Name: ______________________________ (please print)

Signature: ___________________________

# ECE 3455
# Exam 1
# October 9, 2010

## Exam duration: 90 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

***This exam has 9 pages, including the cover sheet. Raise your hand if you are missing a page.***

1 ________________/25

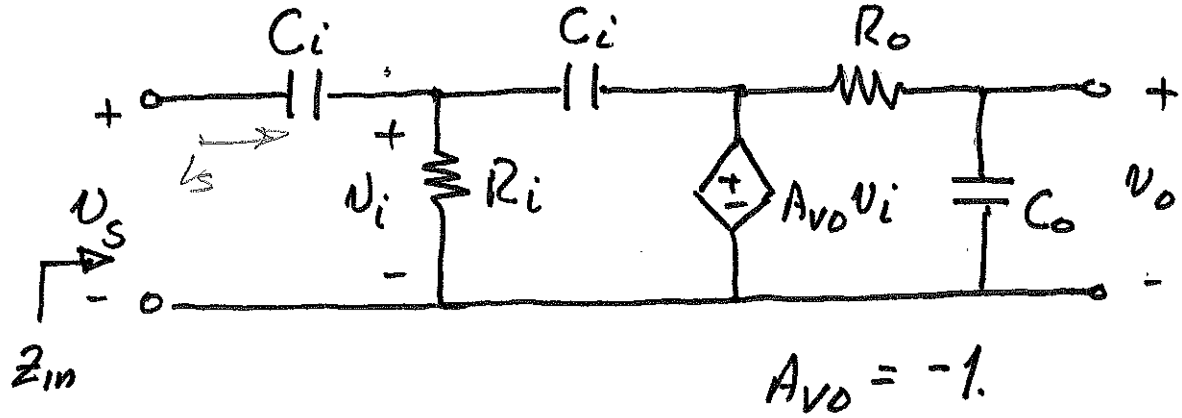
2 ________________/25

3 ________________/25

4 ________________/25

Total ________________/100

1. (**25 points**) Find the input impedance, $Z_{in}$, of the circuit below.




$$\frac{\bar{V}_i}{R_i} + \frac{\bar{V}_i - \bar{V}_s}{1/j\omega C_i} + \frac{\bar{V}_i - A_{vo}\bar{V}_i}{1/j\omega C_i} = 0$$

$$\Rightarrow \bar{V}_i\left(\frac{1}{R_i} + j\omega C_i + 2j\omega C_i\right) = \bar{V}_s \cdot j\omega C_i$$

+10 $$\therefore \frac{\bar{V}_s}{\bar{V}_i} = \frac{1+3j\omega C_i R_i}{j\omega C_i R_i}$$

$$\bar{I}_i = (\bar{V}_s - \bar{V}_i)\, j\omega C_i = \bar{V}_s\left(1 - \frac{\bar{V}_i}{\bar{V}_s}\right) j\omega C_i$$

$$= \bar{V}_s\left(1 - \frac{j\omega C_i R_i}{1+3j\omega C_i R_i}\right) j\omega C_i$$

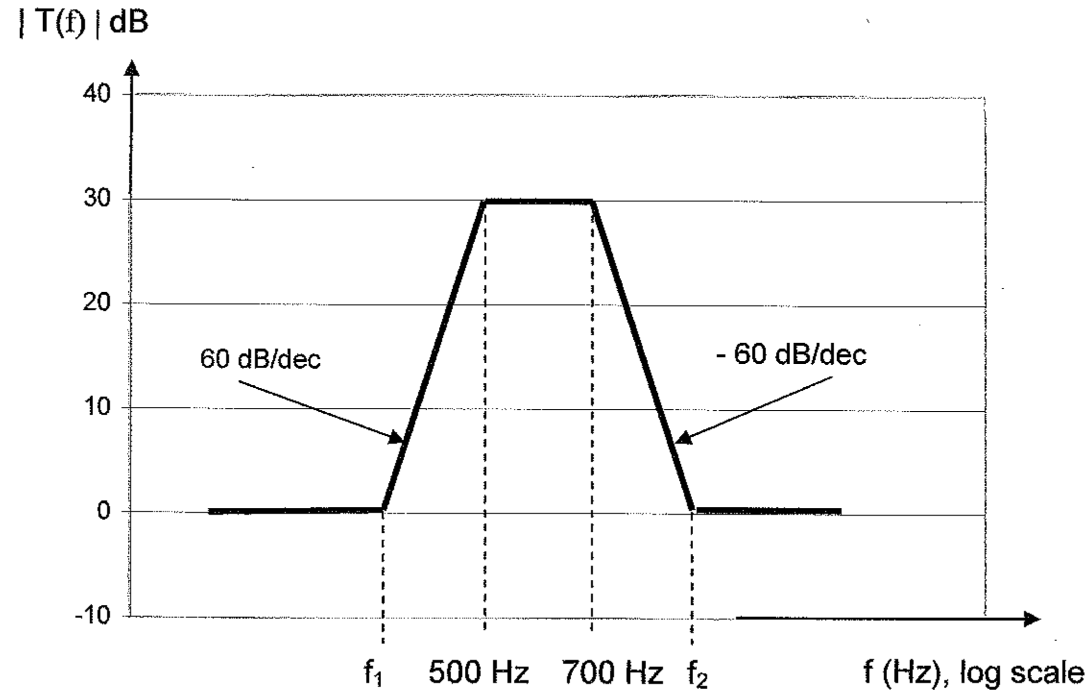
+10 $$= \bar{V}_s\left(\frac{1+2j\omega C_i R_i}{1+3j\omega C_i R_i}\right) j\omega C_i$$

Finally $$\boxed{Z_i = \frac{\bar{V}_s}{\bar{I}_i} = \frac{1}{j\omega C_i}\,\frac{1+3j\omega C_i R_i}{1+2j\omega C_i R_i}}$$

+5

2. (**25 points**) The figure below shows the straight-line approximation to the magnitude Bode plot for a certain filter. As the figure shows, the gain between 500 Hz and 700 Hz is 30 dB, and drops off on either side at 60 dB/dec. The gain levels off at 0 dB at frequencies $f_1$ and $f_2$. There are no poles or zeros outside the region shown in the plot. *Note that the plot is merely a sketch: it is NOT drawn to scale, and should not be used for quantitative calculations.*

a) It is known that the transfer function T(f) is multiplied by a constant that is real and negative. Find the transfer function.
b) What is the phase of the transfer function as f approaches infinity?




The slopes are 60 dB/dec and run for 30 dB, which means that $f_1$ and 500 Hz are separated by 1/2 dec, as are 700 Hz and $f_2$.

So...
$$\frac{500\ Hz}{f_1} = 10^{0.5} \qquad \frac{f_2}{700\ Hz} = 10^{0.5}$$

$$\Rightarrow f_1 \simeq 160\ Hz \qquad f_2 \simeq 2200\ Hz.$$

+5

Poles & Zeros:

+4 $z_1$ at 160 Hz $\qquad$ $p_1$ at 500 Hz

$z_2$ at 2200 Hz $\qquad$ $p_2$ at 700 Hz

3 times each! ($\pm 60$ dB/dec)

$$T(f) = K\,\frac{(1+jf/160)^3(1+jf/2200)^3}{(1+jf/500)^3(1+jf/700)^3}$$

We need K:

+4 For $f \to 0$, $T(f) \to K$, and $|T(f)| \to 0$ dB $\Rightarrow 1$ V/V

So $\underline{\underline{K = -1}}$. (negative since we are told this!)

$$+10 \qquad T(f) = -1 \cdot \frac{(1+jf/160)^3(1+jf/2200)^3}{(1+jf/500)^3(1+jf/700)^3}$$

+2 ii) For f large, $T(f) \to -1 \Rightarrow \angle T(f) \to 180^\circ$

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If we had evaluated $|T(f)|$ at, say, 600 Hz, we would find

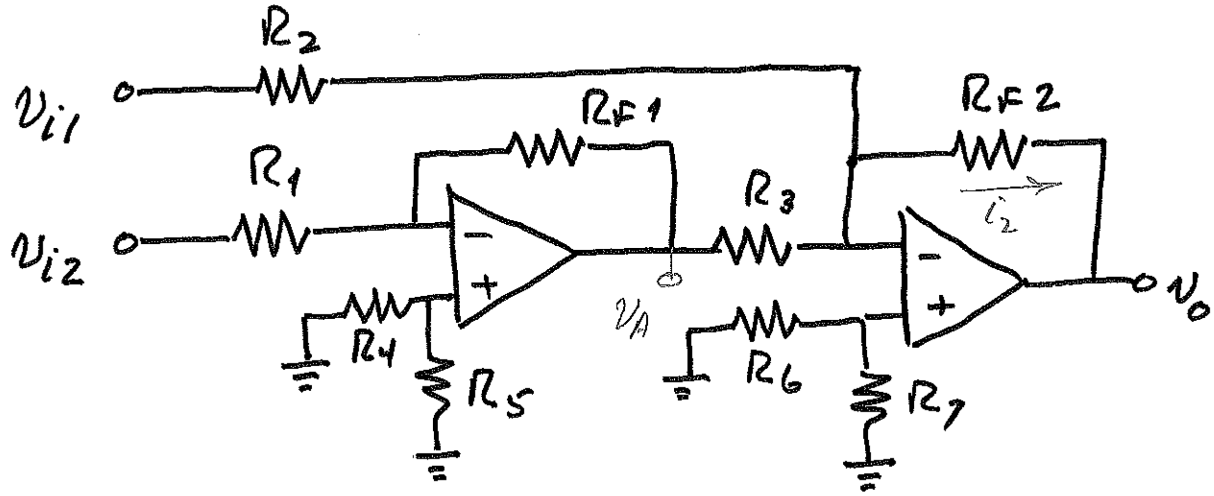
$$|T(f)| = |K| \cdot \left| \frac{(1+j600/160)^3(1+j600/2200)^3}{(1+j600/500)^3(1+j600/700)^3} \right| = |K| \times 2.48 = 10^{30/20}$$

$$\Rightarrow |K| = 4.23$$

Why the discrepancy? Because the break points are so close together that the actual function $|T(f)|$ does not reach 30 dB at 600 Hz: It is about 17.5 dB there. No credit was subtracted if K was found this way.

$A_d(v_{i2} - v_{i1})$

3. (**25 points**) If the resistances in the circuit below are chosen appropriately, the circuit is a differential amplifier, i.e., $v_o = A_d(\cancel{v_{i1} - v_{i2}})$. Choose resistances so that the differential gain $A_d$ is 10, and the input resistance seen by $v_{i1}$ and $v_{i2}$ are each 5 kΩ. Assume the op amps are ideal.




$$v_A = -\frac{R_{F1}}{R_1} v_{i2} \qquad i_2 = \frac{v_A}{R_3} + \frac{v_{i1}}{R_2}$$

$$\boxed{\begin{array}{l} R_1 = R_2 = 5\,k\Omega \\ R_{F1} = R_3 = 10\,k\Omega \\ R_{F2} = 50\,k\Omega \\ R_{4,5,6,7} = \text{anything} \end{array}}$$

$$\therefore\ v_o = -\frac{R_{F2}}{R_3} v_A - v_{i1}\frac{R_{F2}}{R_2}$$

$$= \frac{R_{F2} R_{F1}}{R_1 R_3} v_{i2} - \frac{R_{F2}}{R_2} v_{i1}$$

For $R_i = 5\,k\Omega$ for each input, we need

$$R \equiv R_2 = R_1 = 5\,k\Omega.$$

For $v_o = A_d(v_{i2} - v_{i1})$ we need

$$\frac{R_{F2} R_{F1}}{R R_3} = \frac{R_{F2}}{R} \Rightarrow \frac{R_{F1}}{R_3} = 1$$

Choose $R_{F1} = R_3 = 10\,k\Omega \Rightarrow R_{F2} = 50\,k\Omega$



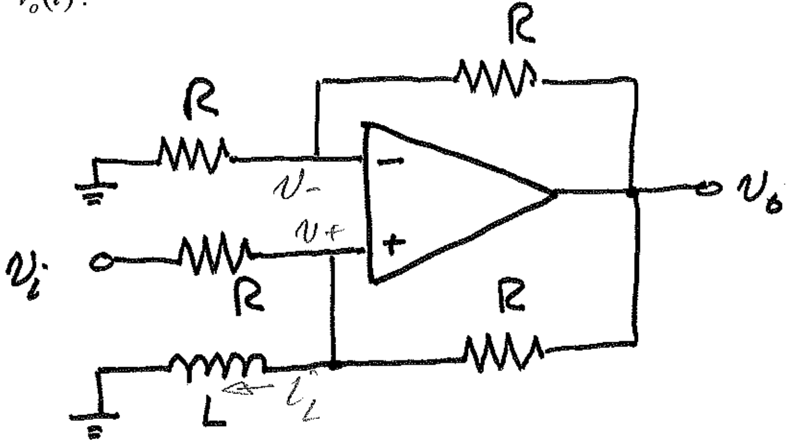
$R_4, R_5, R_6, R_7$ have no effect $\rightarrow 10\,k\Omega$

4. **(25 points)** For the circuit below, assume the op amp is ideal.

i) Find $v_o(t)$ as a function of $v_i(t)$ for arbitrary $v_i(t)$.

ii) Find the transfer function $T(\omega) = \dfrac{\bar{V}_o}{\bar{V}_i}$.

iii) Show that if $v_i(t) = V_m \cos(\omega t)$, your answers to part i) and part ii) predict the same thing for $v_o(t)$.




i)

$$v_- = v_+ = \frac{1}{2} v_o \qquad \frac{\frac{1}{2} v_o - v_i}{R} + \frac{\frac{1}{2} v_o - v_o}{R} + \frac{1}{L}\int \frac{1}{2} v_o \, dt + i_L(0) = 0$$

$$\Rightarrow \quad \frac{-v_i}{R} + \frac{1}{L}\int \frac{1}{2} v_o \, dt + i_L(0) = 0$$

$$-\frac{1}{R}\frac{dv_i}{dt} + \frac{1}{2L} v_o = 0$$

$$\Rightarrow \quad \boxed{v_o = \frac{2L}{R}\frac{dv_i}{dt}}$$

ii)

$$\frac{\frac{1}{2}\bar{V}_o - \bar{V}_i}{R} + \frac{\frac{1}{2}\bar{V}_o - \bar{V}_o}{R} + \frac{\frac{1}{2}\bar{V}_o}{j\omega L} = 0$$

$$\Rightarrow \quad \bar{V}_o = \frac{2j\omega L}{R}\bar{V}_i$$

$$\boxed{\frac{\bar{V}_o}{\bar{V}_i} = \frac{2j\omega L}{R}}$$

Room for extra work

iii)

$v_i = V_m \cos \omega t$

$\frac{dv_i}{dt} = -\omega V_m \sin \omega t$

$\underbrace{v_0 = -\frac{2L\omega}{R} V_m \sin \omega t}$

$\bar{V}_i = V_m \angle 0^\circ$

$\bar{V}_0 = \frac{2j\omega L}{R} V_m \angle 0^\circ$

$= \frac{2\omega L}{R} V_m \angle 90^\circ$

$\Rightarrow v_0 = \frac{2L\omega}{R} V_m \cos(\omega t + 90^\circ)$

$\underbrace{v_0 = -\frac{2L\omega}{R} V_m \sin(\omega t)}$

Q.E.D.