Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Quiz #2
September 25, 2007

Quiz duration: 30 minutes

1. You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
2. Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
3. Show units in intermediate and final results, and in figures.
4. If your work is sloppy or difficult to follow, points will be subtracted.

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The figure on the next page is the magnitude Bode plot for a particular transfer function T(ω).

i) From the plot, estimate the breakpoints, i.e., the poles and zeros, of T(ω). In doing this, you may assume that all slopes are multiples of +/- 20 dB/dec.

ii) Based on the breakpoints you determined, find the transfer function T(ω), including any constants necessary to adjust the magnitude of the plot.

iii) Based on the breakpoints you determined, what is the slope of the ***phase*** Bode plot at ω = 20,000 rad/s?

i) One way to do this is to draw the straight-line approximation to the curve; this has been done on the graph. An estimate of the breakpoints is then given by the intersections of the lines:

$$\begin{cases} \omega_1 \simeq 380 \text{ rad/s} & \omega_2 \simeq 1.8\times10^4 \text{ rad/s} - \text{double pole} \\ \omega_0 = 0 \text{ rad/s} & \omega_3 \simeq 6\times10^5 \text{ rad/s} \end{cases}$$

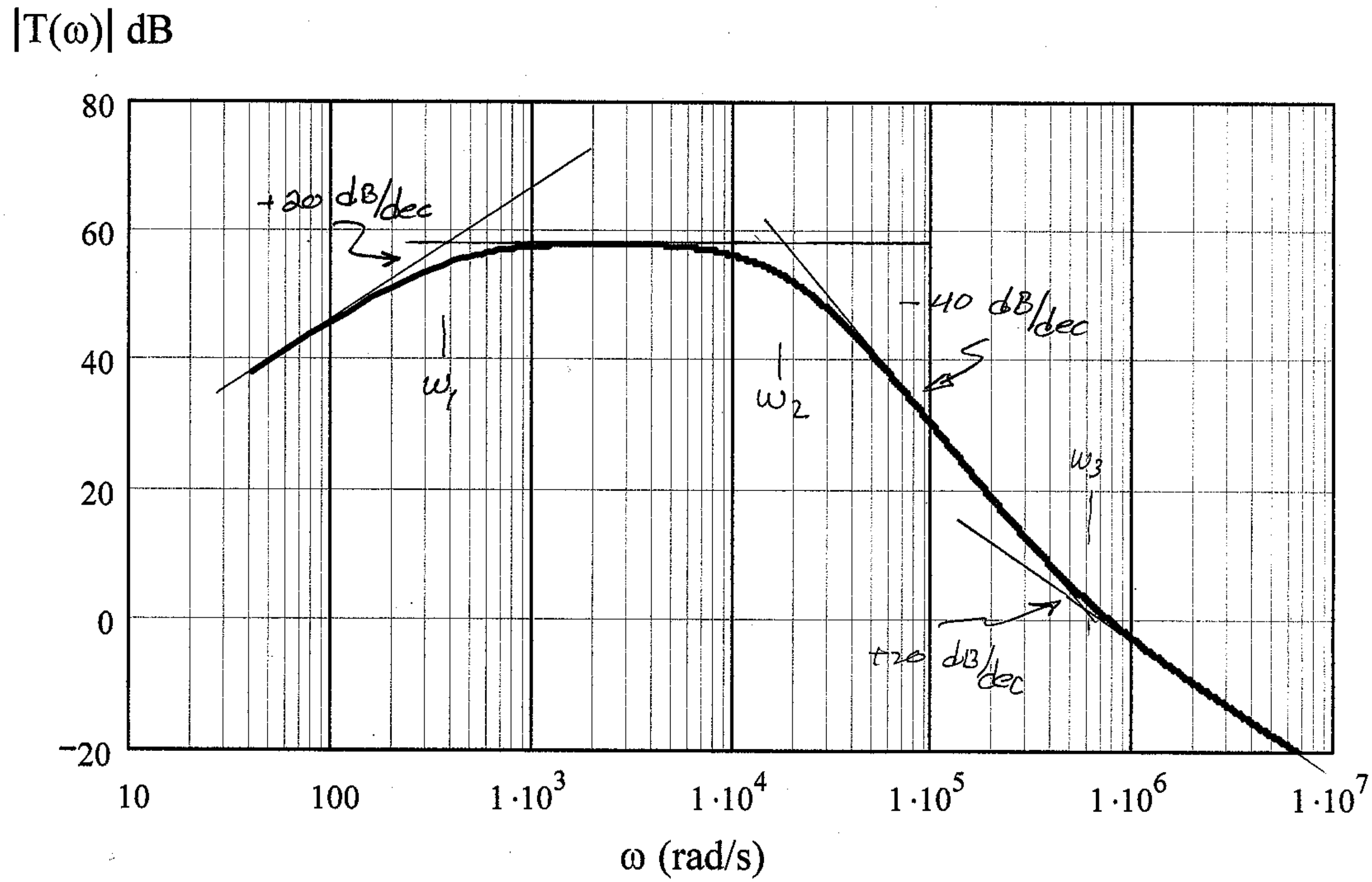
For $4\times10^4 \lesssim \omega \lesssim 4\times10^5$ rad/s, the slope is -40 dB/dec, so $\underline{\underline{\omega_2}}$ is the start of a $\underline{\underline{\text{double pole}}}$. So we have

ii)

$$T(\omega) = K \frac{j\omega_0(1+j\omega/\omega_3)}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)^2}$$

$$= K \frac{j\omega(1+j\omega/6\times10^5)}{(1+j\omega/380)(1+j\omega/1.8\times10^4)^2}$$

The "$j\omega$" in the numerator is the pole at $\omega=0$; we know this exists because of the increasing slope as ω increases at the left-end of the plot.



$\omega_0 = 0$ rad/s (Z is causing increase of 20 dB/dec at low ω)

$\omega_1 \simeq 380$ rad/s $\quad$ $\omega_2 \simeq 4.8 \times 10^4$ rad/s is a double pole

$\omega_3 \simeq 6 \times 10^5$ rad/s

Room for Extra Work

To find K we evaluate at a convenient place.

Choose $\omega = 3000$ rad/s. In that case, $\omega/6\times10^5 = 5\times10^{-3}$, so we ignore this term and

$$|T(\omega)|_{3000\ \text{rad/s}} = \left| K \cdot \frac{j \cdot 3000}{(1+j\,3000/380)(1+j\,3000/1.8\times10^4)^2} \right|$$

$$= K \cdot 367.$$

$$20\log|T(\omega)|_{3000\ \text{rad/s}} \simeq 58 = 20\log K + 20\log 367$$

from graph

$$20\log K = 58 - 20\log 367 \implies K \cong 2.2$$

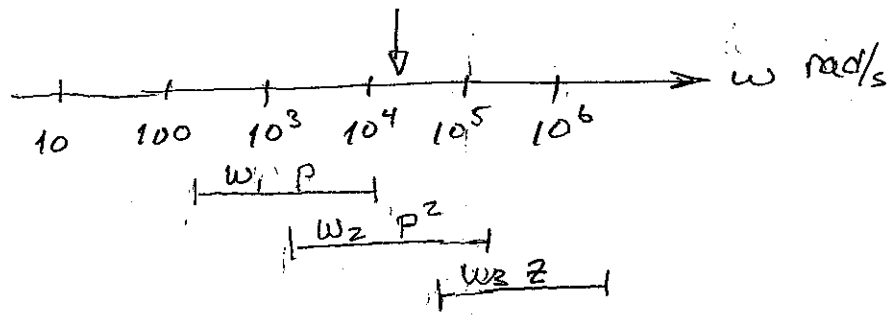
So:

$$\boxed{T(\omega) = 2.2\,\frac{j\omega(1+j\omega/6\times10^5)}{(1+j\omega/380)(1+j\omega/1.8\times10^4)^2}}$$

Check: For $\omega = 10^5$ rad/s, $|T(\omega)| = 26.6 = 28.5$ dB

The plot shows $|T(\omega)| \simeq 30$ dB so we are not too far off.

iii) If we were plotting the phase Bode plot, we might want to annotate the ω-axis like this:



Now at $\omega = 20\,000$ rad/s there are 2 poles operating, so the slope will be $\sim -90°/\text{dec}$

The actual transfer function used in the plot was:

$$T(\omega) = 2.0 \frac{j\omega\left(1 + j\omega/5\times10^5\right)}{\left(1 + j\omega/400\right)\left(1 + j\omega/2\times10^4\right)^2}$$