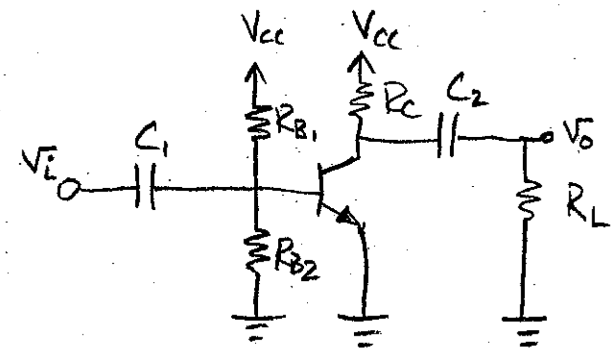
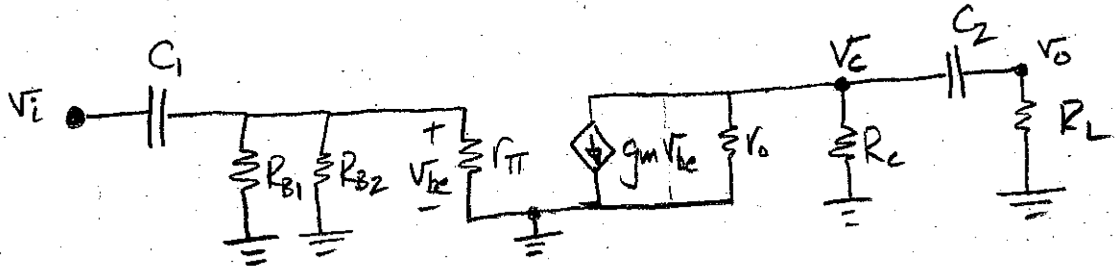
Assume that the transistor in the following circuit is active. Find an expression for the small signal gain (v_o/v_i), the input impedance (Z_{in}), and the output impedance (Z_{out}).

Be sure to draw the small signal equivalent circuit and it is recommended that you use the hybrid pi model. Reduce your expressions and express them in terms of the circuit components (resistors and capacitors), g_m or β, r_o, and r_π.





$$V_{be} = \frac{V_i \cdot (R_{B1} \| R_{B2} \| r_\pi)}{\frac{1}{j\omega C_1} + R_{B1} \| R_{B2} \| r_\pi} = \frac{V_i \; j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)}{1 + j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)}$$

$$V_c = -g_m V_{be} \cdot r_o \| R_c \| \left(\frac{1}{j\omega C_2} + R_L\right)$$

$$V_o = \frac{V_c}{R_L + \frac{1}{j\omega C_2}} \cdot R_L = \frac{V_c \, j\omega C_2 R_L}{1 + j\omega C_2 R_L}$$

and so:

$$\frac{V_o}{V_i} = -\frac{j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)}{1 + j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)} \cdot \frac{j\omega C_2 R_L}{1 + j\omega C_2 R_L} \cdot g_m \left[r_o \| R_c \| \left(\frac{1}{j\omega C_2} + R_L\right)\right]$$

while this is an acceptable answer, the term $r_o \| R_c \| (\frac{1}{j\omega C_2} + R_L)$ is not particularly useful in understanding the frequency response of the circuit.

For this quiz, the expression is ok. Please note, though, that if we call $R_C' = R_C \| r_o$, which is useful because typically $r_o \gg R_C$ and so $R_C' \cong R_C$, we can rewrite

$$r_o \| R_C \| \left(\frac{1}{j\omega C_2} + R_L\right) = R_C' \| \left(\frac{1}{j\omega C_2} + R_L\right) = \frac{R_C'\left(\frac{1}{j\omega C_2} + R_L\right)}{R_C' + R_L + \frac{1}{j\omega C_2}}$$

$$= \frac{R_C'(1 + j\omega C_2 R_L)}{1 + j\omega C_2(R_C' + R_L)}$$

and so:

$$\frac{v_o}{v_i} = -g_m \cdot \frac{j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)}{1 + j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)} \cdot \frac{j\omega C_2 R_L R_C'}{1 + j\omega C_2 (R_C' + R_L)}$$

$$= -g_m (R_L \| R_C') \cdot \frac{j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)}{1 + j\omega C_1 (R_{B1} \| R_{B2} \| r_\pi)} \cdot \frac{j\omega C_2 (R_C' + R_L)}{1 + j\omega C_2 (R_C' + R_L)}$$

Hence, our bandpass gain is $-g_m (R_L \| R_C')$ and we have two corners in our high-pass response, one at $\omega_1 = \frac{1}{(R_{B1} \| R_{B2} \| r_\pi) C_1}$ and the other at $\omega_2 = \frac{1}{(R_C' + R_L) C_2}$

Z_{in} & Z_{out} can be written as:

$$Z_{in} = \frac{1}{j\omega C_1} + R_{B1} \| R_{B2} \| r_\pi = \frac{1 + j\omega [R_{B1} \| R_{B2} \| r_\pi] C_1}{j\omega C_1}$$

$$Z_{out} = \frac{1}{j\omega C_2} + r_o \| R_C = \frac{1 + j\omega [r_o \| R_C] C_2}{j\omega C_2}$$

So, as $\omega \to 0$ Z_{in} & $Z_{out} \Rightarrow \infty$ (they block DC)

for high frequencies, $Z_{in} \sim R_{B1} \| R_{B2} \| r_\pi$
and $Z_{out} \sim r_o \| R_C = R_C'$