e. g. see the example above for optical communication link.*
- Wide ranging application areas:
  - science and engineering
  - business, finance, any process simulation with some stochastic aspects or properties
  - can be used even if the problem is not stochastic, such as Monte Carlo integration (a very soft approach of numerical integration)

Topics

- Concepts introduction: noises and random events
- Descriptive statistics, probability
  - Distribution functions, probability density func (pdf), cumulative density func (cdf). Multivariate.
  - Examples and applications
- Numerical simulations and Monte Carlo
- Bayes' theorem and intro to Bayesian decision theory
  - Review cluster classifications, ROC concepts, Kalman's filter
- Introduction to stochastic calculus (*if have time*)
  - Ito calculus & finance applications: Black-Scholes model



PHILOSOPHICAL
TRANSACTIONS,
GIVING SOME
ACCOUNT
OF THE
Present Undertakings, Studies, and Labours,
OF THE
INGENIOUS,
IN MANY
Considerable Parts of the WORLD.

VOL. LIII. For the Year 1763.

LONDON:
Printed for L. DAVIS and C. REYMERS,
Printers to the ROYAL SOCIETY,
against Gray's-Inn Gate, in Holbourn.
M.DCC.LXIV.

http://library.bayesia.com/display/FAQ/Bayesian+Belief+Network+Definition

Bayes' rule

4.2 Bayes' rule (theorem)

Bayes' theorem:

Since: $P[A \mid B] = \frac{P[A \cap B]}{P[B]}$ and $P[B \mid A] = \frac{P[A \cap B]}{P[A]}$, we have:

$P[A \mid B]\,P[B] = P[B \mid A]\,P[A]$ or:

$$P[B \mid A] = \frac{P[A \mid B]\,P[B]}{P[A]}$$

Foundation of:

- Bayesian inference
- Bayesian decision theory
- Bayesian belief network

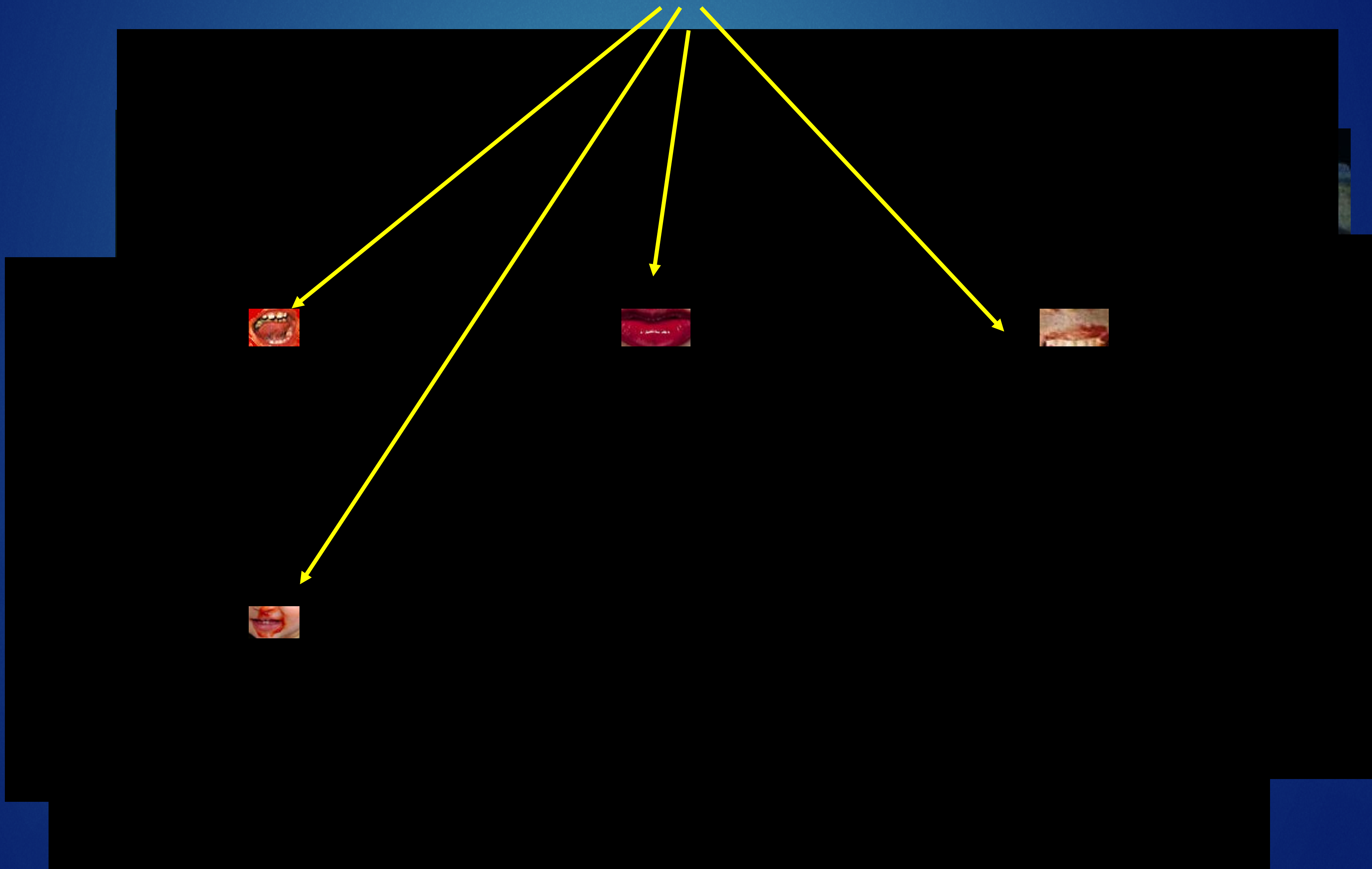
Suppose we play a game of 5 cards. The hand is said to be red or black if a majority of the cards is red or black. You are supposed to bet red or black. What is the probability for you to have a red or black hand?

```
number of first card red=  351
      Number of red hands= 506 ratio:  0.693676
number of first card black=  341
      Number of black hands= 494 ratio:  0.690283
both = 0.69198
```

Using Bayes' rule:
Knowing 1 card (red or black) improves our chance of calling correctly from 50% (pure chance) to $\frac{1133}{1666}$ =68%

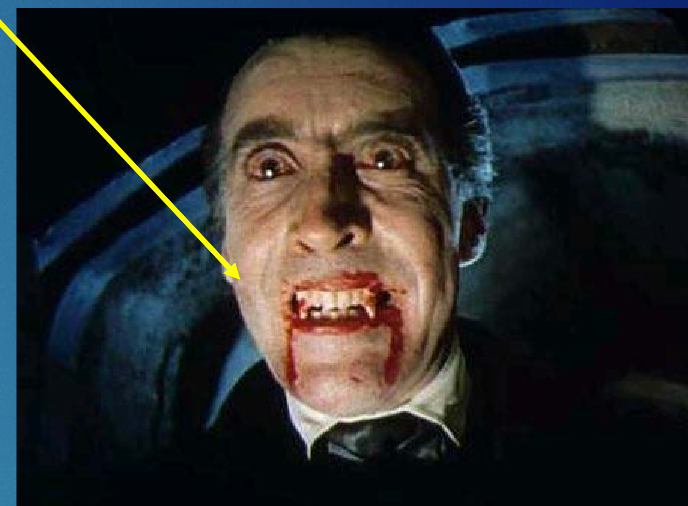
Bayesian inference

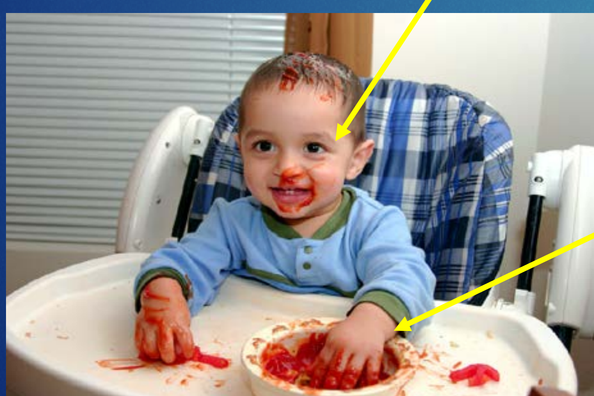
What is the red stuff on these mouths?



Bayesian inference

What is the red stuff on these mouths?





If this were french fries, what would you infer?
If this were spaghetti, what would you infer?

Bayes' rule and Bayesian decision (or inference)

conditional probability: *probability of A, given (or if) B* P[A|B]

3.2 Bayes' rule (theorem)

Bayes' theorem:

Since: $P[A \mid B] = \frac{P[A \cap B]}{P[B]}$ and $P[B \mid A] = \frac{P[A \cap B]}{P[A]}$, we have:

$P[A \mid B]\, P[B] = P[B \mid A]\, P[A]$ or:

$$P[B \mid A] = \frac{P[A|B]\, P[B]}{P[A]}$$

in the previous slide:

P[red stuff=blood|Dracula] > P[red stuff=ketchup|Dracula]

inference: the red stuff on Dracula lips is blood

P[red stuff=ketchup|toddler] > P[red stuff=blood|toddler]

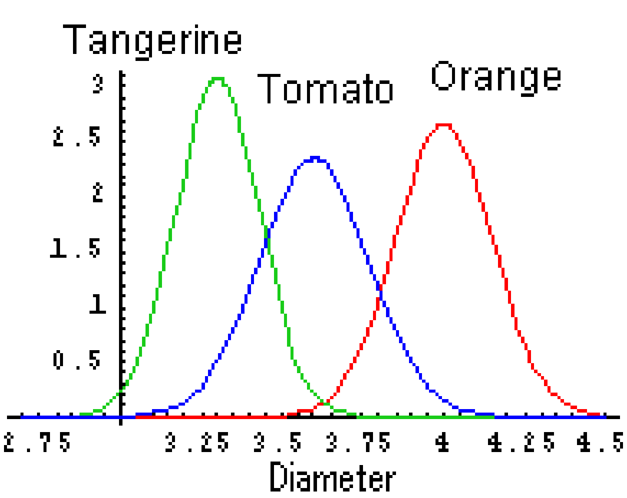
inference: the red stuffs on toddler lips is ketchup

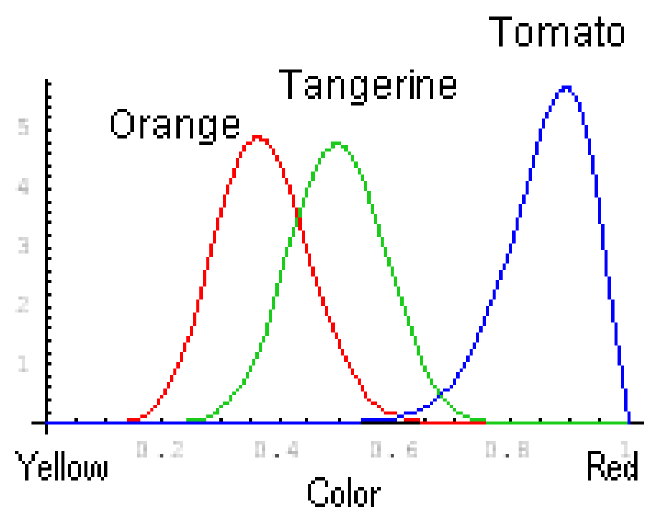
Bayesian classification: example

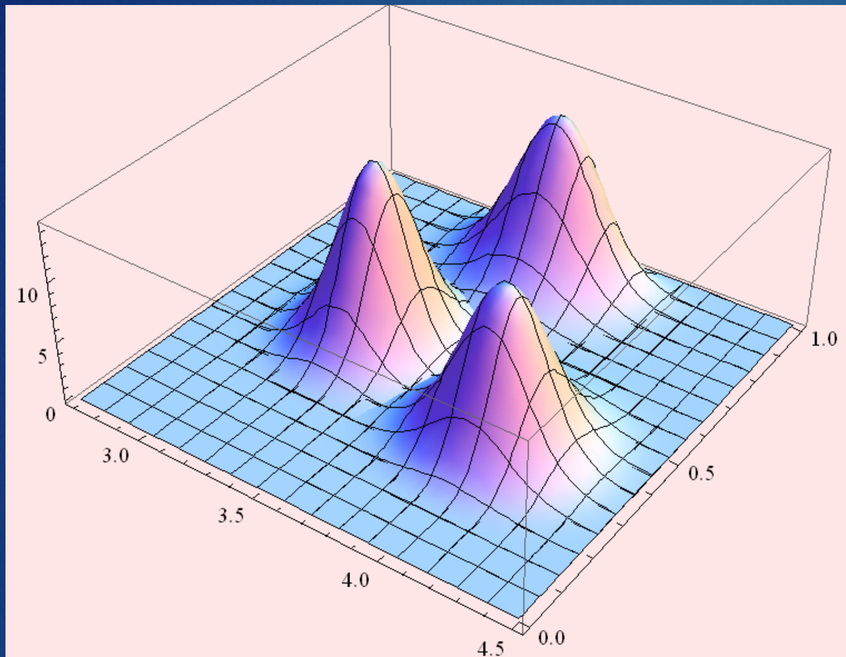
Tangerines Oranges Tomatoes

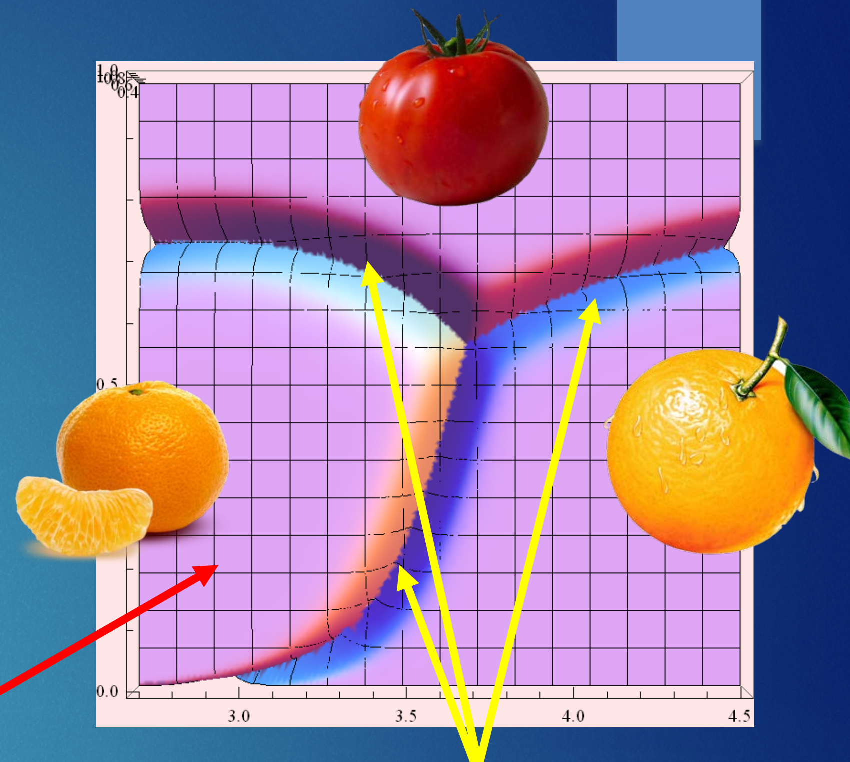
Features:

- color: orange – red-ness: γ
- size: diameter d



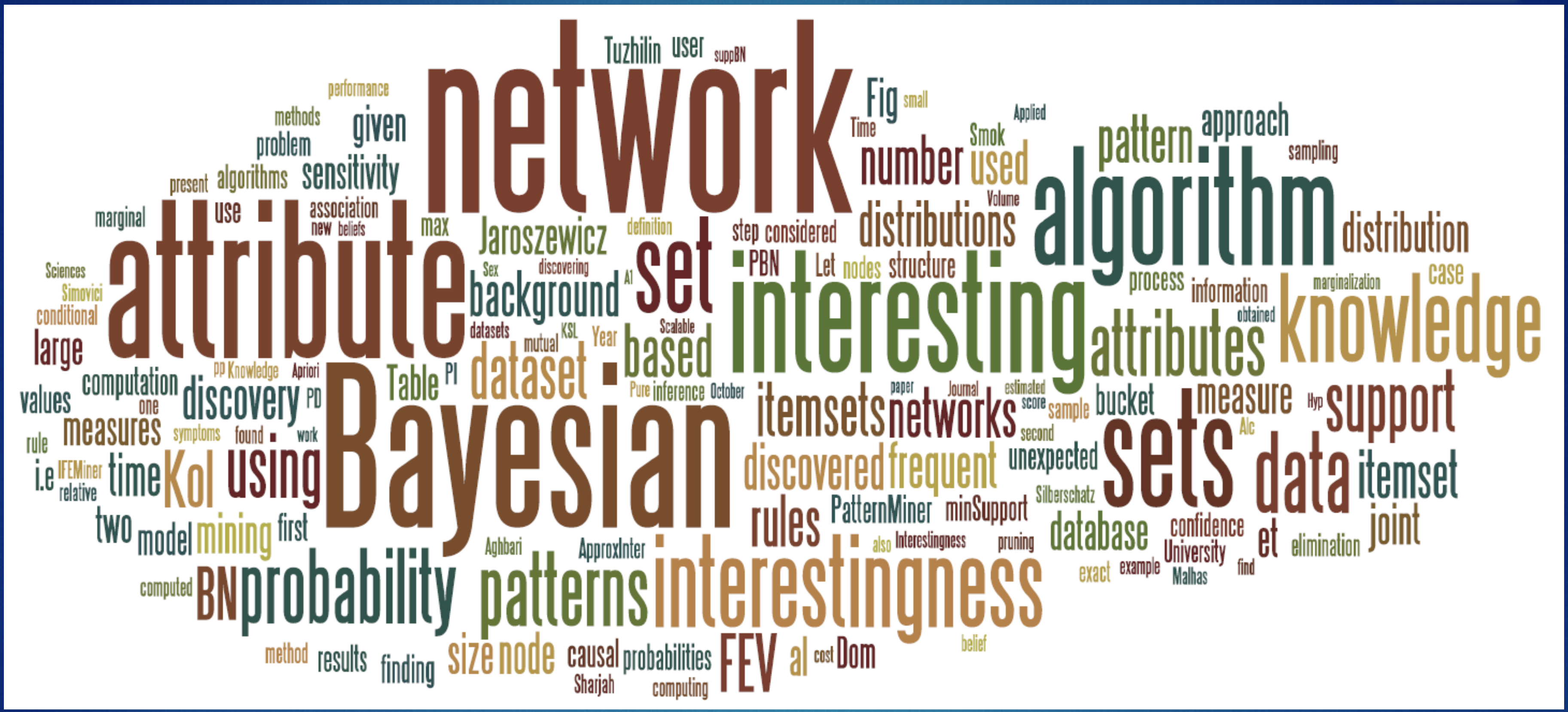






Discriminant boundaries: where the probabilities for two or more categories are equal.

Bayesian decision or Bayesian classification: to mark the region with the highest probability for each category: orange, tomato, or tangerine



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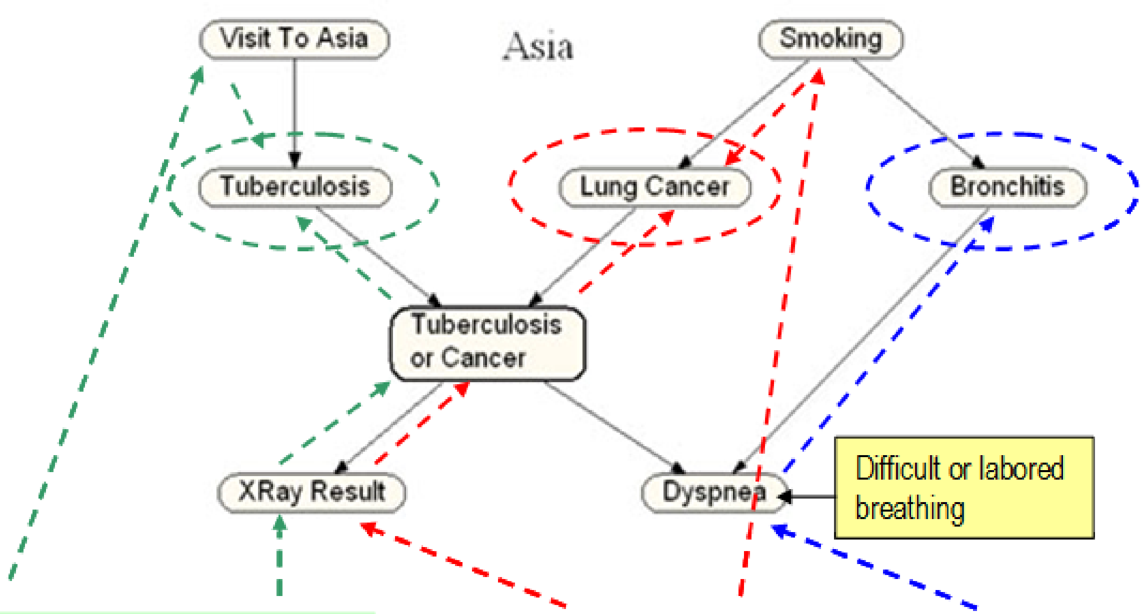
11.1 Introduction

The following materials are adapted form Nortica, a BBN software from Norsys. The example below is fr

Steffen L. and David J. Spiegelhalter (1988) "Local computations with probabilities on graphical structures and their application to expert systems" in Journ

Society B, 50(2), 157-194.



Case A: Mr. Smith's chest X-Ray is highly abnormal: it may indicate **tuberculosis or cancer**. It is learnt that he came back from Asia; hence **his risk of tuberculosis is more probable than that of cancer**

Case B: Mr. Doe's chest X-Ray is highly abnormal: it may indicate **tuberculosis or cancer**. He is a chain smoker, hence **his risk of cancer is more probable than that of tuberculosis**

Case C: Ms. Andrews' suffers acute dyspnea: it may indicate just **bronchitis** or **tuberculosis, cancer**. She doesn't smoke and has is not involved unhealthy environment. Hence, it more probable that **she has bronchitis rather than cancer or tuberculosis**

homework

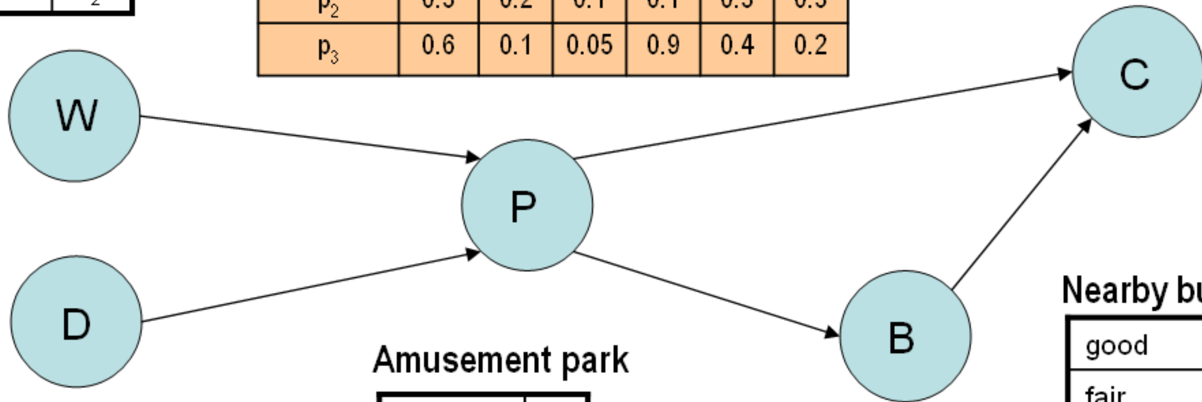
Weather:

good	w_1
bad	w_2

Weather	w_1 (good)			w_2 (bad)		
Occasion	wkd	wke	hol	wkd	wke	hol
p_1	0.1	0.7	0.85	0.	0.3	0.5
p_2	0.3	0.2	0.1	0.1	0.3	0.3
p_3	0.6	0.1	0.05	0.9	0.4	0.2

Area crimes

vandals	c_1
pickpockets	c_2
shoplift	c_3



Amusement park

high	p_1
medium	p_2
low	p_3

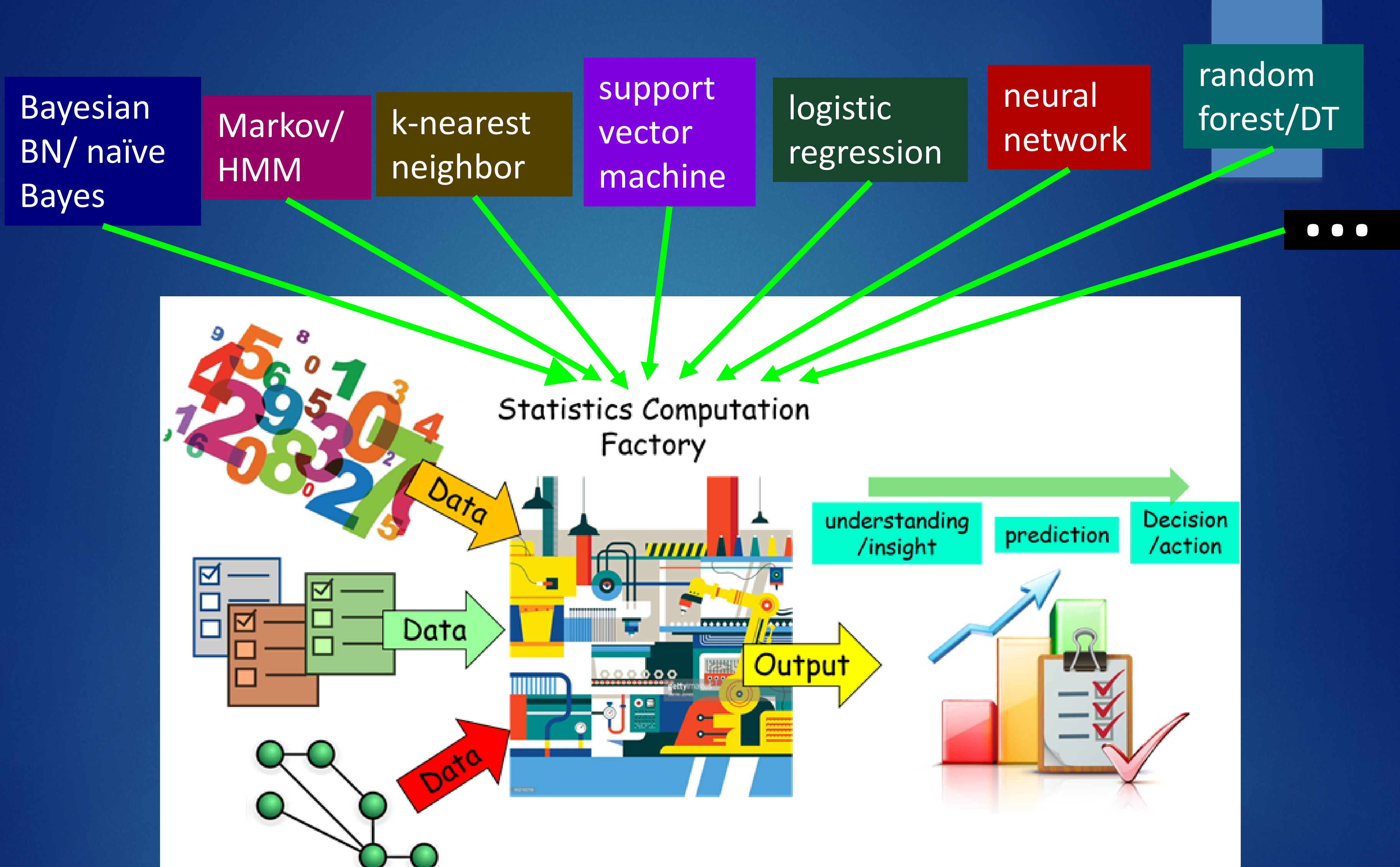
Park att	p_1	p_2	p_3
b_1	0.8	0.2	0
b_2	0.2	0.6	0.25
b_3	0.	0.2	0.75

Nearby business

good	b_1
fair	b_2
bad	b_3

Day/Occasion:

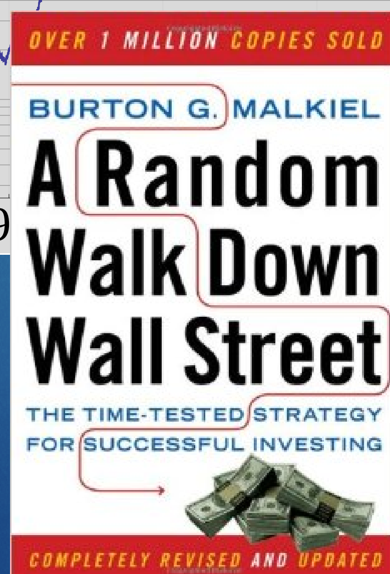
wkday	d_1
wkend	d_2
holid	d_3

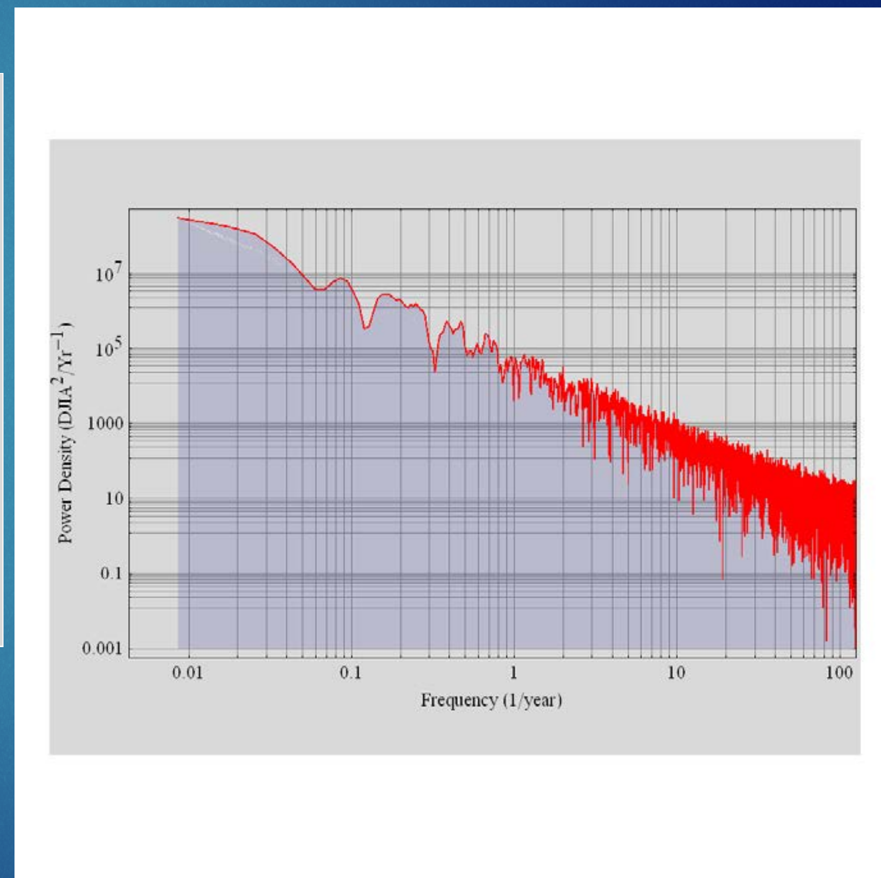


even without prior knowledge, classification (inference, decision) can be done solely based on data

PSD of random-walk: 1/f^2 noise

- Security market is Brownian motion-like (or drunkard random walk)
- As expected, 1/f^2 behavior



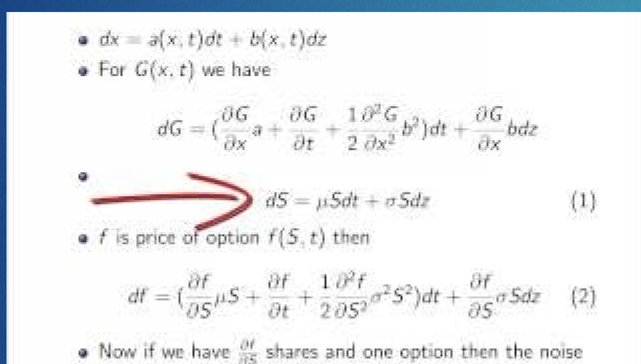


Application– finance and investment

For entertainment only - do not invest your money with this

Brownian motion – Itô calculus – and Black-Scholes option pricing theory

see Mathematica lecture



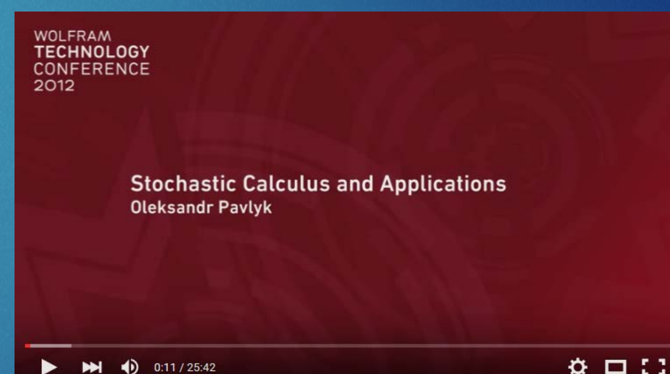
- $dx = a(x,t)dt + b(x,t)dz$
- For $G(x,t)$ we have

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}b\,dz$$

- $$dS = \mu S dt + \sigma S dz \qquad (1)$$
- f is price of option $f(S,t)$ then

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz \qquad (2)$$

- Now if we have $\frac{\partial f}{\partial S}$ shares and one option then the noise



Summary

- Stochastic phenomena are ubiquitous in many fields, and numerical methods are essential for their study.
- Specifically, Monte Carlo method greatly benefits from modern computer capability.
- Applications:
  - scientific – engineering
  - data science, statistical learning, AI
  - finance analytics
  - cryptography, information