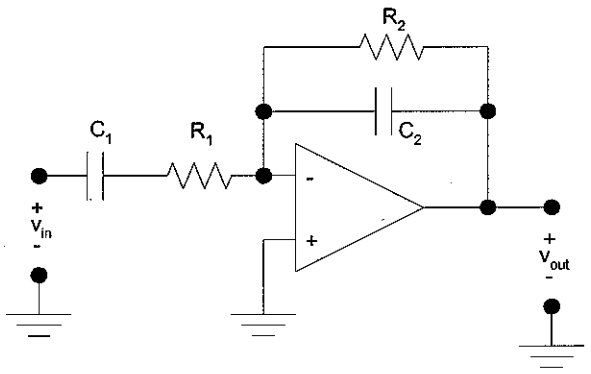
(a) Derive the transfer function, $H_1(j\omega)=v_{out}/v_{in}$, for the circuit below. Assume ideal op amps.

(b) Given that $\frac{1}{R_1C_1} = 10\ rad/s$, $\frac{1}{R_2C_2} = 1{,}000\ rad/s$, and $R_1 = R_2$, plot the straight-line approximations to the magnitude and phase Bode plots for this circuit using the semilog graph paper on the next two pages.



(a) This is an inverting Amplifier with

$$H_1(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$$

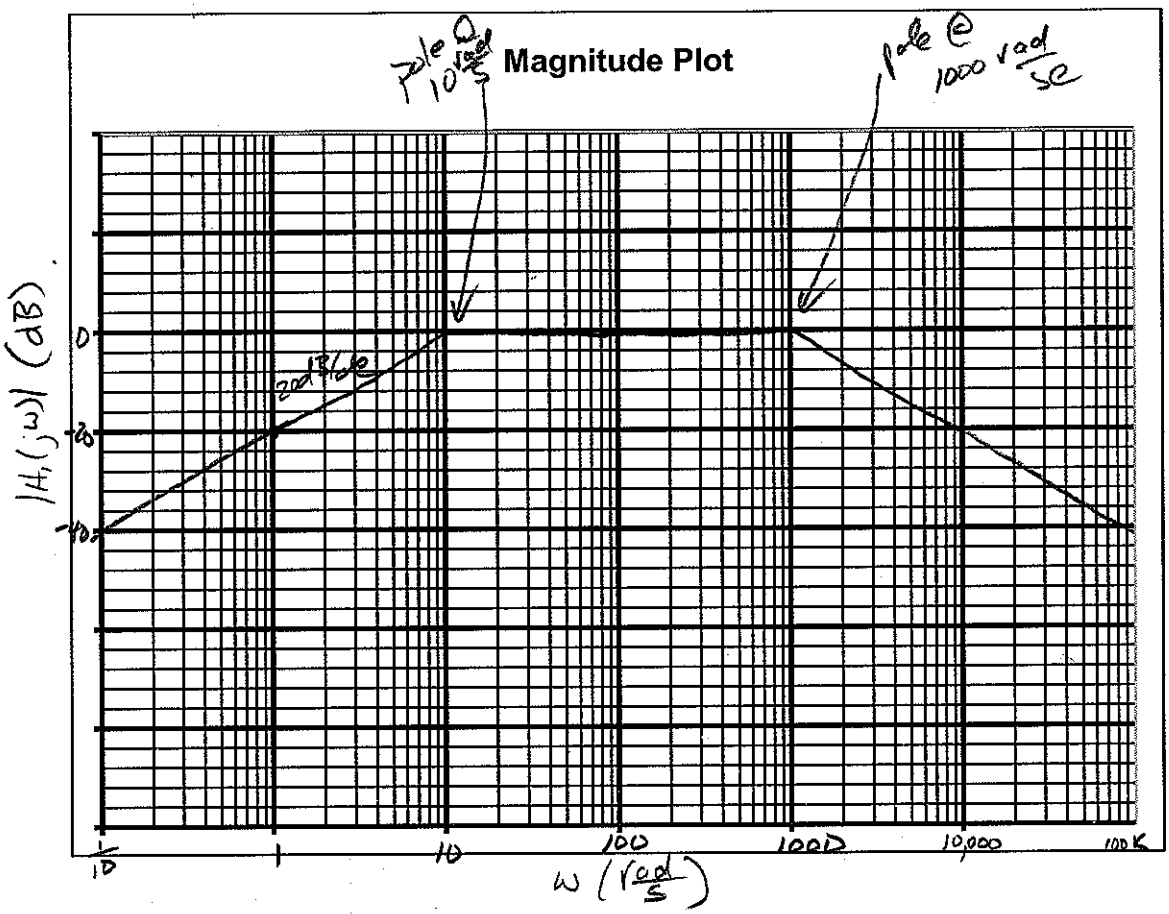
$$Z_2 = R_2 \| \frac{1}{j\omega C_2} = \frac{1}{\frac{1}{R_2} + j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

$$H_1(j\omega) = -\frac{Z_2}{Z_1} = -\frac{\frac{R_2}{1 + j\omega R_2 C_2}}{\frac{1 + j\omega R_1 C_1}{j\omega C_1}} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

- or -

$$H_1(j\omega) = -\frac{R_2}{R_1} \cdot \frac{j\omega C_1 R_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$



(b) Rewrite $H_1(j\omega)$ w/ $\frac{1}{R_1C_1} = 10\ \frac{rad}{s}$, $\frac{1}{R_2C_2} = 1K\ \frac{rad}{s}$ & $R_1 = R_2$

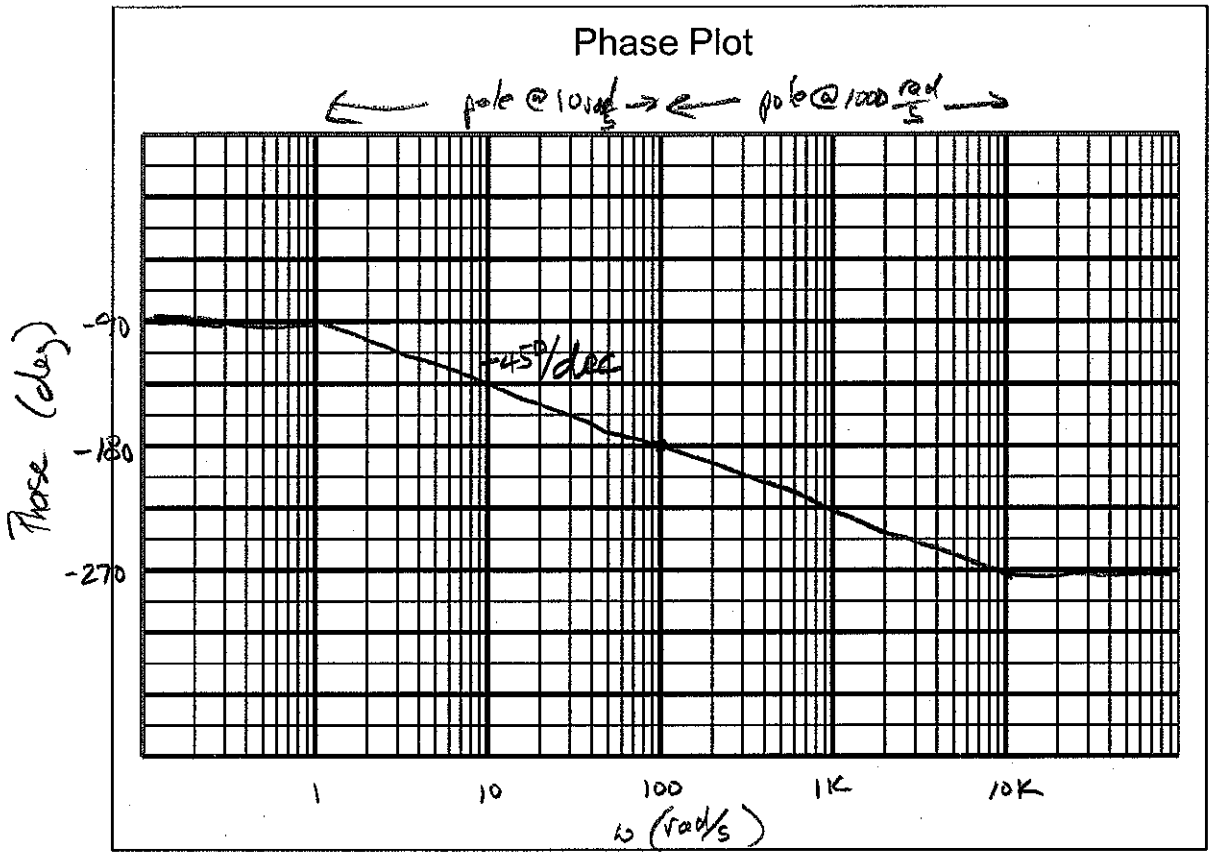
$$H_1(j\omega) = \frac{-j\frac{\omega}{10}}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{1000}\right)}$$

One zero @ $\omega = 0$, two poles @ $\omega = 10$ & $1000\ \frac{rad}{sec}$

$$H_1\left(1\ \tfrac{rad}{sec}\right) = \frac{-j\frac{1}{10}}{(1+j(.1))(1+j(.1))} = \frac{-j}{10}$$

$$|H_1(1)| = \frac{1}{10} \rightarrow -20dB$$

$$\angle H_1(1) = \frac{-j}{10} \rightarrow -90^\circ$$


Phase Plot
pole @ 10 rad/s
pole @ 1000 rad/s
-45°/dec
-90
-180
-270
Phase (deg)
1
10
100
1K
10K
ω (rad/s)