Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Final Exam
December 11, 2010

Exam duration: 170 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 12 pages, including the cover sheet. Raise your hand if you are missing a page.

1 _____________/45

2 _____________/35

3 _____________/35

4 _____________/45

5 _____________/40

Total _______________/200

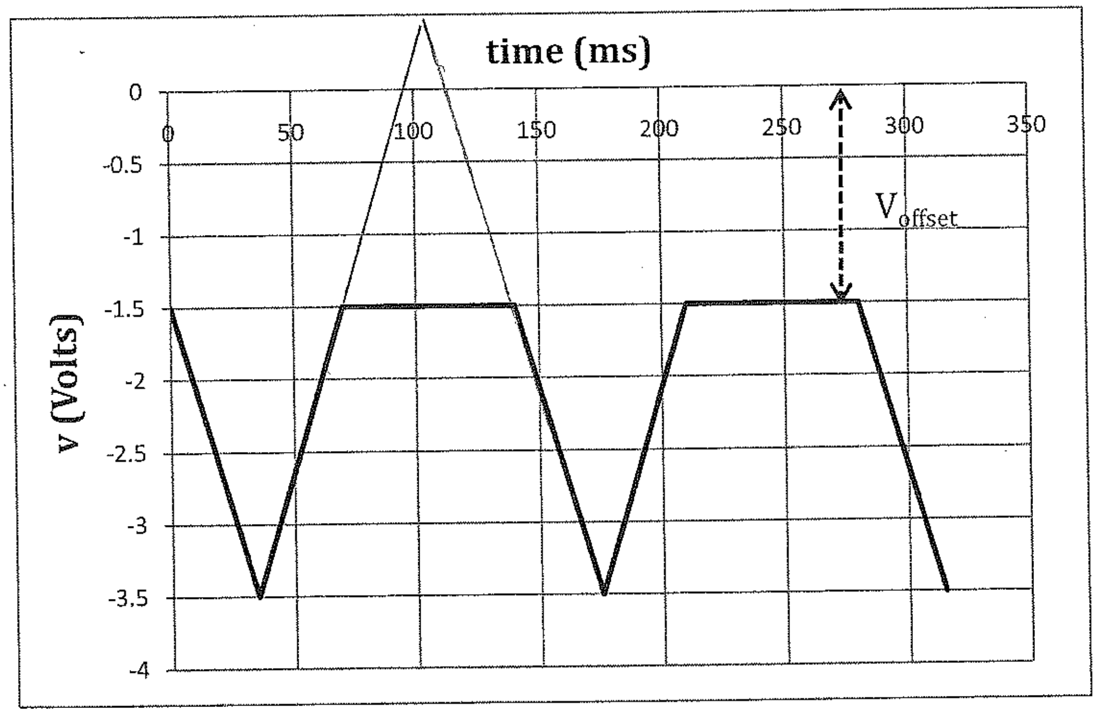
Note: the exam score will be normalized to 100.

1. (**45 points**) Design a circuit that produces the output shown in the figure below. To do so, assume that you have an unlimited number of the following parts available.

- Ideal dual-power-supply op amps.
- 9 V batteries.
- Ideal BJTs with a β of 100 and a $V_{CE,sat}$ of 0.3 V.
- Diodes that can be modeled using a constant voltage drop model with V_{th} = 0.7 V.
- Ideal transformers.
- Capacitors of any value less than or equal to 10 μF; resistors of any value less than or equal to 500 kΩ.; potentiometers of any number of turns and any resistance.

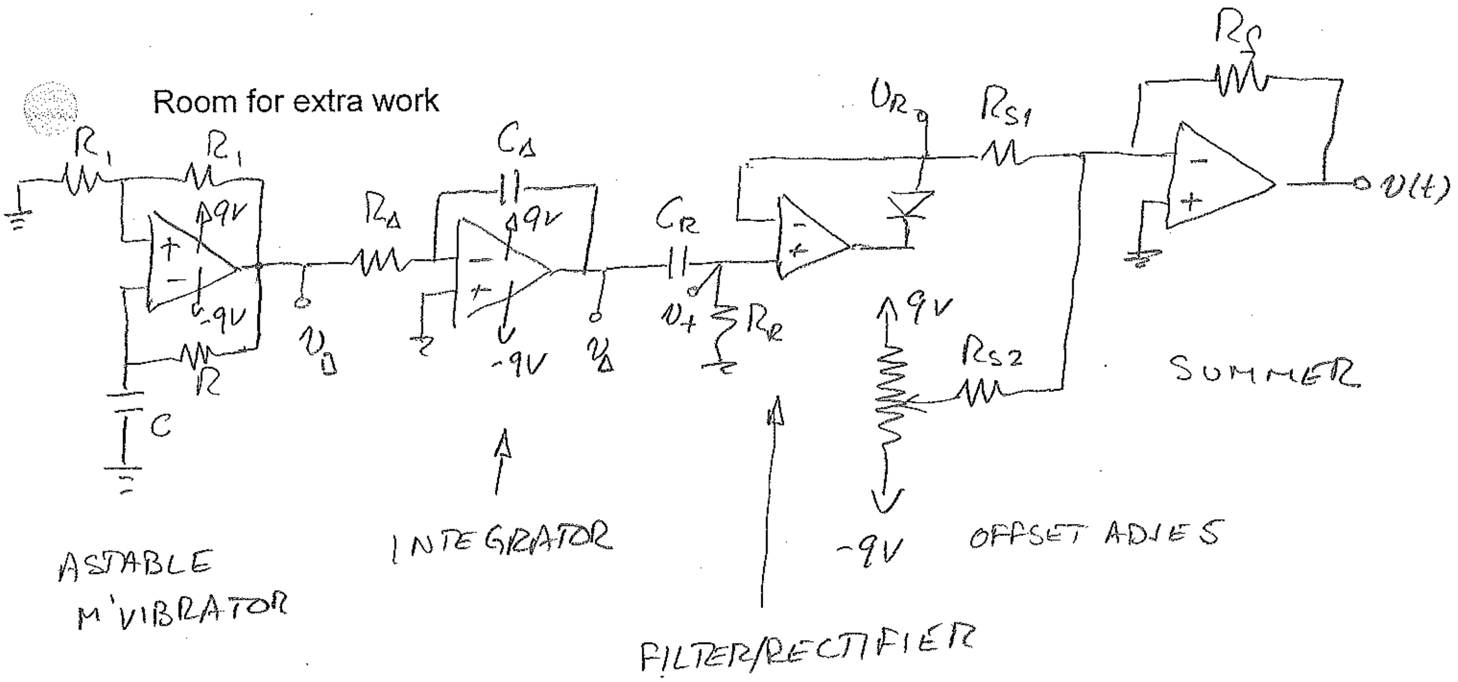
As an additional specification, the value labeled V_{offset} must be adjustable between -3 to +3 V. (In the figure it is set at -1.5 V.)

To receive full credit, you must provide a brief explanation (a few words or a quick sketch) of what each part of your circuit is intended to do.



There are several ways this could be done. Here is one...
Evidently this is a half-wave reftified triangle wave with amplitude 4 V and period 140 ms.

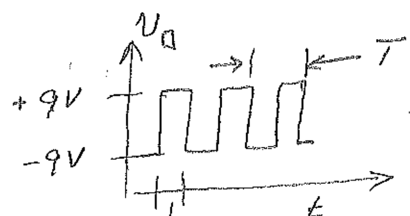
Room for extra work



Astable mv: $T = 2RC \ln \dfrac{1+\beta}{1-\beta}$ $\quad R_1 = 10\,k\Omega \Rightarrow \beta = \dfrac{R_1}{R_1+R_1} = 0.5$

$T = 0.140\ s \Rightarrow RC = \dfrac{0.140}{2\ln\frac{3/2}{1/2}} = 0.0637\ s^{-1}$

$\boxed{R = 100k \Rightarrow C = 0.64\ \mu F}$

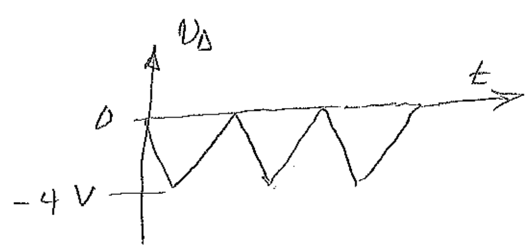


Integrator: $\quad v_\Delta = -\dfrac{1}{R_\Delta C_\Delta}\displaystyle\int_0^{T/2} v_\square\, dt = -\dfrac{1}{R_\Delta C_\Delta}\, 9 \cdot \dfrac{T}{2} = -4\,V$

The integration gives the peak-peak value, which is 4 V. We have assumed that $v_\square(t=0) = +9\,V$, so our v_Δ is negative.

$R_\Delta C_\Delta = 9\,\dfrac{T}{2}\,\dfrac{1}{4} = 0.1575\ s^{-1}$

$\boxed{R_\Delta = 100\,k\Omega \Rightarrow C_\Delta = 1.58\ \mu F}$

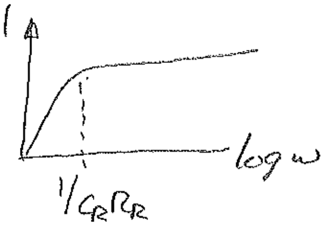


p. 2

Room for Extra Work

Filter: To remove dc, we use a low pass filter. We need the breakpoint to be below the lowest frequency component, which is $f_0 = 1/T = 7.14\ Hz$. We have

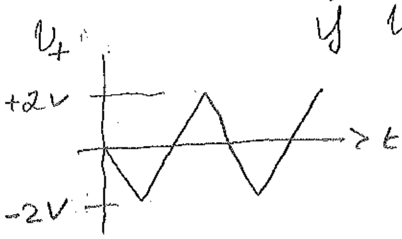
$$T(\omega) = \frac{\bar{V}_+}{\bar{V}_\Delta} = \frac{R_R}{R_R + 1/j\omega C_R} = \frac{j\omega C_R R_R}{1 + j\omega C_R R_R}$$

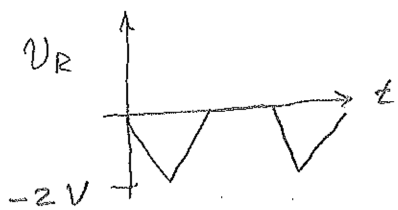


We want $\frac{1}{C_R R_R} \ll 7.14$. So we can just make $C_R R_R$ large, but if we say $\frac{1}{C_R R_R} = 0.714$ and

choose $\boxed{R_R = 100\,k\Omega \Rightarrow C_R = 7.14\,\mu F}$

Rectifier: This is just a superdiode that outputs 0 if $v_\Delta > 0$.





Summer: We don't need gain for v_R, so $R_f = R_{S1}$.

Offset: With the pot set for 9V, we want $-3V$ at $v(t)$.

So $$-\frac{R_f}{R_{S2}} = -3 \Rightarrow R_f = 3R_{S2}.$$

Choose $\boxed{R_{S1} = 10\,k\Omega \quad R_f = 10\,k\Omega \quad R_{S2} = 3.33\,k\Omega}$

2. (**35 points**) The transfer function for a certain filter has the following characteristics.

i) A double zero at 0 rad/s.
ii) A pole at 800 rad/s.
iii) A double pole at 8,000 rad/s.
iv) A double zero at 40,000 rad/s.
v) A gain of 54 dB at 4200 rad/s.

+14 a) Find the transfer function T(ω) for this filter. Assume that any constants necessary to adjust the gain are positive and real.
+21 b) On the page that follows, draw the straight-line approximation to the phase Bode plot. Include all non-zero poles and zeroes.

+2 ea. = 8 +2 12

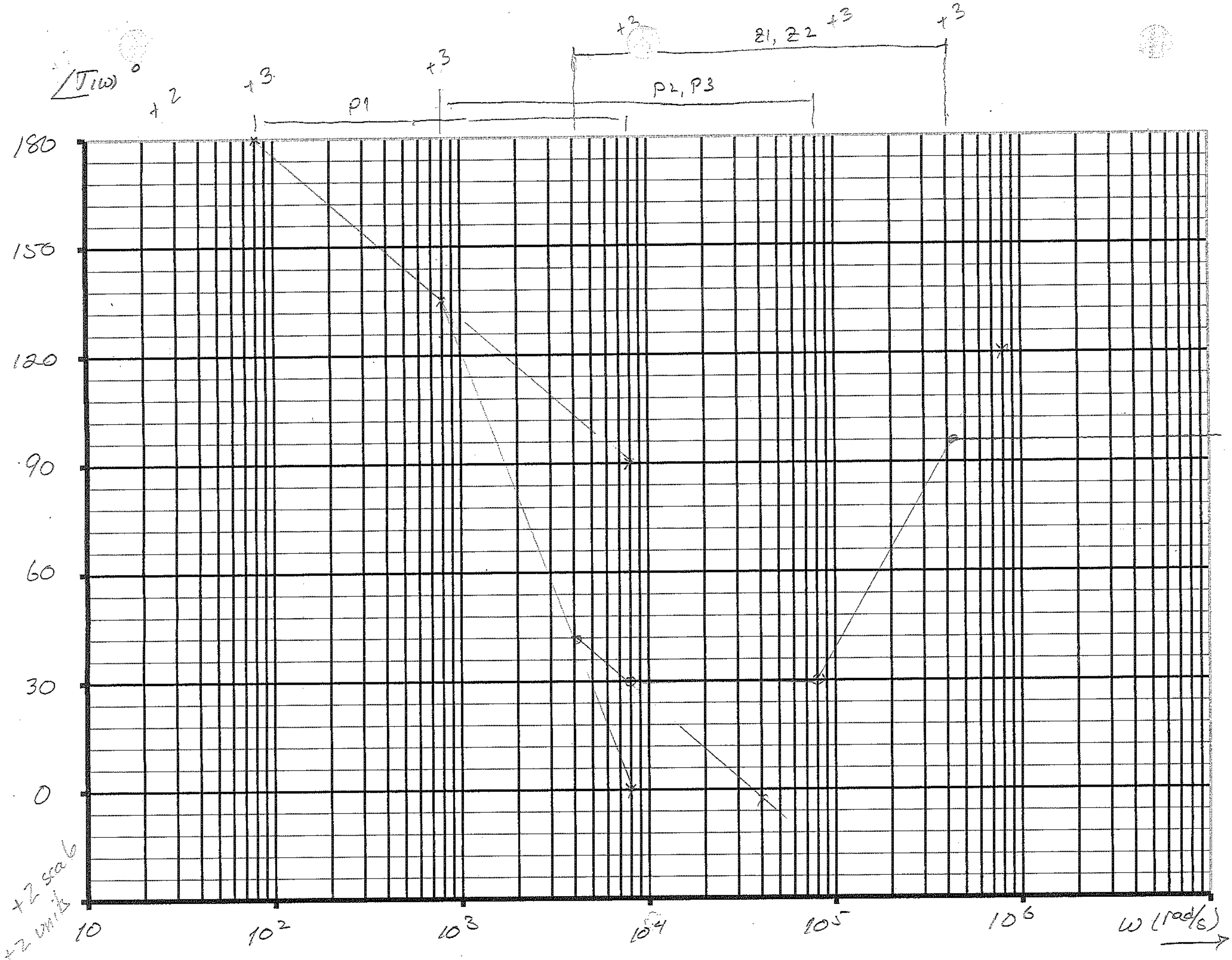
$$\boxed{T(\omega) = K \cdot \frac{(j\omega)^2 (1 + j\omega/40000)^2}{(1 + j\omega/800)(1 + j\omega/8000)^2}}$$

$$|T(\omega = 4200 \text{ rad/s})| = 54\,dB \rightarrow 502 \text{ V/V}$$

$$T(\omega = 4200 \text{ rad/s}) = \frac{(j\,4200)^2 (1 + j\,4200/40000)^2}{(1 + j\,4200/800)(1 + j\,4200/8000)^2}$$

$$= 2.616 \times 10^6 \angle 57.37^\circ \text{ V/V}$$

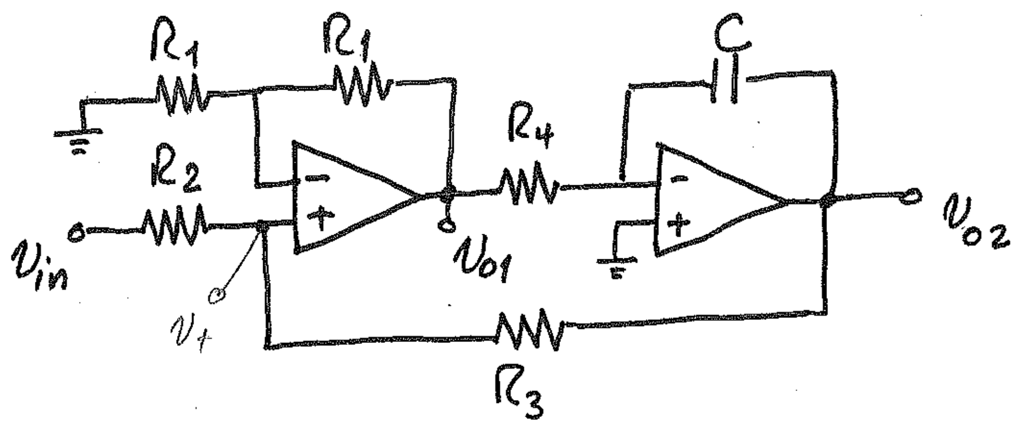
+4 So $k \cdot 2.616 \times 10^6 = 502 \Rightarrow \boxed{K = 1.92 \times 10^{-4}}$



$\angle T(j\omega)$ °
+2
+3
+3
+3
+3
+3
P1
P2, P3
Z1, Z2
180
150
120
90
60
30
0
10
10^2
10^3
10^4
10^5
10^6
ω (rad/s)
+2 scale
+2 units

3. (**35 points**) The circuit below is intended to be a dual-purpose filter. Assume ideal op amps in solving it.

a) Find the transfer function $T_1(\omega) = \mathbf{V_{01}/V_{in}}$.
b) Find the transfer function $T_2(\omega) = \mathbf{V_{02}/V_{in}}$.
c) Identify the breakpoint(s) for each of $T_1(\omega)$ and $T_2(\omega)$, and indicate the nature of the filtering action (low pass, high pass, band pass, something else...)



a), b)

+15

$$\frac{\bar V_{02}}{\bar V_{01}} = -\frac{1/j\omega C}{R_4} = \frac{-1}{j\omega C R_4}$$

$$\bar V_{01} = 2\bar V_+ \Rightarrow \bar V_+ = \frac{1}{2}\bar V_{01}$$

$$\frac{\bar V_+ - \bar V_{in}}{R_2} + \frac{\bar V_+ - \bar V_{02}}{R_3} = 0 \Rightarrow \bar V_+ = \frac{R_3}{R_2+R_3}\bar V_{in} + \frac{R_2}{R_2+R_3}\bar V_{02}$$

simplify ... $\bar V_+ = \alpha \bar V_{in} + \beta \bar V_{02}$

$$\bar V_+ = \frac{1}{2}\bar V_{01} = \alpha \bar V_{in} - \beta\frac{\bar V_{01}}{1+j\omega CR_4}$$

$$\bar V_{01}\left(1 + \frac{2\beta}{j\omega CR_4}\right) = 2\alpha \bar V_{in}$$

+5 +15

$$\boxed{T_1(\omega) = \frac{\bar V_{01}}{\bar V_{in}} = \frac{2\alpha j\omega CR_4}{2\beta + j\omega CR_4}}$$

$$\boxed{T_2(\omega) = \frac{\bar V_{02}}{\bar V_{in}} = \frac{-2\alpha}{2\beta + j\omega CR_4}}$$

Room for extra work

c) $T_1(\omega)$ has a zero at 0 and a pole at

$$\omega_{p1} = \frac{2\beta}{CR_4} = \frac{2R_2/(R_2+R_3)}{CR_4}$$

+5

This is a high-pass filter.

$T_2(\omega)$ has a pole at

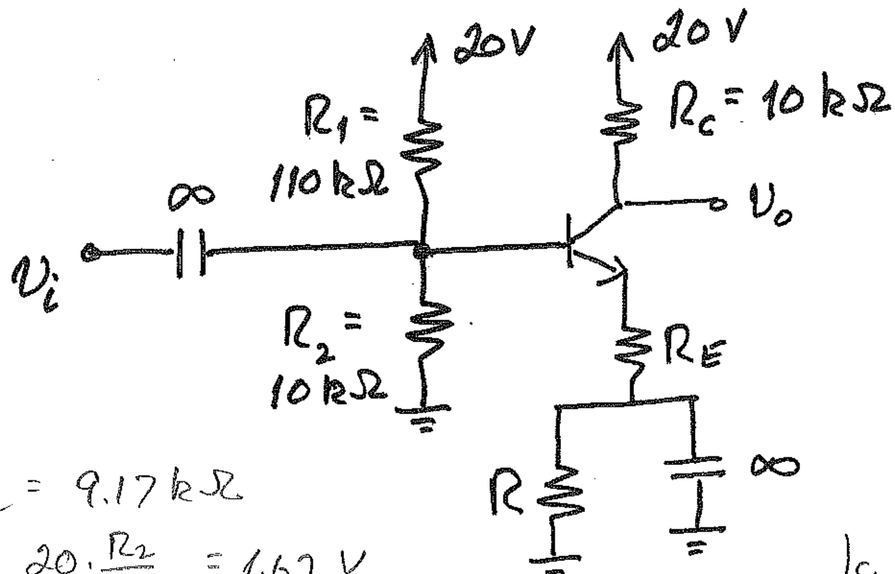
$$\omega_{p2} = \frac{2\beta}{CR_4} = \frac{2R_2/(R_2+R_3)}{cR_4}$$

+5

This is a low-pass filter with the same breakpoint as the high-pass.

4. (**45 points**) The circuit below is intended to be a high gain voltage amplifier. Find values of R_E and R that will complete the design. The BJT has $\beta = 60$ and $V_{CE,sat} = 0.3$ V. The specifications are as follows.

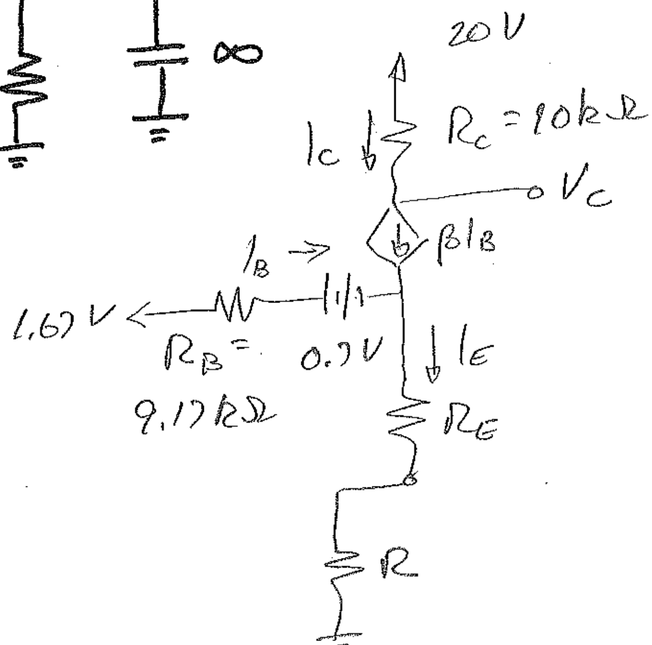
i) The amplifier is to operate over the frequency range 20 Hz to 20 kHz.
ii) The gain in the pass band v_o/v_i is to be 34 dB.
iii) The dc value of the collector voltage should be 10 V.



34 dB ⇒ 50 V/V

$R_B = R_1 \| R_2 = 9.17\text{ k}\Omega$

$V_{Th} = 20 \cdot \frac{R_2}{R_1 + R_2} = 1.67\text{ V}$



+ 8

+ 2
I_C

DC Analysis :

We are given $V_C = 10\text{ V}$

$\Rightarrow \quad I_C = \beta I_B = 1\text{ mA}.$

Then $I_B = \frac{I_C}{\beta} = 16.67\mu A$

$I_E = (\beta+1) I_B = 1.017\text{ mA}$

This allows us to determine only the sum $R_E + R$:

$-1.67 + I_B R_B + 0.7 + (\beta+1) I_B (R_E + R) = 0$

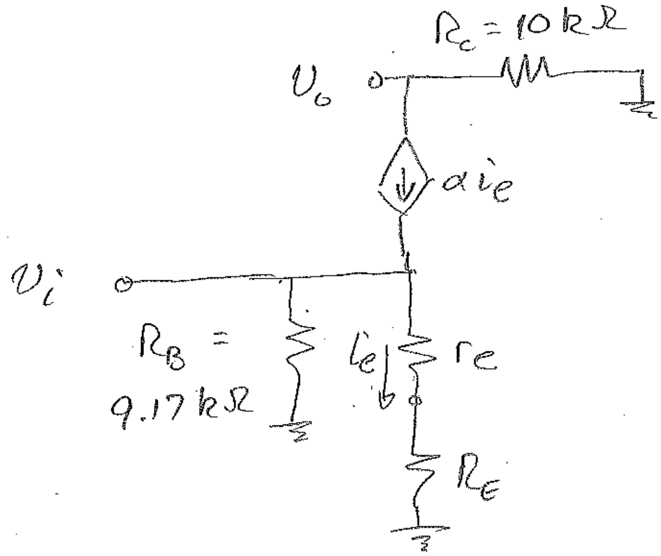
$(V_E = 1.67 - 0.7 - I_B R_B = 0.817\text{ V})$

Room for extra work

$$R_E + R = \frac{1.67 - 0.7 - I_B R_B}{(\beta+1) I_B} = 804\ \Omega \qquad +10$$

To go farther we need the ac model.

+10



$$r_e = \frac{V_T}{I_E} = \frac{0.025}{1.017\,mA} = 24.6\ \Omega$$

$$\alpha = \frac{\beta}{\beta+1} = 0.984$$

$$v_o = -\alpha i_e R_c \qquad i_e = v_i/(r_e + R_E)$$

+10

$$\frac{v_o}{v_i} = \frac{-\alpha R_c}{r_e + R_E} \Rightarrow \frac{\alpha R_c}{r_e + R_E} = 50$$

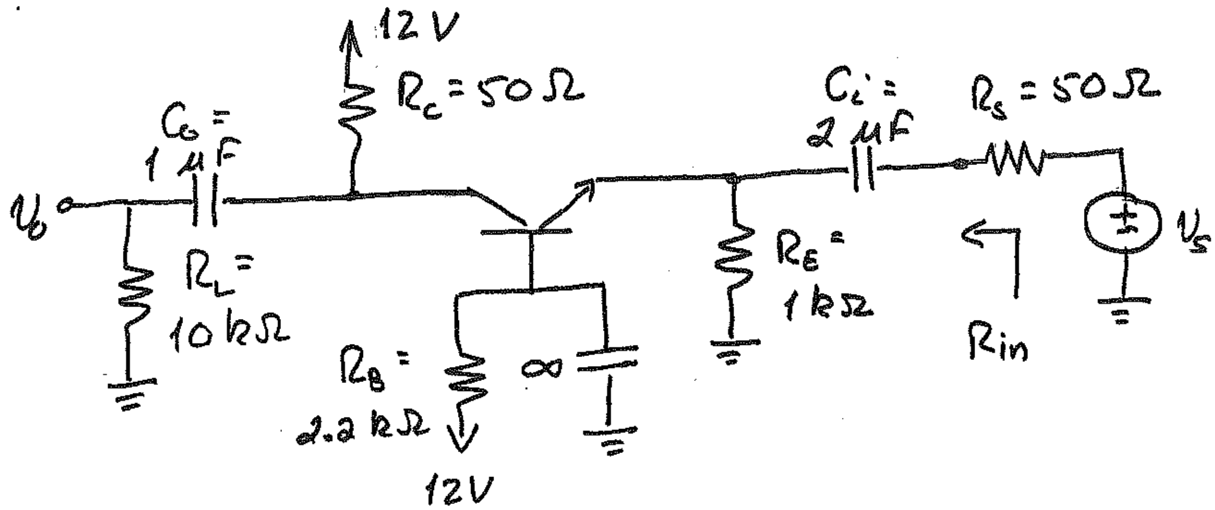
$$\therefore\ r_e + R_E = \frac{\alpha R_c}{50} = 197\ \Omega$$

$$\therefore\ \boxed{R_E = 172\ \Omega \Rightarrow R = 632\ \Omega} \qquad +5$$

So the operating frequency range is irrelevant!

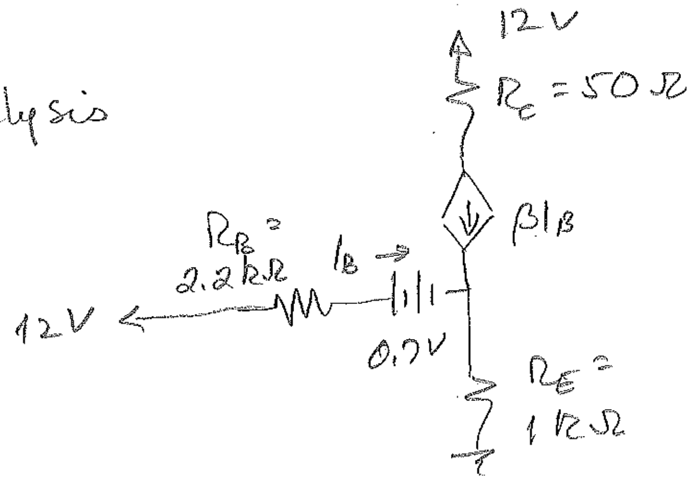
5. (**40 points**) The circuit below is a common base amplifier. You may assume that it is operating in the linear region. It is characterized by $\beta = 100$ and $V_{CE,sat} = 0.3$ V.

a) Find the gain v_o/v_s in the pass band.
b) Find the input resistance R_{in} in the pass band.



DC Analysis

+8



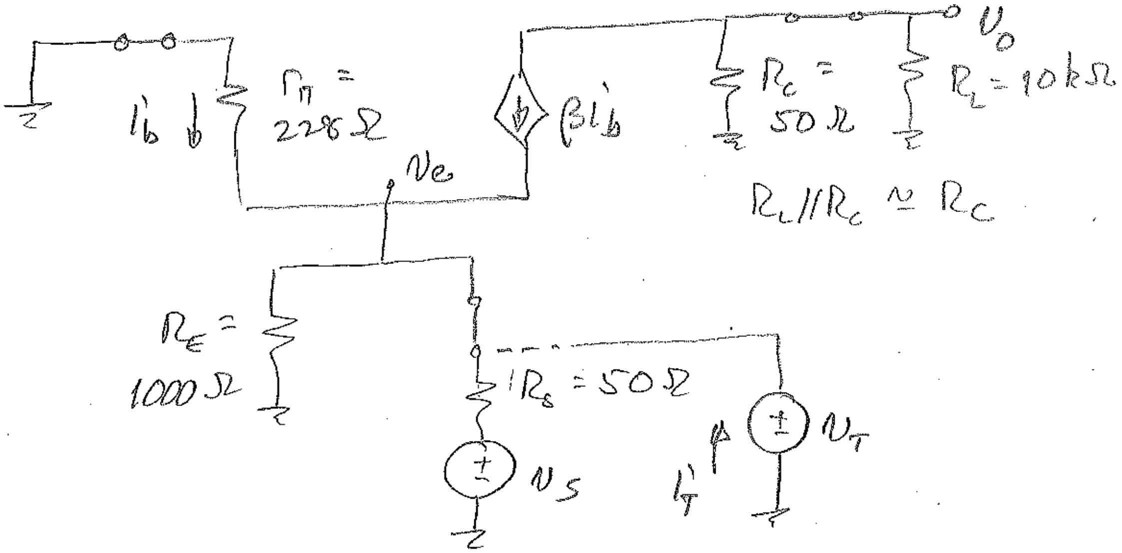
$$r_\pi = \frac{V_T}{I_B} = 228\,\Omega$$

+2

$$I_B = \frac{12-0.7}{2.2\,k\Omega + (\beta+1)(1000)} = 0.1095\ mA$$

In the pass band, $C_i \rightarrow$ short, $C_o \rightarrow$ short. Also, the base capacitor is infinite, so it is also a short at ac.

+12



a) Replace v_s, R_s with a test voltage. Then

+8

$$i_T' = \frac{v_T}{R_E} - (\beta+1)\, i_b' \qquad i_b' = -\frac{v_T}{r_\pi}$$

$$\therefore\ i_T' = v_T\left(\frac{1}{R_E} + \frac{\beta+1}{r_\pi}\right)$$

$$\boxed{R_{in} = \frac{R_E\, r_\pi}{(\beta+1)R_E + r_\pi} = 2.25\,\Omega}$$

b)

$$v_o = -\beta i_b' \cdot R_C // R_L \qquad (\beta+1)\frac{v_e}{r_\pi} + \frac{v_e}{R_E} + \frac{v_e - v_s}{R_s} = 0$$

+10

$$i_b' = -\frac{v_e}{r_\pi} \qquad v_e\left(\frac{\beta+1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_s}\right) = \frac{v_s}{R_s}$$

$$\frac{v_e}{v_s} = \frac{1}{50}\cdot\left(\frac{101}{228} + \frac{1}{1000} + \frac{1}{50}\right)^{-1}$$

$$= 4.31\times10^{-2}$$

$$\frac{v_o}{v_s} = \frac{\beta}{r_\pi}\cdot\frac{v_e}{v_s} R_C // R_L$$

$$= \frac{100}{228}\cdot 4.31\times10^{-2}(50)$$

$$\boxed{\frac{v_o}{v_s} = 0.945\,\frac{V}{V}}$$