Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Exam 1
March 1, 2008

Exam duration: 90 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 10 pages, including the cover sheet. Raise your hand if you are missing a page.

1 ________________/25

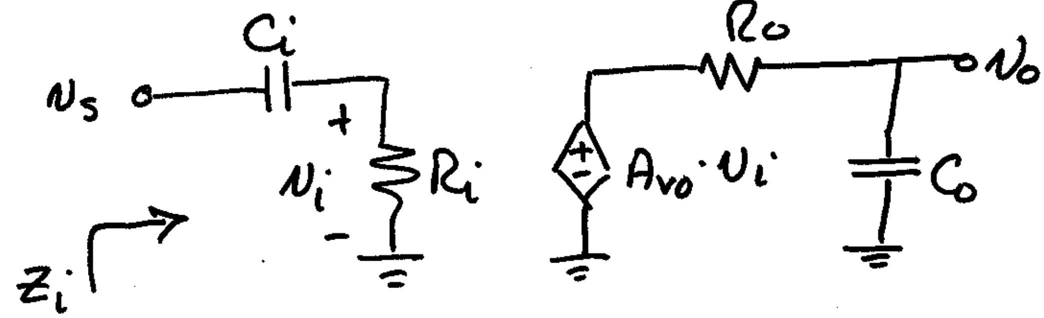
2 ________________/25

3 ________________/25

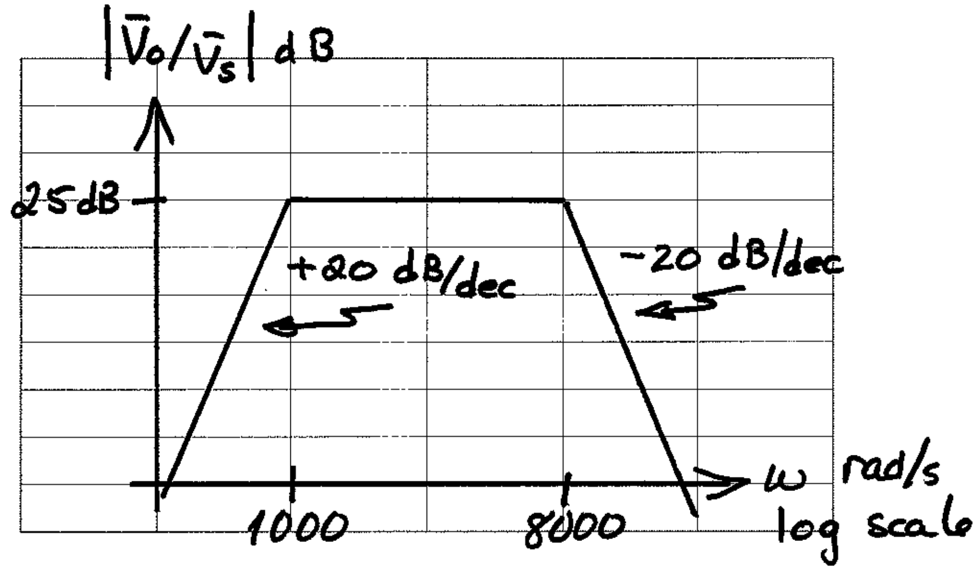
4 ________________/25

Total ________________/100

1. **(25 points)** We want to design a band-pass filter based on the circuit shown below. The Bode plot indicates schematically the specifications for frequency response that the amplifier is required to have.



a) Find the transfer function, $\mathbf{V_o}/\mathbf{V_s}$ for the circuit.
b) Choose values for the resistors and capacitors, and for A_{vo}, such that (i) the input impedance Z_i approaches 22 kΩ at very high frequencies and (ii) the frequency response shown in the Bode plot is obtained.



a)

$$\bar{V}_o = A_{vo}\cdot\bar{V}_i\cdot\frac{1/j\omega C_o}{1/j\omega C_o + R_o} = A_{vo}\bar{V}_i\cdot\frac{1}{1+j\omega C_o R_o}$$

$$\bar{V}_i = \bar{V}_s\cdot\frac{R_i}{R_i + 1/j\omega C_i} = \bar{V}_s\cdot\frac{j\omega C_i R_i}{1+j\omega C_i R_i}$$

$$\therefore\quad \frac{\bar{V}_o}{\bar{V}_s} = A_{vo}\frac{j\omega C_i R_i}{(1+j\omega C_i R_i)(1+j\omega C_o R_o)}$$

Room for extra work

So we have a zero at 0 rad/s ; one pole each at $1/C_iR_i$ and $1/C_oR_o$ rad/s.

b) For $\omega \to \infty$, $Z_i \to R_i = 22\ k\Omega$.

So $1/C_iR_i = 1000$ rad/s $\Rightarrow C_i = \dfrac{1}{22000 \cdot 1000}$

$= 45{,}45\ nF$.

$R_i = 22\ k\Omega$
$C_i = 45{,}45\ nF$

$1/C_oR_o = 8000$ rad/s

We have no constraint on C_o or R_o so we'll choose $R_o = 100\ \Omega$

$\therefore\ 1/C_oR_o = 8000$ rad/s $\Rightarrow C_o = \dfrac{1}{8000 \cdot 100}$

$= 1{,}25\ \mu F$

$R_o = 100\ \Omega$
$C_o = 1{,}25\ \mu F$

Let's evaluate $\bar{V}_o/\bar{V}_s$ at $\omega = 3000$ rad/s $\Rightarrow \dfrac{\bar{V}_o}{\bar{V}_s} = 25\ dB$

$$\left.\frac{\bar{V}_o}{\bar{V}_s}\right|_{\omega = 3000\ rad/s} = A_{vo} \cdot \frac{j(3000)(1/1000)}{(1 + j\,3000 \cdot 1/1000)(1 + j\,3000/8000)}$$

$$\left.\frac{\bar{V}_o}{\bar{V}_s}\right|_{\omega = 3000\ rad/s} = A_{vo} \cdot (0.8877 - j0.0329)$$

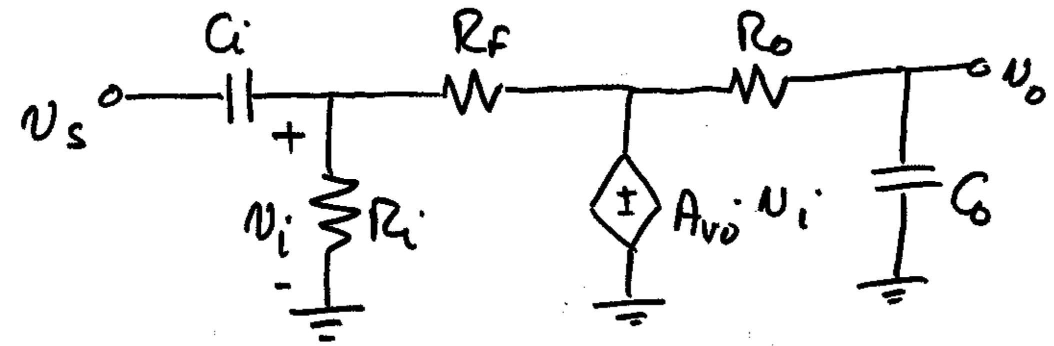
$$= A_{vo} \cdot 0.8883 \angle -2.12^\circ$$

So $\quad 20 \log \left| \frac{\bar{V}_o}{\bar{V}_s} \right| = 20 \log A_{vo} \cdot 0.8883 = 25$

$$\log A_{vo} \cdot 0.8883 = \frac{25}{20} = \log A_{vo} + \log 0.8883$$

$$\Rightarrow \boxed{A_{vo} = 20.02}$$

2. **(25 points)** A feedback resistor, R_F, is added to the circuit of Problem 1, as shown below. Find the transfer function V_o/V_s with R_F in place. (Just find the transfer function; you do not need to re-calculate R, C, and A_{vo} to get the proper Bode plot behavior.)



$$\bar{V}_o = A_{vo}\bar{V}_i \frac{1}{1+j\omega C_o R_o}$$

$$\frac{\bar{V}_i}{R_i} + \frac{\bar{V}_i - \bar{V}_s}{1/j\omega C_i} + \frac{\bar{V}_i(1-A_{vo})}{R_F} = 0$$

$$\bar{V}_i = \bar{V}_s \frac{j\omega C_i \bar{V}_s}{1/R_i + j\omega C_i + \frac{1-A_{vo}}{R_F}}$$

$$= \bar{V}_s \frac{j\omega C_i R_i R_F}{R_F + R_i(1-A_{vo}) + j\omega C_i R_i R_F}$$

$$\therefore \frac{\bar{V}_o}{\bar{V}_s} = A_{vo} \frac{j\omega C_i R_i R_F}{(R_F + R_i(1-A_{vo}) + j\omega C_i R_i R_F)(1+j\omega C_o R_o)}$$

3. **(25 points)** The transfer function for a particular amplifier is shown below.

a) State clearly the poles and zeros of the transfer function.
b) On the paper provided on the next page, plot the **phase** Bode plot for this transfer function.

$$T(\omega) = -\frac{6.7x10^{-2} * (1 + j\omega/500)^2 (2 + j\omega/4000)}{(1 + j\omega/800)(10000 + j\omega)^2 (j\omega)}$$

ZERO'S: $\omega = 500$ rad/s DOUBLE ZERO

$2 = \omega/4000 \Rightarrow \omega = 8000$ rad/s

POLES $\omega = 800$ rad/s $\omega = 10\,000$ rad/s DOUBLE POLE

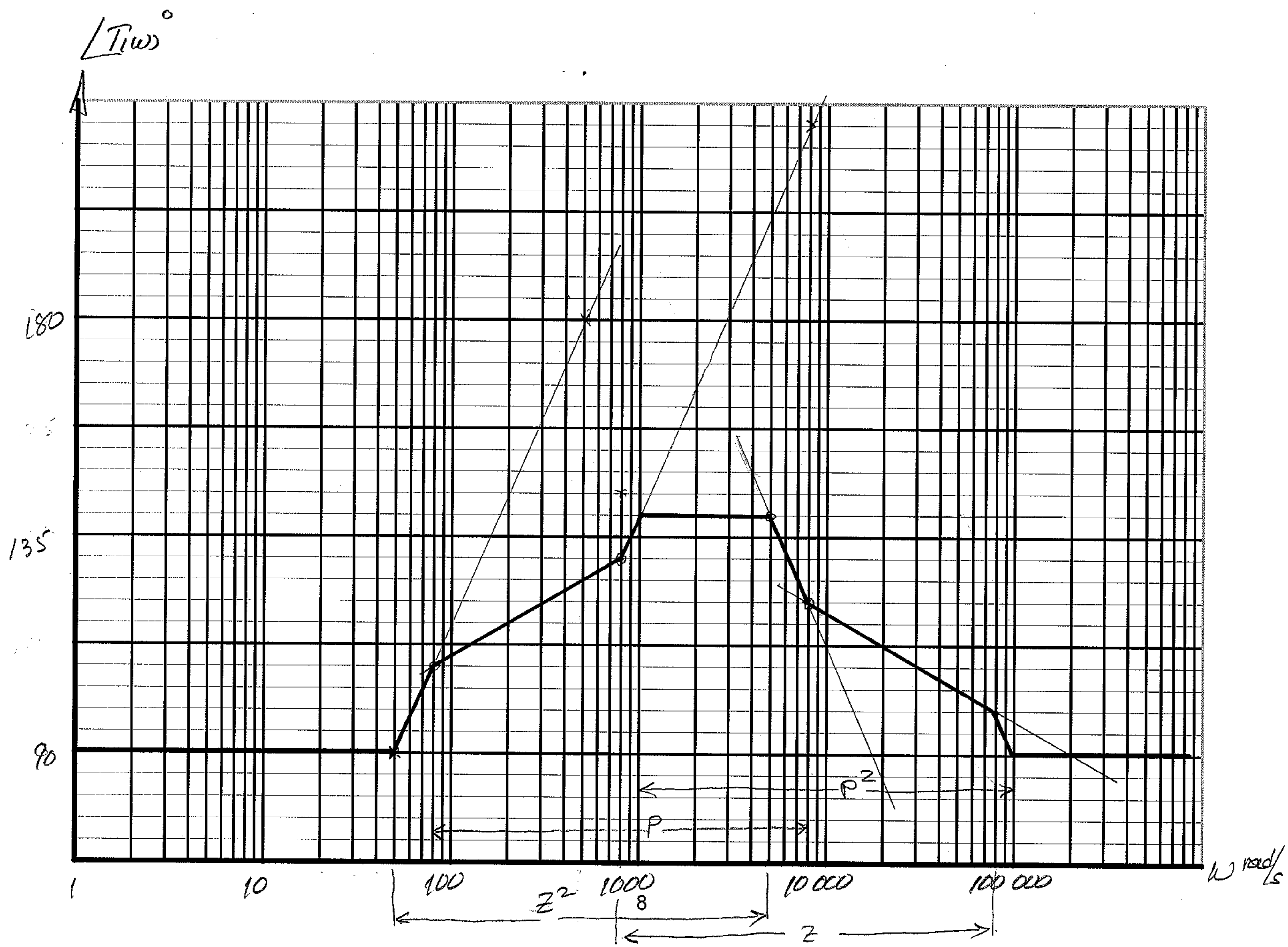
$\omega = 0$

For $\omega \to 0$, $T(\omega) \to \dfrac{-6.7\times10^{-2}(1)^2(2)}{(1)(10\,000)^2(j\omega)}$

$\Rightarrow \angle T(\omega) \to \angle -1/j = 90^\circ$

$\omega \to \infty$ $T(\omega) \to \dfrac{-6.7\times10^{-2}(j\omega/500)^2(j\omega/4000)}{(j\omega/800)(j\omega)^2(j\omega)}$

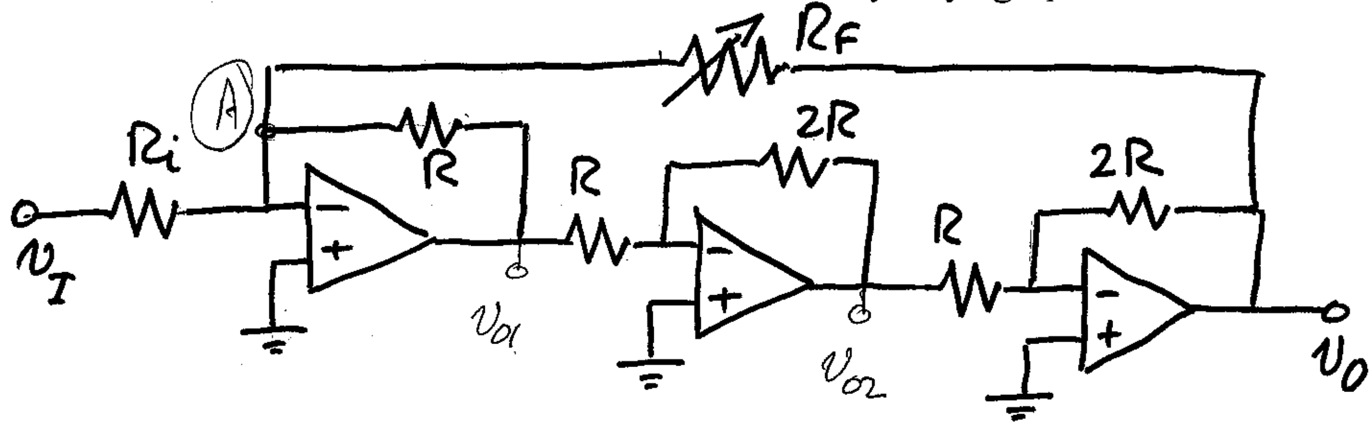
$\angle T(\omega) \to \angle -1/j = 90^\circ$



∠T(jω)°
180
135
90
1
10
100
1000
10 000
100 000
ω rad/s
P
P^2
Z^2
8
Z

4. **(25 points)** The op amps in the circuit below are "ideal". By varying R_F, a range of voltage gain (v_O/v_I) can be obtained.

a) Find the gain v_O/v_I in terms of the various resistances.
b) Considering values of R_F that are positive or 0, what are the maximum and minimum values of gain that can be obtained by varying R_F?



$$v_{O2} = -\frac{2R}{R} v_{O1} \qquad v_O = -\frac{2R}{R} v_{O2} = 4 v_{O1}$$

$$\Rightarrow v_{O1} = \frac{1}{4} v_O$$

KCL at (A)

$$\frac{v_{O1}}{R} + \frac{v_I}{R_i} + \frac{v_O}{R_F} = 0$$

$$\frac{v_O}{4R} + \frac{v_I}{R_i} + \frac{v_O}{R_F} = 0$$

$$v_O \frac{R_F}{4R} + v_I \frac{R_F}{R_i} + v_O = 0$$

$$v_O\left(1 + \frac{R_F}{4R}\right) = -\frac{R_F}{R_i} v_I$$

$$\boxed{\frac{v_O}{v_I} = \frac{-R_F/R_i}{1 + R_F/4R}}$$

Room for extra work

b) Consider extremes in R_F:

$$R_F = 0 \Rightarrow \boxed{\frac{v_O}{v_I} = 0}$$ (output is shorted to ground via R_F)

$$R_F = \infty \Rightarrow v_{O1} = -v_I \frac{R}{R_i}$$

(no connection through R_F)

$$v_{O2} = -2v_{O1} = 2v_I \frac{R}{R_i}$$

$$v_O = -2v_{O2} = -4v_I R/R_i$$

$$\boxed{\frac{v_O}{v_I} = -\frac{4R}{R_i}}$$

We can look at this mathematically as well:

$$\frac{v_O}{v_I} = \frac{-1/R_i}{1/R_F + 1/4R}$$

$$R_F \to \infty \Rightarrow \frac{v_O}{v_I} = -\frac{1/R_i}{1/4R} = -\frac{4R}{R_i} \checkmark$$