Name: SOLUTIONS (please print)

Signature: ______________________

ECE 3455
Exam 1
October 6, 2007

exam duration: 90 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 8 pages, including the cover sheet. Raise your hand if you are missing a page.

1 ______________/35

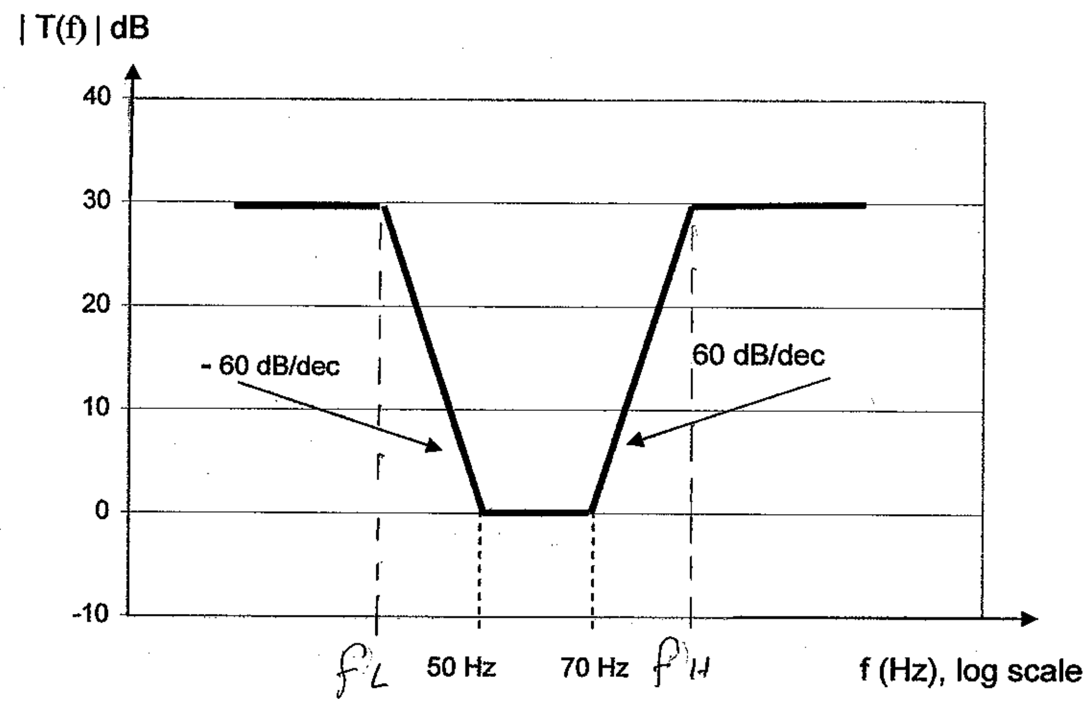
2 ______________/30

3 ______________/35

Total ______________/100

1. **(35 points)** The figure below shows the straight-line approximation to the magnitude Bode plot for a "notch" filter. As the figure shows, the gain between 50 Hz and 70 Hz is 0 dB; the gain increases by 60 dB/dec for frequencies above 70 Hz and below 50 Hz, but maximizes at 30 dB. There are no poles or zeros outside the region shown in the plot. *Note that the plot is merely a sketch: it is NOT drawn to scale, and should not be used for quantitative calculations.*

a) Find the transfer function T(f) for this notch filter. Assume that any constants necessary to adjust the magnitude are real and positive.
b) Using the paper provided on the next page, draw the straight-line approximation to the **Phase** Bode plot for this transfer function.



a) we need 3 poles at f_L, 3 zero's at 50 Hz, 3 more zero's at 70 Hz, and 3 more poles at f_H.

But what are f_L and f_H? Since $T(f)$ increases at 60 dB/dec, it reaches 30 dB after ½ decade. The frequency factor covered in ½ decade is ...

Room for Extra Work

...

$10^{0.5} \simeq 3.16$. So $f_L = \frac{50}{3.16} \cong 16$ Hz

$f_H = 70 \times 3.16 \simeq 220$ Hz

Therefore

$$T(f) = K \cdot \frac{(1+jf/70)^3(1+jf/50)^3}{(1+jf/16)^3\,(1+jf/220)^3}$$

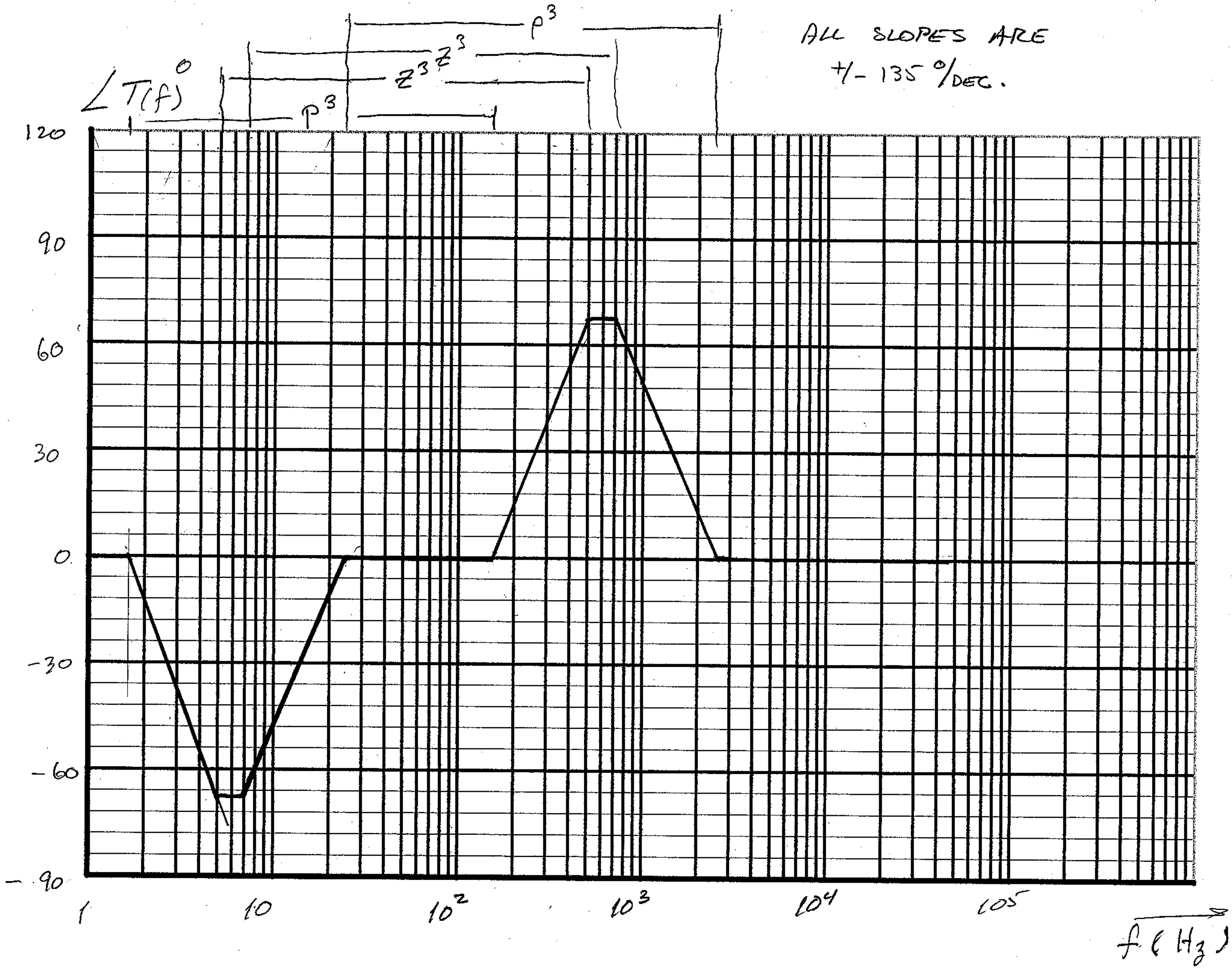
We need to evaluate this at a single point to find K. Using $f=0$ seems convenient:

$$T(f=0) = K = 30\ \text{dB} \quad \Rightarrow \quad \underline{K = 10^{30/20} = 31.6}$$

---

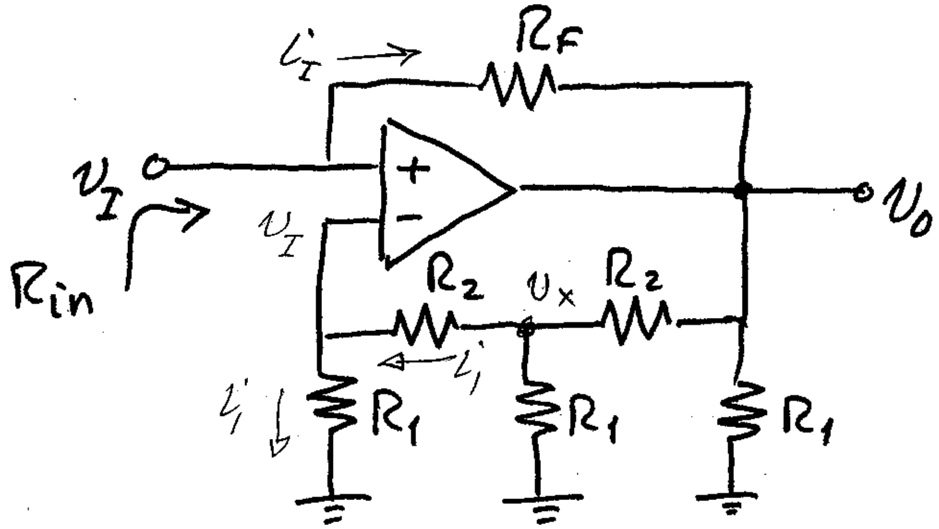
b) See plot...

Also $f \to 0 \Rightarrow \angle T(f) \to 0$


∠T(f)°
p³
z³ z³
p³
ALL SLOPES ARE
+/- 135 °/DEC.
120
90
60
30
0
-30
-60
-90
1
10
10²
10³
10⁴
10⁵
f (Hz)

2. **(30 points)** Assuming the op amp in the circuit below is ideal, do the following.

a) Find the gain v_O/v_I in terms of the resistances given.
b) Find the input resistance R_{in} seen by the source, for $R_2 = 2R_1$.



we have negative feedback, so...

$$\frac{v_x}{R_1} + \frac{v_x - v_I}{R_2} + \frac{v_x - v_O}{R_2} = 0$$

We need another equation to eliminate v_x:

$$i_1 = \frac{v_x - v_I}{R_2} = \frac{v_I}{R_1} \Rightarrow v_x = v_I\left(1 + \frac{R_2}{R_1}\right)$$

we might have seen this from the start: at the point labeled v_x, the output is just for the non-inverting configuration.

Solving this together with the first equation:

Room for extra work

$$v_x\left(\frac{1}{R_1}+\frac{2}{R_2}\right)-\frac{v_I}{R_2}=\frac{v_O}{R_2}$$

$$v_I\left[\left(1+\frac{R_2}{R_1}\right)\left(\frac{1}{R_1}+\frac{2}{R_2}\right)-\frac{1}{R_2}\right]=\frac{v_O}{R_2}$$

$$\frac{v_O}{v_I}=\left(1+\frac{R_2}{R_1}\right)\left(\frac{R_2}{R_1}+2\right)-1$$

$$\boxed{\frac{v_O}{v_I}=1+3\frac{R_2}{R_1}+\left(\frac{R_2}{R_1}\right)^2}$$

Input resistance:

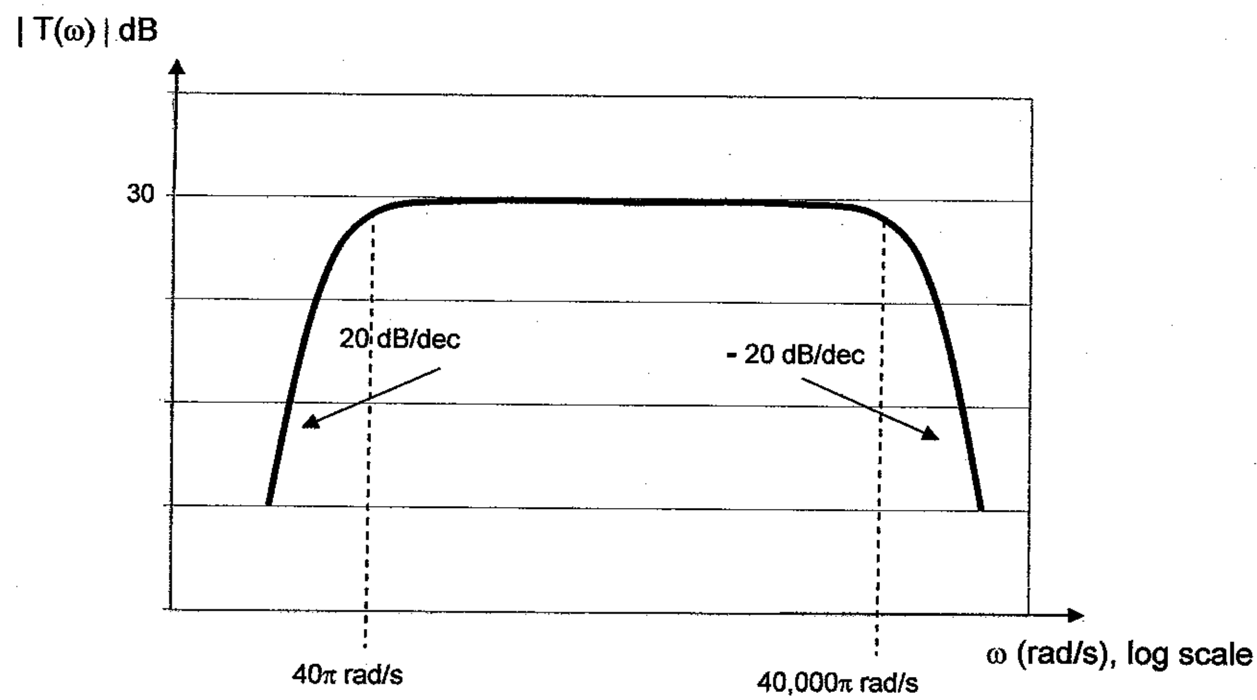
$$R_{in}=\frac{v_I}{i_I}\qquad i_I=\frac{v_I-v_O}{R_F}$$

For $R_2=2R_1$, $v_O=11\,v_I \Rightarrow i_I=-10/R_F$

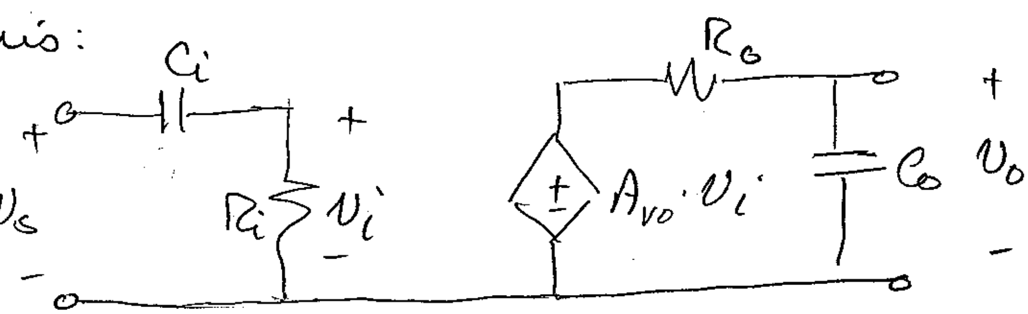
$$\therefore\ \boxed{R_{in}=-\frac{R_F}{10}}$$

3. **(35 points)** The figure below shows the magnitude Bode plot for a band pass filter. As the figure shows, the gain in the "pass band", that is, between 40π rad/s and 40,000π rad/s, is 30 dB; the gain rolls off at 20 dB/dec outside the pass band. There are no poles or zeros outside the region shown in the plot. *Note that the plot is merely a sketch: it is NOT drawn to scale, and should not be used for quantitative calculations.*

a) Draw a circuit diagram for a voltage amplifier that would produce the Bode plot shown. That is, choose capacitance and resistance values such that the breakpoints are as shown, and choose an appropriate gain parameter.
b) Find the transfer function T(ω) for your circuit diagram.
c) For any convenient frequency in the middle of the pass band, find both the input and output impedance for your circuit. State the frequency at which you evaluated the impedances.



Let's try this:



Room for extra work

At low ω, $C \to \infty$, so

$v_i \to 0$ because of the open circuit at the input.

At high ω, $C \to 0 \Rightarrow v_o \to 0$. So this circuit should give us what we want.

$$T(\omega) = A_{vo} \cdot V_i \cdot \frac{1}{1+j\omega C_o R_o}$$

$$V_i = \frac{j\omega C_i R_i}{1+j\omega C_i R_i}$$

$$\boxed{T(\omega) = A_{vo} \cdot \frac{j\omega C_i R_i}{1+j\omega C_i R_i} \cdot \frac{1}{1+j\omega C_o R_o}}$$

This transfer function has a zero at 0, and poles at $1/C_iR_i$ and $1/C_oR_o$. So if we choose the poles correctly, we can get the Bode plot shown

$1/C_iR_i = 40\pi$ choose $R_i = 10\,k\Omega$

$$\Rightarrow \quad C_i = \frac{1}{4\pi \cdot 10000} \cong 8\,\mu F$$

$1/C_oR_o = 40\,000\,\pi$ choose $R_o = 1\,k\Omega$

$$\Rightarrow \quad C_o = \frac{1}{40\,000\,\pi \cdot 1000} \cong 8\,nF$$

We need a gain parameter, too:

$\omega = 20\,000$ rad/s $\Rightarrow$ $T(\omega) \simeq A_{vo}$:

$$T\Big|_{\omega = 20\,\mathrm{krad/s}} = A_{vo} \frac{20\,000 \cdot C_i R_i}{1 + 20\,000 \cdot C_i R_i} \cdot \frac{1}{1 + 20000\, C_o R_o}$$

$$\simeq A_{vo} = 30\ \mathrm{dB}$$

$$\Rightarrow \underbrace{A_{vo} \simeq 31.6}$$

c) $Z_{in} = 1/j\omega C_i + R_i$ $\qquad Z_{out} = 1/j\omega C_o \,//\, R_o$

$$= \frac{R_o}{1 + j\omega C_o R_o}$$

at $\omega = 20\,000$ rad/s,

$$Z_{in} = \frac{1}{j \cdot 20000 \cdot 8\times10^{-6}} + 10\ \mathrm{k\Omega} \simeq 10\,000\ \Omega$$

$$Z_{out} = \frac{1000}{1 + j\,20000 \cdot 8\times10^{-9} \cdot 1000} \simeq 1000\ \Omega.$$

So in the region over which $T(\omega) \simeq$ constant at 30 dB, there is no effect due to the capacitors. As a result, $Z_{in} \simeq R_i$ and $Z_{out} \simeq R_o$.