Name: ______________________________ (please print)

Signature: ___________________________

ECE 3355 – Exam #1
October 5, 2019

Keep this exam closed and face up until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. Show all units in solutions, intermediate results, and figures.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam.

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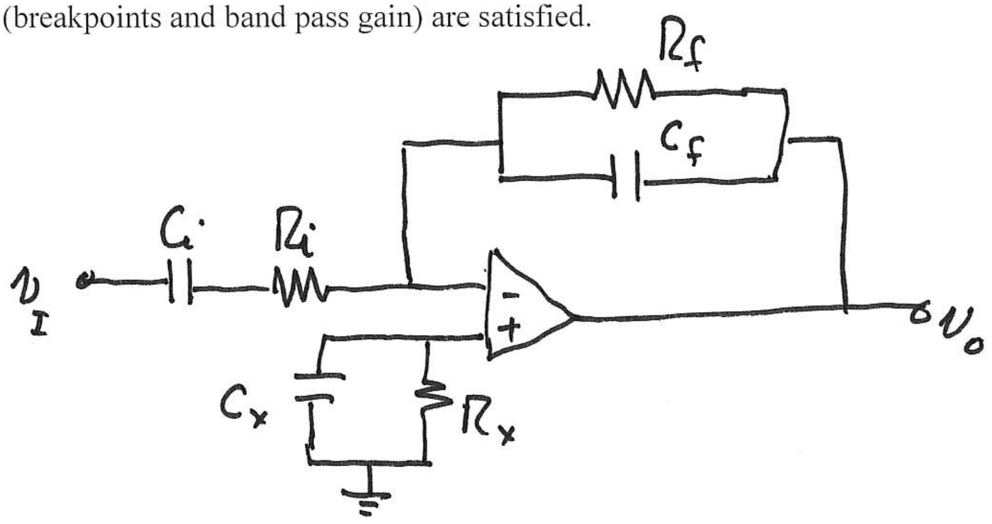
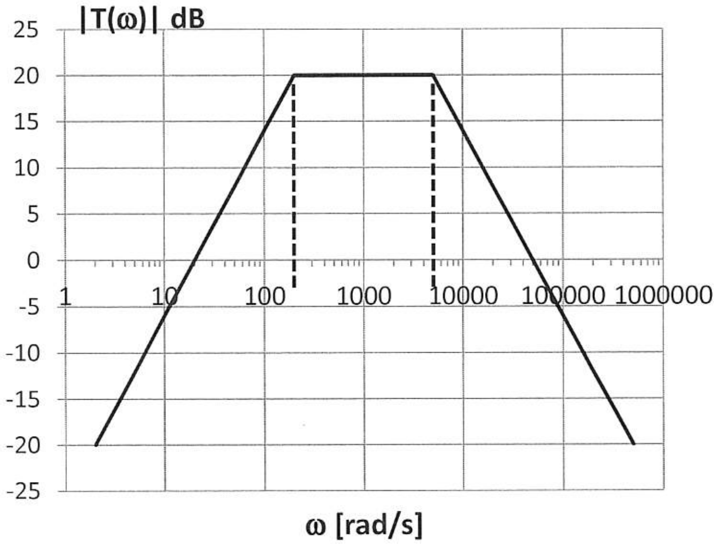
__________________/100

Room for extra work

1. (35 points) The circuit below is intended to be a band-pass filter, with a magnitude Bode plot as shown in the graph.

a) Find the transfer function T(ω) = **V₀/Vᵢ**.
b) Choose values for the capacitors and resistors so that the Bode plot specifications (breakpoints and band pass gain) are satisfied.





Room for extra work

2. (30 points)

a) Using the graph paper on the next page, draw the straight-line approximation to the phase Bode plot for a transfer function with the following properties.

- zeros at $\omega = 200$ and 800 rad/s
- poles at $\omega = 0$ and 5000 rad/s
- a magnitude of 40 dB at high frequencies

b) Write the transfer function $T(\omega)$ that has these characteristics.

3. (35 points)

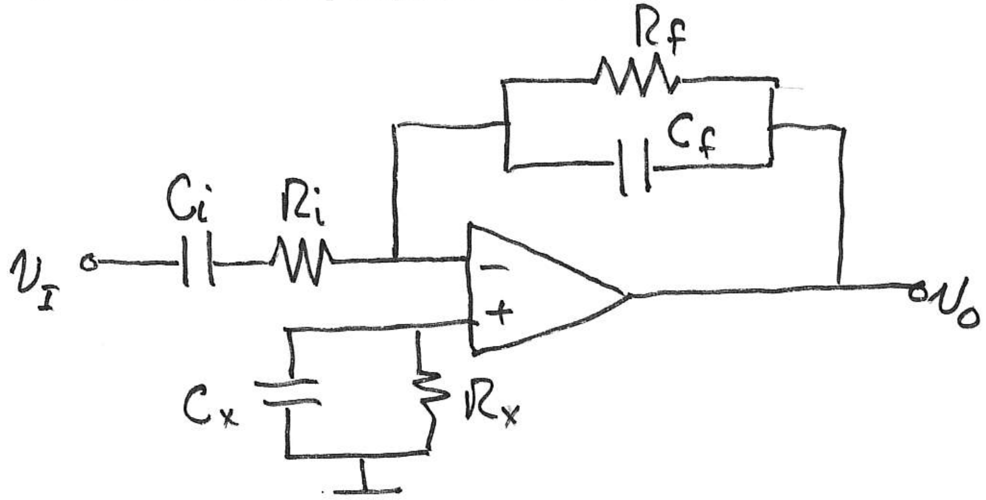
a) Using any components you choose, design a low-pass filter with a breakpoint of 2000 rad/s, a high-frequency roll-off (the rate at which the magnitude Bode plot decreases) of -20 dB/dec, and a gain at low frequencies of 20 dB.

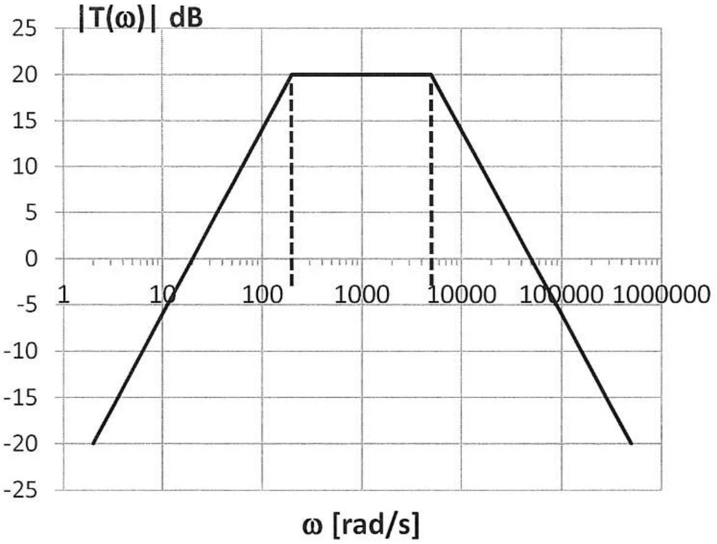
b) By extending your design (or creating a new one) create a low-pass filter with the same breakpoint and low frequency gain, but a high-frequency roll-off of 40 dB/dec.

Room for extra work

1. (35 points) The circuit below is intended to be a band-pass filter, with a magnitude Bode plot as shown in the graph.

a) Find the transfer function T(ω) = **V**$_o$/**V**$_i$.
b) Choose values for the capacitors and resistors so that the Bode plot specifications (breakpoints and band pass gain) are satisfied.





a) C_x, R_x are not relevant since there is no current at the input. The rest of the circuit is in the inverting configuration ...

Room for extra work

$$T(\omega) = \frac{\tilde{V}_o}{\tilde{V}_i} = \frac{-R_f // 1/j\omega C_f}{1/j\omega C_i + R_i} = -\frac{R_f}{1+j\omega C_f R_f} \cdot \frac{j\omega C_i}{1+j\omega C_i R_i}$$ +15

So we have a zero at $1/C_i R_f$

poles at $1/C_f R_f$, $1/C_i R_i$

b)

There is no way to decide which of the poles corresponds to which $1/RC$, so we'll pick something:

① $1/C_i R_i = 200$ rad/s ② $1/C_f R_f = 5000$ rad/s +4 +4

In the passband, $\omega_{PB} >> 1/C_i R_i$ and $\omega_{PB} << 1/C_f R_f$ so

$$T(\omega_{PB}) \cong \frac{-j\omega C_i R_f}{(1)(j\omega C_i R_i)} \Rightarrow |T(\omega_{PB})| = ③\ \frac{R_f}{R_i} = 20\text{dB} = 10\ \frac{V}{V}$$ +6

So we have 3 constraints ①, ②, ③. $C_i = 10\, C_f$

Choose $\underline{R_i = 1\text{k}\Omega}$ $\underline{R_f = 10\text{k}\Omega}$ +2

So $\underline{C_f} = \frac{1}{R_f(5000)} = \underline{0.02\,\mu F}$ +2

$\underline{C_i} = \frac{1}{R_i \cdot (200)} = \underline{5\,\mu F}$ +2

If we had chosen $1/C_i R_i = 5000$ rad/s, $1/C_f R_f = 200$ rad/s,

then $$T(\omega_{PB}) \simeq \frac{-j\omega C_i R_f}{(j\omega C_f R_f)(1)} \Rightarrow |T(\omega_{PB})| = \frac{C_i}{C_f} = 10$$

Choose $\underline{C_f = 1\mu F} \Rightarrow \underline{C_i = 10\ \overset{4}{\mu}F}$ and $\underline{R_f = 5\text{ k}\Omega}$

$\underline{R_i = 20\ \Omega}$

2. (30 points)

a) Using the graph paper on the next page, draw the straight-line approximation to the phase Bode plot for a transfer function with the following properties.

- zeros at ω = 200 and 800 rad/s
- poles at ω = 0 and 5000 rad/s
- a magnitude of 40 dB at high frequencies

b) Write the transfer function T(ω) that has these characteristics.

a) I have indicated at the top of the graph the range over which the zeros and poles are active.

The pole at 0 means that at low ω, $\angle T(\omega) \rightarrow -90^\circ$, but it does create a slope on the plot.

We can check that our plot is correct for $\omega \rightarrow \infty$ after we write the transfer function:

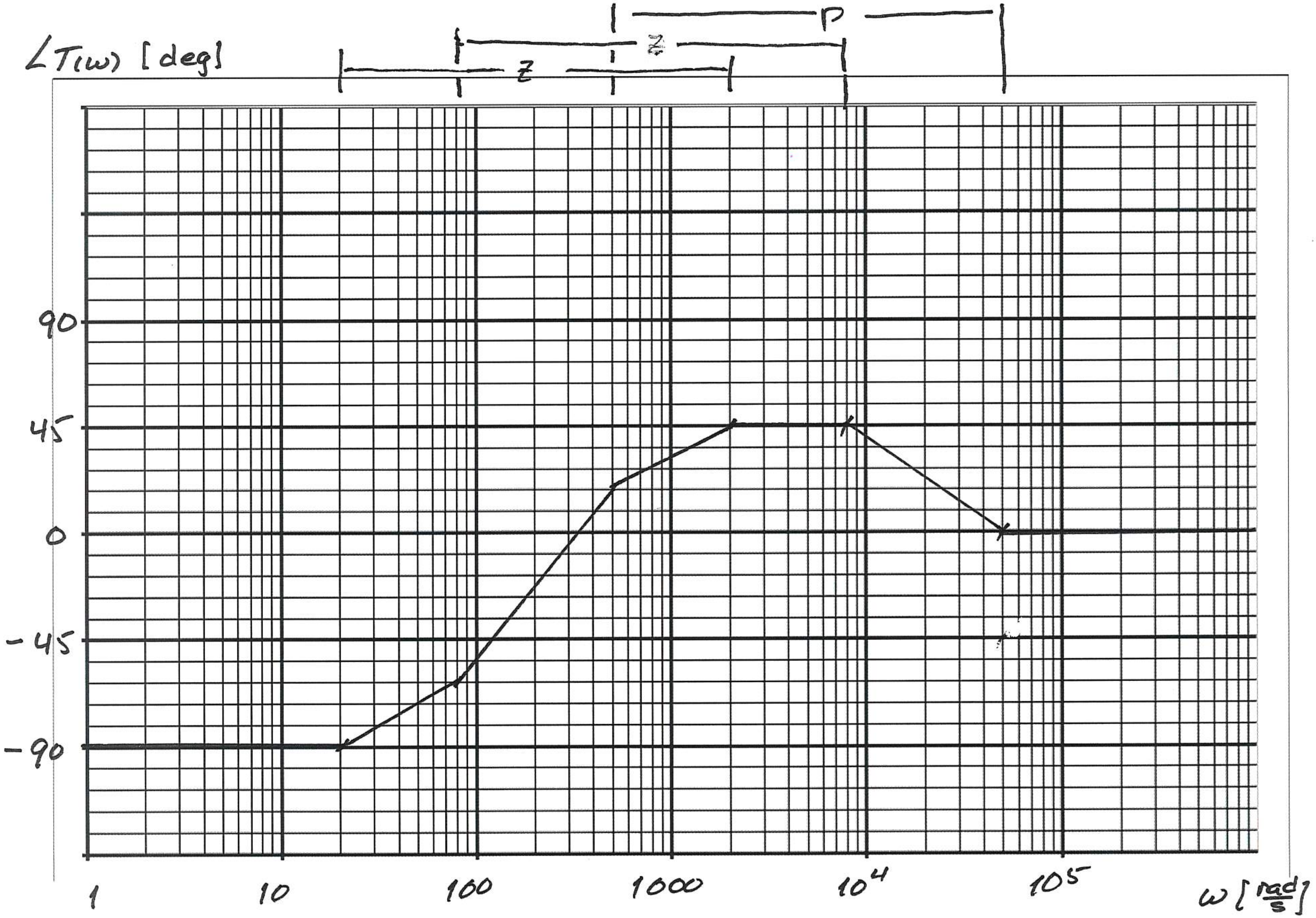
b) $$T(\omega) = K \frac{(1+j\omega/200)(1+j\omega/800)}{j\omega(1+j\omega/5000)}$$ + 10

$$\omega \rightarrow \infty \Rightarrow \quad T(\omega) \rightarrow K \cdot \frac{j\omega/200 \cdot j\omega/800}{j\omega \cdot j\omega/5000} = K \cdot 0.03125$$

So $0.03125 \cdot K = 40\text{dB} = 100 \Rightarrow \boxed{K = 3200}$ + 4

Also note that $\omega \rightarrow \infty \Rightarrow T(\omega) = 100$

$\Rightarrow \angle T(\omega) = 0^\circ$, as we have on our plot.



starting phase +1

breakpoints +1 +1 +1 +1 +1 +1

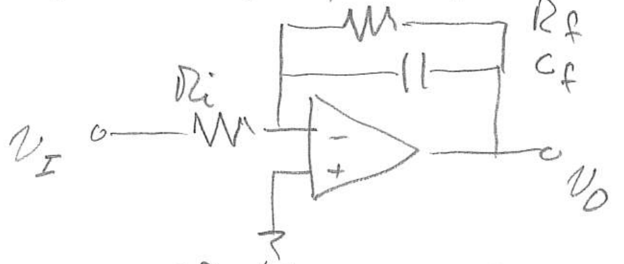
slope +1 +1 +1 +1 +1 +1

3. (35 points)

20 a) Using any components you choose, design a low-pass filter with a breakpoint of 2000 rad/s, a high-frequency roll-off (the rate at which the magnitude Bode plot decreases) of -20 dB/dec, and a gain at low frequencies of 20 dB.

15 b) By extending your design (or creating a new one) create a low-pass filter with the same breakpoint and low frequency gain, but a high-frequency roll-off of 40 dB/dec. (−40 dB/dec)

a) The following op amp will do the job:



$$T(\omega) = \frac{V_o}{V_i} = \frac{-R_f/(1+j\omega C_f R_f)}{R_i} = -\frac{R_f}{R_i}\frac{1}{1+j\omega C_f R_f}$$

Breakpoint $\omega_0 = 1/C_f R_f = 2000$ rad/s

$\omega \to 0 \Rightarrow T(\omega) \simeq \frac{R_f}{R_i} = 20\ dB = 10\ \frac{V}{V}$

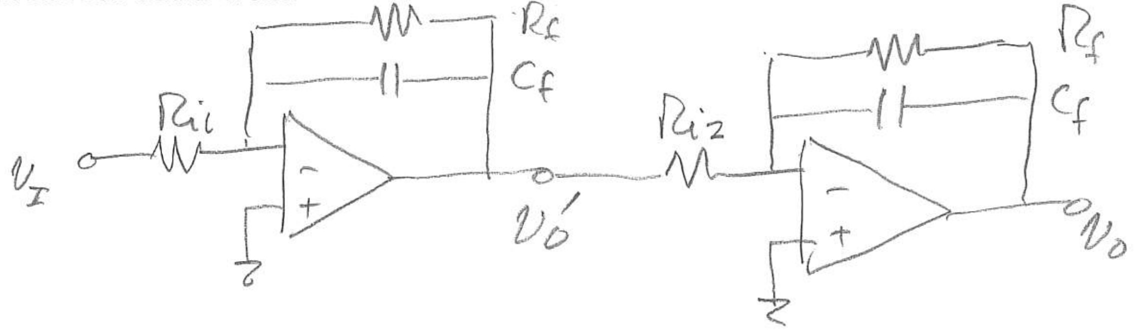
$R_i = 10 R_f$ Choose $R_f = 1\ k\Omega \Rightarrow R_i = 10\ k\Omega$

$$C_f = \frac{1}{2000 \cdot R_f} = 50\ nF$$

b) We can cascade two of these:

↗

Room for extra work



$$\bar{V}_o' = -\bar{V}_i \frac{R_f}{R_{i1}} \frac{1}{1+j\omega C_f R_f} \qquad \bar{V}_o = -\bar{V}_o' \cdot \frac{R_f}{R_{i2}} \frac{1}{1+j\omega C_f R_f}$$

$$T(\omega) = \frac{\bar{V}_o}{\bar{V}_i} = \frac{R_f \cdot R_f}{R_{i1} R_{i2}} \cdot \frac{1}{(1+j\omega C_f R_f)^2}$$

$R_f = 1\,k\Omega$
$C_f = 50\,nF$

So now we have a double pole at 2000 rad/s which gives a slope −40 dB/dec.

$$|T(\omega \to 0)| = \frac{R_f \cdot R_f}{R_{i1} R_{i2}} = 10 \Rightarrow R_{i1} \cdot R_{i2} = \frac{R_f^2}{10} = 10^5$$

So we can leave R_i at $R_i = 10\,k\Omega$, but R_{i2} has to be $10\,\Omega$.

A better choice might be $R_{i1} = 1\,k\Omega$, $R_{i2} = 100\,\Omega$.

$$\frac{R_f^2}{R_i^2} = 10 = \frac{R_f}{R_i} = \sqrt{10} \sim 3$$