Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Final Exam
December 5, 2007

Exam Rating (out of 5 chili peppers): 

Exam duration: 170 minutes

- You may have ***two*** 8 ½ x 11 in. "crib" sheets, written on both sides, during the exam. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

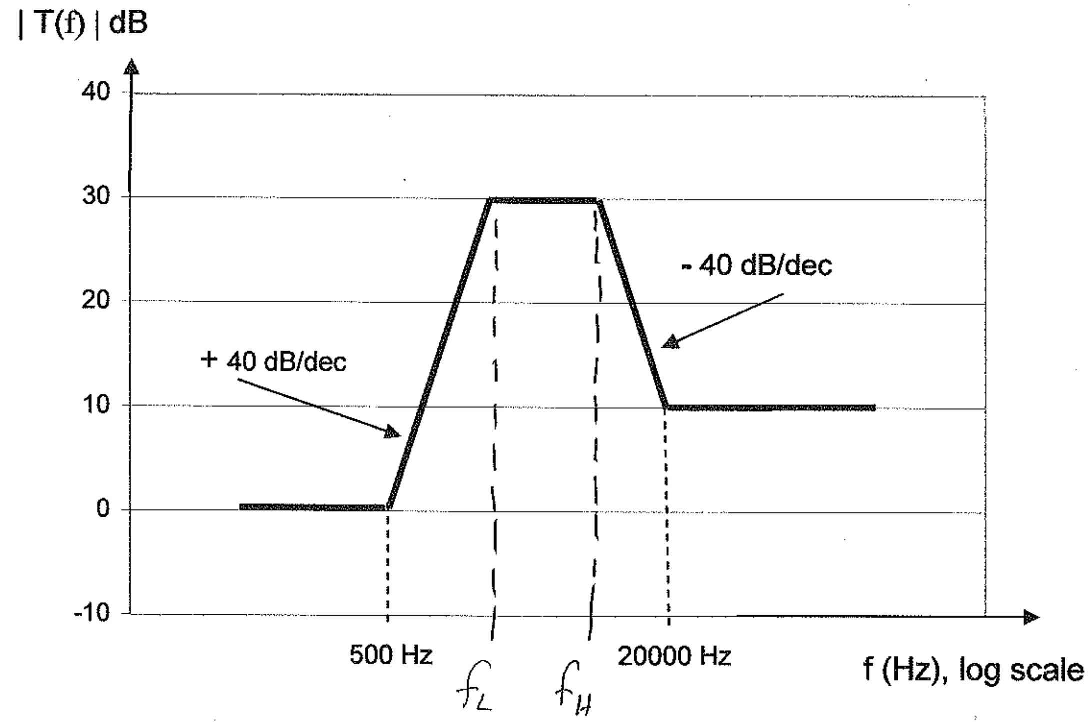
This exam has 11 pages, including the cover sheet. Raise your hand if you are missing a page.

1	________________/40
2	________________/50
3	________________/60
4	________________/40
5	________________/10
Total	________________/200

Please provide <u>your</u> chili pepper rating: 1 2 3 4 5

1. **(40 points)** The figure below shows the straight-line approximation to the magnitude Bode plot for a certain filter. There are no poles or zeros outside the region shown in the plot. *Note that the plot is merely a sketch: it is NOT drawn to scale, and should not be used for quantitative calculations.*

a) Find the transfer function T(f) for this filter. Assume that any constants necessary to adjust the magnitude are real and positive.
b) Using the paper provided on the next page, draw the straight-line approximation to the **Phase** Bode plot for this transfer function.



We need to identify f_L and f_H. We note that at f_L T(f) has increased by 30 dB at a rate of 40dB/dec - this requires $30/40 = 0.75$ decade,
Also, f_H to 20 000 Hz requires $20\,dB/(40\,dB/dec) = 0.5$ dec.

+3 So $f_L = 500 \times 10^{0.75} = 2812$ Hz

+3 $f_H = \dfrac{20000}{10^{0.5}} = 6325$ Hz

+6 Thus $T(f) = K \cdot \dfrac{(jf + 500)^2 (jf + 20\,000)^2}{(jf + 2812)^2 (jf + 6325)^2}$

We need K: $T(f=0) = 1$ (0 dB)

$$= K \cdot \frac{500^2 \times 20000^2}{2812^2 \times 6325^2}$$

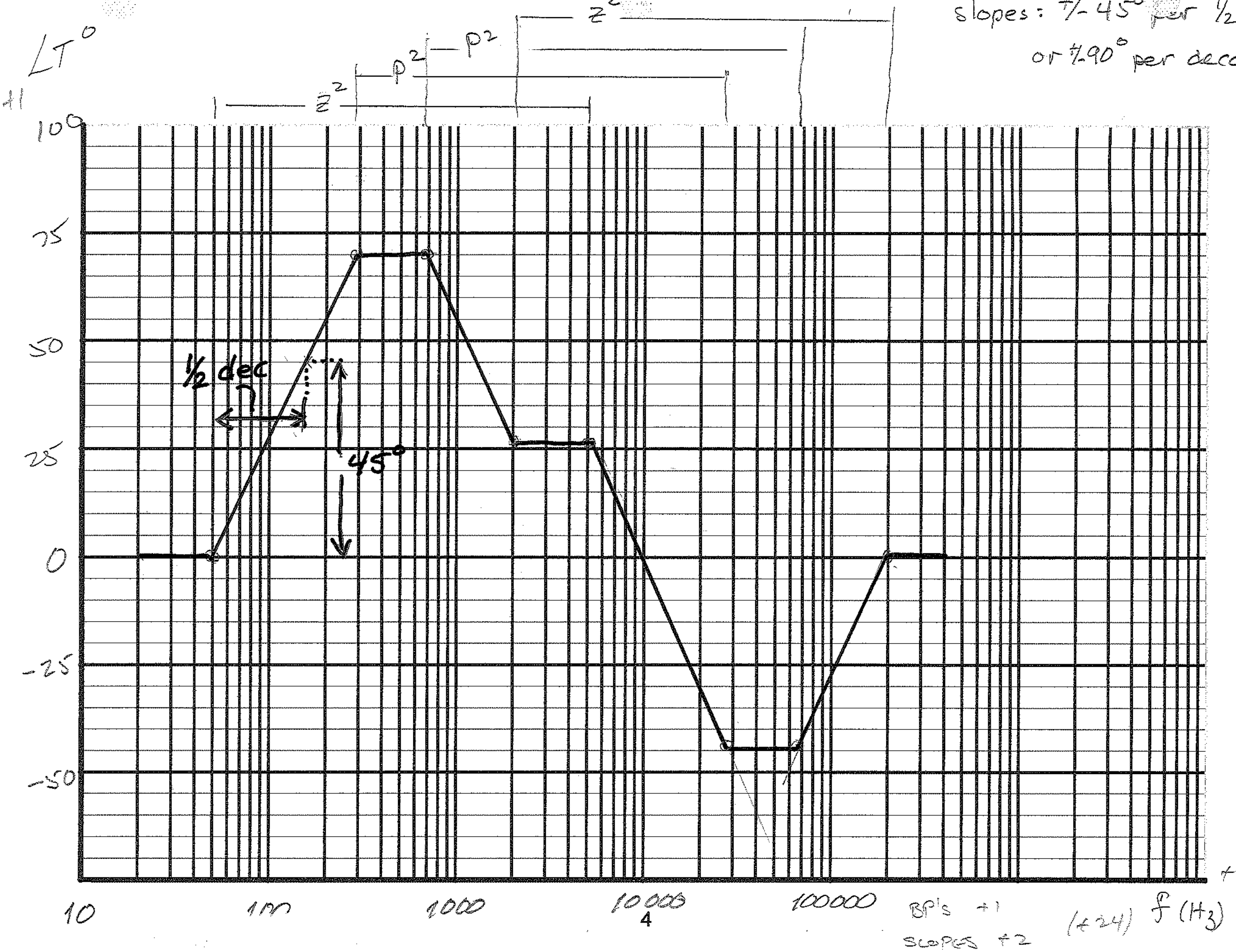
+2 $\Rightarrow \underbrace{K = 3.16}$

- So we have

$$\left.\begin{array}{l} \text{2 zeros at } 500 \\ \text{2 poles at } 2812 \\ \text{2 poles at } 6325 \\ \text{2 zeros at } 20000 \end{array}\right\} Hz$$

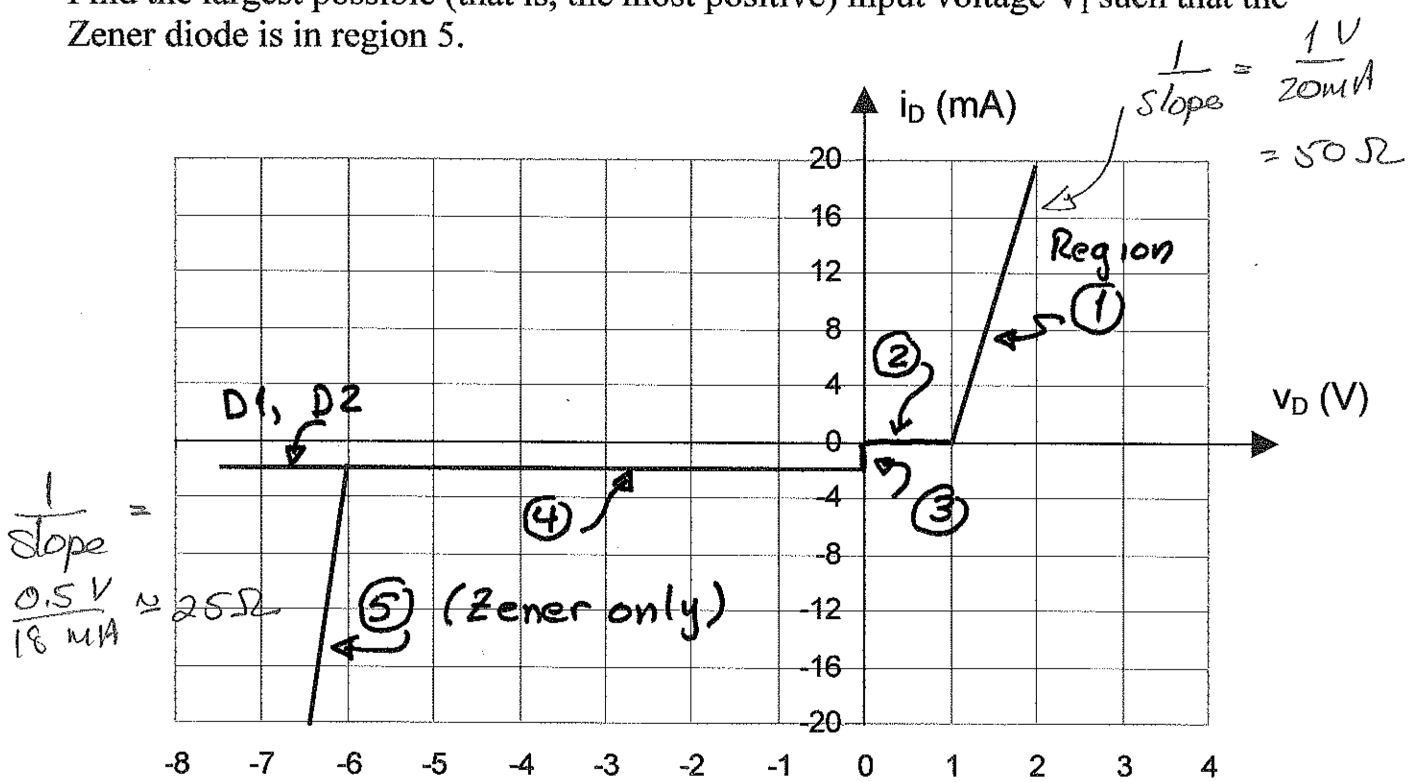
Phase Bode plot follows.

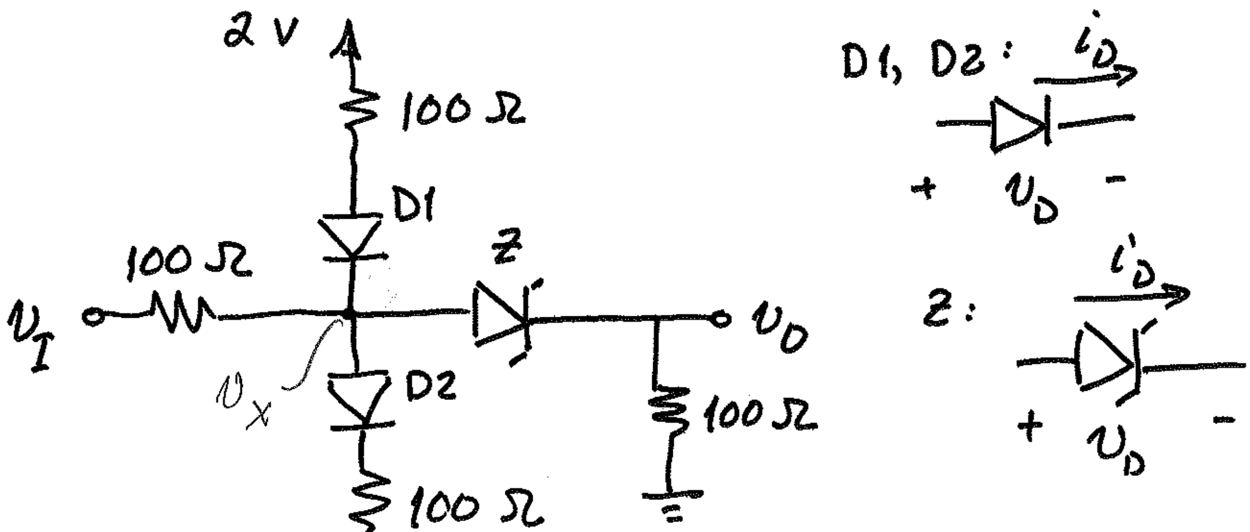
slopes: +/- 45° per ½ dec
or +/-90° per decade

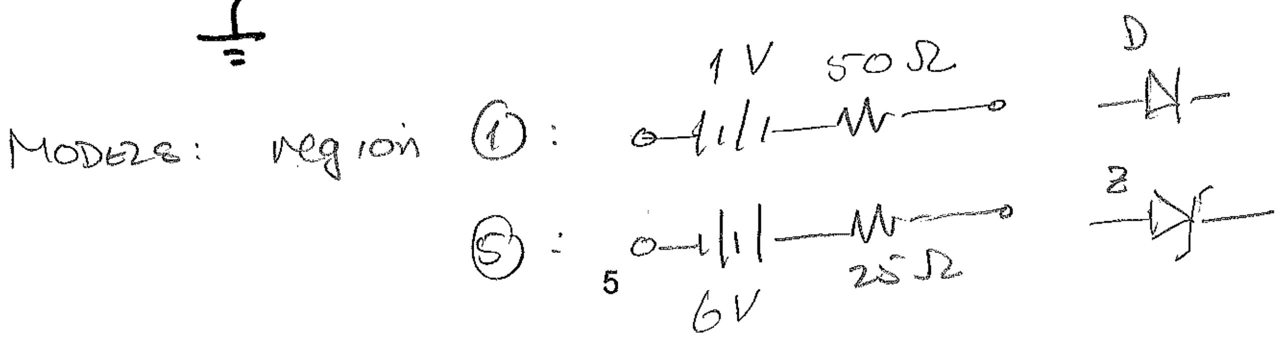


2. **(50 points)** The graph shows the current-voltage characteristics for the diodes in the circuit below. Diodes D1 and D2 and Zener Diode Z have the characteristics indicated.

Find the largest possible (that is, the most positive) input voltage V_I such that the Zener diode is in region 5.







Room for Extra Work

Approach: If Z is in region ⑤, clearly v_x is negative. We want to find v_I such that Z is "just" entering ⑤ since this will correspond to the most positive v_x and thus the most positive v_I. So we will set $v_D = -6\,V$ and $i_D = -2\,mA$ for the Zener.*

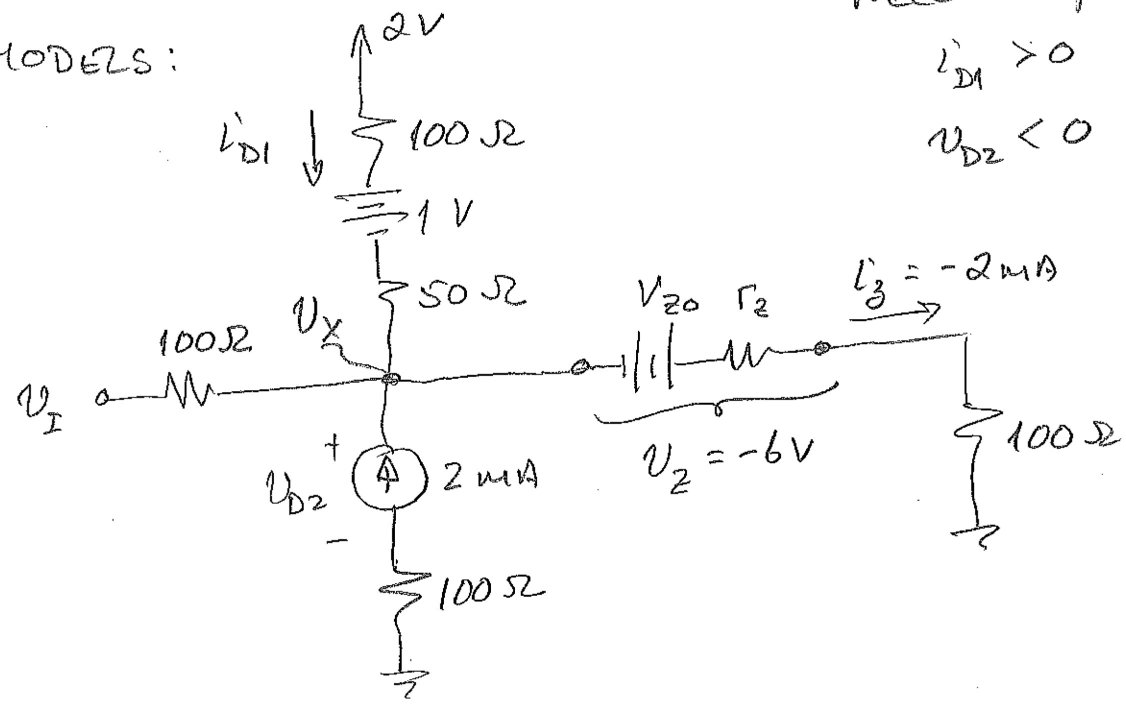
What about D1, D2? Surely if $v_x < 0$, D1 is in ① and D2 is in ④.

Need to prove:

$i_{D1} > 0$

$v_{D2} < 0$

MODELS:



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* Note that because we know v_z and i_z, we do not need V_{Z0} and r_z. We can find them if we like:

$r_z = 25\,\Omega$ from the graph

$V_{Z0} = -6\,V + 0.002(25) = -5.95\,V$

We see that $v_X = -6 - 0{,}002(100) = -6.2V$. Then

$$\frac{v_X - v_I}{100} + \frac{v_X - 2 + 1}{150} - 0{,}002 - 0{,}002 = 0$$

$$1.50(v_X - v_I) + v_X - 1 - 0.6 = 0$$

$$v_I = \frac{2.5\,v_X}{1.5} - \frac{1.6}{1.5}$$

Analysis + 12

$$v_X = -6.2V \Rightarrow \underline{v_I = -11.40\ V.}$$

D1, D2:

$$i_{D1} = \frac{2 - 1 - v_X}{150} = 48\ mA\ ✓$$

+ 6

+ 6

$$v_X = v_{D2} - 0{,}002(100)$$

$$\Rightarrow v_{D2} = v_X + 0{,}2 = -6\ V\ ✓$$

Z:

+ 12

v_I + 4

Incorrect diode state / prove wrong } − 8

all else correct

0 · r_Z − 5

0 $r_{D1, D2}$ +4

− 8

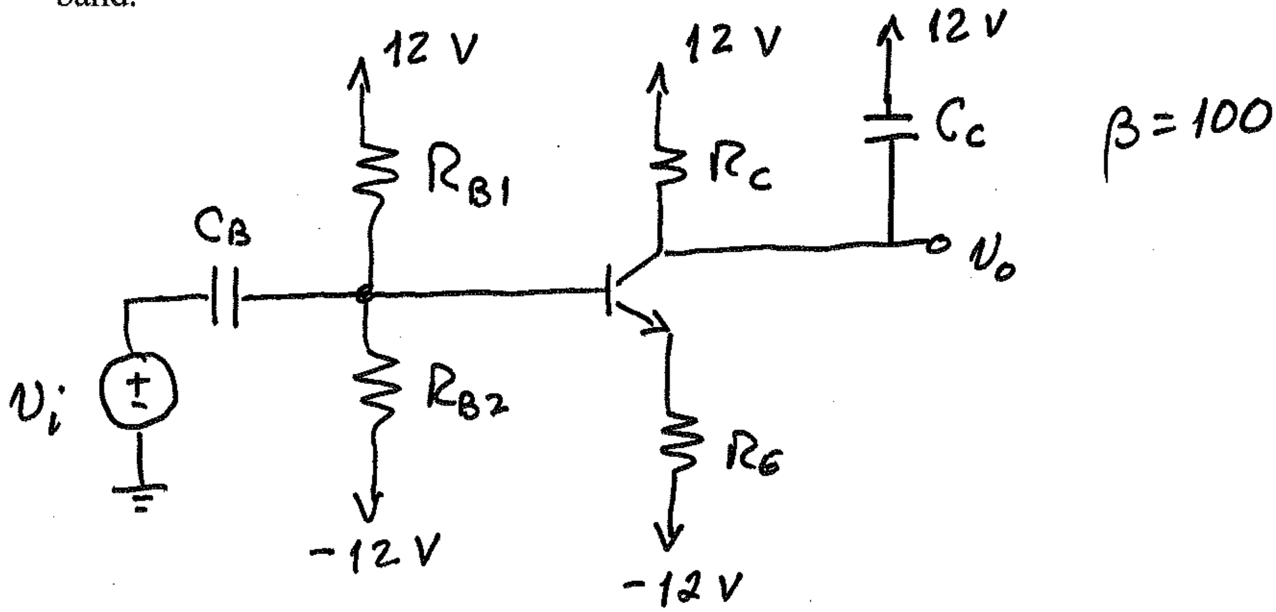
model error
(I_S wrong dir.)

− 15

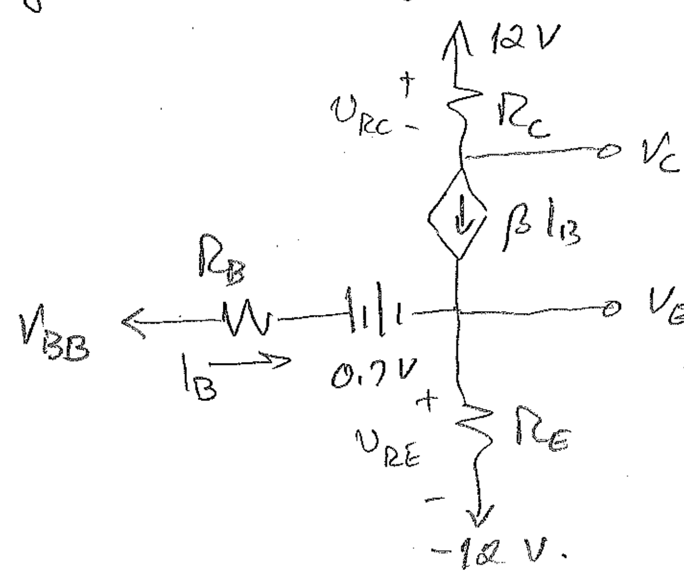
put $V_Z = -6$ V $I_Z = 0$ A
or
fail to see correct Z state

3. **(60 points)** A BJT circuit is built using a four-resistor scheme and 12 V power supplies, as shown below. We wish to design the circuit so that the transfer function V_o/V_i corresponds to a band pass filter with cut-off frequencies at 100 Hz and at 20,000 Hz.

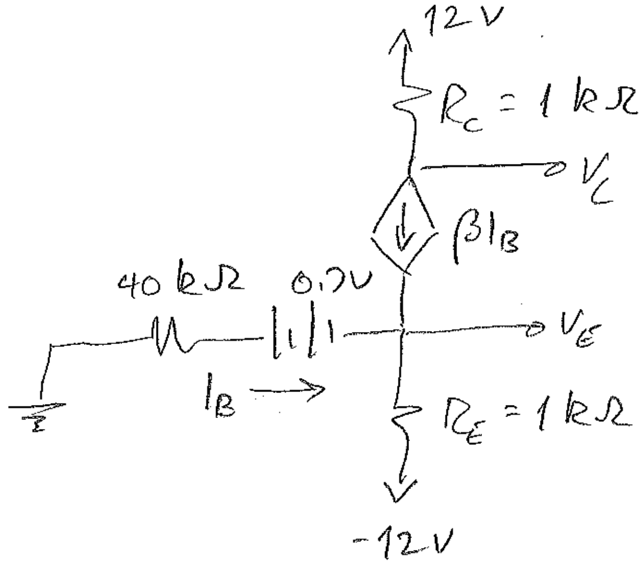
i) Choose resistor and capacitance values such that the BJT is biased in the linear region and the circuit provides the filtering response described above. Be careful to prove that the BJT is correctly biased.
ii) For the circuit you designed, find the voltage gain in the pass band.
iii) For the circuit you defined, find the input and output resistance in the pass band.



i)
DC analysis: Thevenizing the base gives



We can choose $V_{BB} = 0$, which makes $R_{B1} = R_{B2}$.



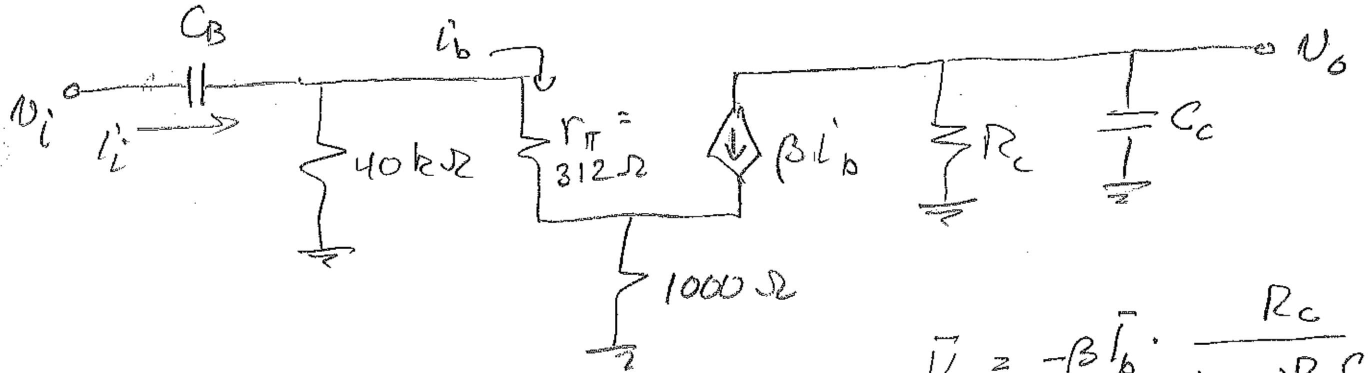
$$V_C = 12 - \beta I_B \cdot 1000$$
$$= 4\ V$$

$$V_E = 101\, I_B \cdot 1000 - 12$$
$$= -3.92\,V$$

$$V_{CE} = 7.92\ V \quad \checkmark$$

AC analysis

$$r_\pi = \frac{V_T}{I_B} = \frac{25\times 10^{-3}}{80\times 10^{-6}} = 312\ \Omega$$

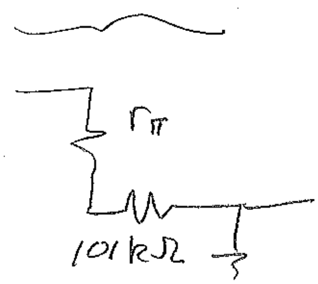


$$\bar{V}_o = -\beta \bar{I}_b \cdot \frac{R_C}{1 + j\omega R_C C_C}$$

MILLER'S DUAL

$101\ k\Omega + r_\pi$

$\cong 101\ k\Omega$



$$R_B' \equiv 40\,k\Omega \,//\, 101\,k\Omega \cong 29\,k\Omega$$

$$\bar{I}_i = \frac{\bar{V}_i}{1/j\omega C_B + R_B'} = \bar{V}_i \cdot \frac{j\omega C_B}{1 + j\omega C_B R_B'} \qquad \bar{I}_b = \bar{I}_i \cdot \frac{R_B}{R_B + r_\pi + 101 R_E}$$

$$T(\omega) = \frac{\bar{V}_o}{\bar{V}_i} = -\beta R_C \frac{j\omega C_B}{1 + j\omega C_B R_B'} \cdot \frac{1}{1 + j\omega R_C C_C} \cdot \frac{R_B}{R_B + r_\pi + 101 R_E}$$

page 2

So we have a zero at 0 and poles at $1/R_C C_C$ and $1/R_B' C_B$ as we anticipated. We'll take

$$\begin{cases} 1/C_B R_B' = 2\pi \times 100 \Rightarrow C_B = (2\pi(100)R_B')^{-1} = 0.055\ \mu F \\ 1/R_C C_C = 2\pi \times 20000 \Rightarrow C_C = (2\pi(20000)R_C)^{-1} \simeq 8\ nF \end{cases}$$

(pretty small!)

ii)

In the pass band: $v_o/v_i \simeq -\frac{R_C}{R_E} = -1.$

$C_B \to$ short; $C_C \to$ open $\Rightarrow$

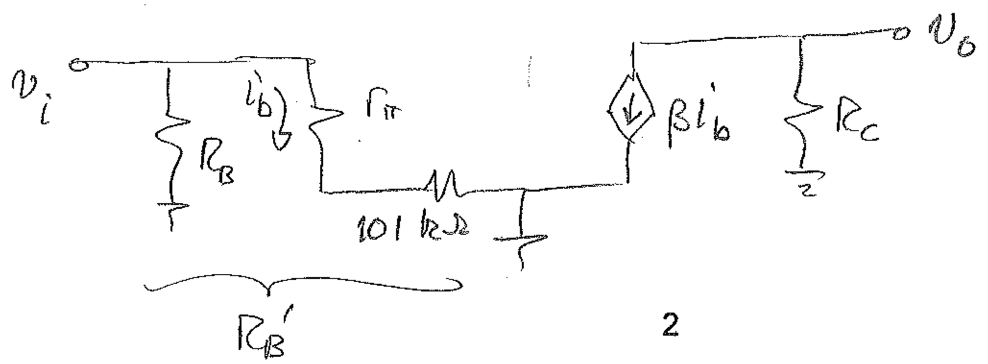
$v_o = -\beta i_b' R_C \quad i_b' = \frac{v_i}{r_\pi + 101 R_E} \Rightarrow \frac{v_o}{v_i} = -\beta \frac{R_C}{r_\pi + 101 R_E} \simeq -\frac{R_C}{R_E}$

iii)

In the pass band, $\underline{R_i = R_B' = 29\ k\Omega}$

$\underline{R_o = R_C = 1\ k\Omega}$

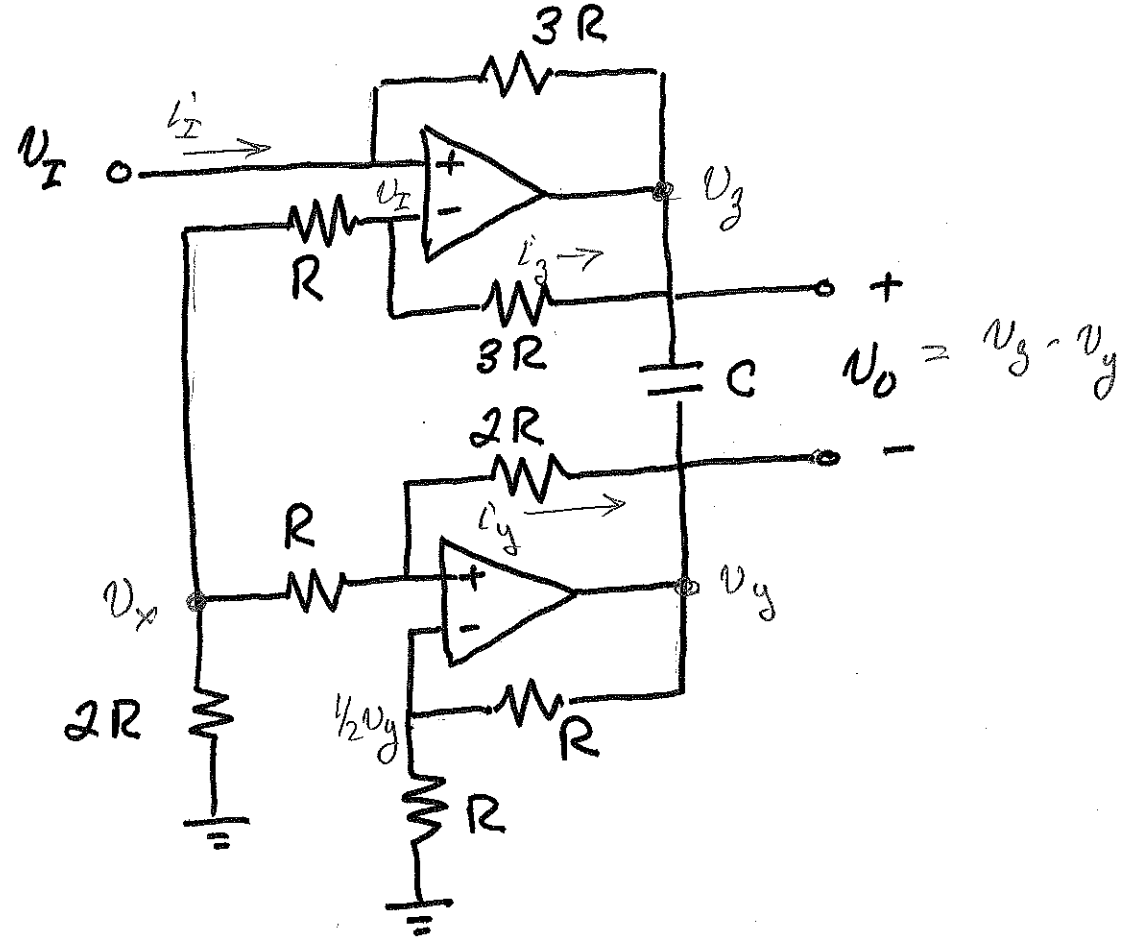
PASS BAND:



4. **(40 points)** The op amps in the circuit below may be considered ideal.

+32 i) Find $v_O(t)$ in terms of $v_I(t)$.

+8 ii) Find the input impedance seen by the source $v_I(t)$.



Node voltage at v_x: $\dfrac{v_x}{2R} + \dfrac{v_x - v_I}{R} + \dfrac{v_x - \frac{1}{2}v_y}{R} = 0$ ①

KVL: $v_y - \frac{1}{2}v_y + \underbrace{\dfrac{v_x - \frac{1}{2}v_y}{R}}_{i_y} \cdot 2R = 0$ ②

② $\Rightarrow$ $-\frac{1}{2}v_y + 2v_x = 0$

$\Rightarrow v_y = 4v_x$ Putting this result in ①:

Room for extra work

$$v_x + 2(v_x - v_I) + 2(v_x - 2v_x) = 0$$

$$\Rightarrow \quad v_x = 2v_I$$

$$\Rightarrow \quad v_y = 8\,v_I$$

$$i_3' = \frac{v_x - v_I}{R} = \frac{v_I}{R}$$

KVL: $$v_3 - v_I + 3R\left(\frac{v_I}{R}\right) = 0 \Rightarrow v_3 = -2\,v_I$$

$$v_0 = v_3 - v_y = -2v_I - 8v_I = \underline{\underline{-10\,v_I}}$$

The capacitor has no effect so $\underline{v_o(t) = -10\,v_I(t)}$,

ii) $$i_I' = \frac{v_I - v_3}{3R} = \frac{v_I + 2v_I}{3R} = \frac{v_I}{R}$$

$$\Rightarrow \quad \frac{v_I}{i_I'} = R_{in} = R$$

$v_o = v_3 - v_y$ + 4

For general mess:
NV for v_x + 6
Other useful eqns + 4 ea.

Simple non-inv. + 2
VDR for v_x − 4
3eqn 4 unk − 6

10 Gerally ok but major error in v_{o1} or v_{o2} / v_3 v_y − 10

5. **(10 points)** *Answer the following question in a few clear, complete, grammatically correct English sentences. Credit will be subtracted if your answer is unclear.*

What is "biasing"? What does it mean to "bias an amplifier" or to "bias a BJT"?

Generally, an amplifier is used to increase the amplitude of a small signal. [There may be frequency-dependent filtering going on as well.] In that case, we would like the amplified signal to ~~be~~ an accurate replica of the input signal. This requires that the amplifier have linear v_O vs. v_I characteristics.

In general, amplifier $v_O - v_I$ characteristics are not linear, but they do have a linear region, i.e. a range of v_I over which the characteristics are linear.

The purpose of biasing is to put the v_O vs. v_I characteristics in a linear region. Then, if the signal amplitude is not too large, the output will be an accurate reproduction of the input.